The Russell–Peano connection

by Gregory H. Moore

Hubert C. Kennedy. *Peano: Life and Works of Giuseppe Peano.* (Studies in the History of Modern Science, vol. 4.) Dordrecht, Holland: D. Reidel Publishing Company, 1980. Pp. xii, 230; portrait. Cloth: US $34.00; paper: US $14.95.

BERTRAND RUSSELL NEVER wavered in acknowledging his intellectual debt to Giuseppe Peano. In many ways the contribution that Russell made to the foundations of mathematics, culminating in *Principia Mathematica*, strongly bears Peano's mark. Yet Russell was no *tabula rasa* when in August 1900 he met Peano at the International Congress of Philosophy in Paris. For five years Russell had published on foundational questions relating largely to geometry and rational mechanics. It is against this background that Peano's influence on Russell must be seen.

However, the book under review, the first full-length biography of Peano, should be judged by broader standards as well. In it Hubert Kennedy has endeavoured to present a balanced and detailed portrait of Peano's life, work, and times. Ten years in preparation, this biography resulted from an extensive search of both the published literature and a vast quantity of unpublished material at various archives. While the first half of the book largely concerns Peano's contributions to mathematics and logic, the second half details his efforts to promote an international auxiliary language: *Latino sine flexione* (Latin without inflections). When Russell met Peano in 1900, the first period had all but ended. Peano had made his principal contributions to mathematics, and by 1903 he was increasingly occupied with promoting *Latino sine flexione*. In this respect Peano resembled another of Russell's correspondents, Louis Couturat, who turned from logic to a passion for the international auxiliary language *Ido*.

Except for its concluding chapter, the book is organized chronologically. As a result, Peano's contributions to mathematics are interspersed with events in his personal and professional life. Although this juxtaposition is potentially an asset, the reader soon concludes that Peano's personal life had little effect on either his publications or his professional activities at the University of Turin.

Among mathematicians, Peano is known principally for two contributions: the Peano curve, which showed that a continuous real function could occupy area, and the Peano postulates for the natural numbers. Although

historians of mathematics are agreed that Richard Dedekind invented these postulates at an earlier date than Peano, Kennedy labours to substantiate Peano's independent formulation. In addition, Kennedy rightly stresses certain major contributions for which Peano has not always received proper credit. Among these are Peano's axiomatization of the concept of vector space and his demonstration of the theorem that a differential equation of the form $y' = f(x, y)$ has a solution whenever f is continuous.

Since it was Peano's symbolic logic that primarily influenced Russell, the development of that logic is of particular interest. Peano first discussed symbolic logic—after the fashion of George Boole, as modified by C. S. Peirce and Ernst Schröder—in a book of 1888 devoted to Hermann Grassmann's ideas on vector calculus. The following year Peano published his *Arithmetices principia*, which contained the symbols ϵ and $\supset$ for set membership and set inclusion respectively. It was this distinction, not observed by Schröder, which Russell considered to be among Peano's foremost advances. It should be noted, however, that this distinction (though not the use of symbolism) had already been made by Georg Cantor in his set theory for well over a decade.

The second major advance that Russell adopted from Peano also occurred in the *Arithmetices principia*. This was the introduction of quantifiers. Since Aristotle, logicians had investigated the notions of "all" and "some". However, such logicians usually did not recognize the importance of separating particular judgments, such as "Socrates is mortal", from general judgments such as "All men are mortal". Peano underlined that distinction by introducing a symbolism for the universal quantifier: implication was subscripted with the variable to be universally quantified. Later he provided anotation for the existential quantifier as well. While granting that Gottlob Frege had proposed a symbolism for universal quantification prior to Peano, neither Kennedy nor Russell seems to acknowledge that Peirce had also introduced such a quantifier before Peano. In fact, Peirce's universal and existential quantifiers were separate from logical connectives such as implication and hence lent themselves more readily to an analysis of the underlying logical structure than did Peano's. This is of interest precisely because much of Russell's early research in mathematical logic was devoted to recasting the theory of relations, on which Peirce and Schröder had laboured diligently, within the framework of Peano's symbolism.

Here it is worthwhile to inquire, as Kennedy does not, what intellectual baggage Russell brought to his initial encounter with Peano. First of all, Russell already possessed a profound interest in logic and a growing desire to create a new foundation for mathematics—as evidenced by an 1899 draft of *The Principles of Mathematics*. Secondly, Russell's interest in these questions had a distinctly philosophical overlay. In particular, there were pronounced Kantian influences on his geometrical writings and distinctly Hegelian influences on his papers concerning rational mechanics. One aspect of this philosophical overlay, which cannot be pursued here, was Russell's predisposition to seek paradoxes in mathematics, a predisposition evident for some years before he met Peano.[1]

Despite the rich harvest of facts found in this biography, it also contains a failure of scholarship that is difficult to comprehend in an experienced historian such as Kennedy. The least serious form of this failure occurs when he quotes (p. 32) from a published work, such as Felix Hausdorff's *Grundzüge der Mengenlehre*, and gives no page reference. Here a reader might hope to recover the page with a modicum of work. It is more serious when Kennedy cites an author, such as Ugo Cassina (pp. 32, 104), and provides no reference at all. Perhaps the work referred to is among the nineteen listed under Cassina's name in the bibliography, and perhaps not. Since Kennedy informs us elsewhere that he had many conversations with Cassina regarding Peano, there may be no published reference at all. The reader is left in ignorance. The most serious fault, however, is that Kennedy gives the reader no indication of precisely what unpublished sources, oral and written, he consulted. The preface lists ten libraries which he visited, and mentions as well interviews with Cassina and three of Peano's nieces. Yet Kennedy never informs us what documents he used in those libraries and what information he obtained from those interviews. Consequently it is all but impossible for a reader to check Kennedy's claims, except where the reader is already familiar with the matter at hand.

In this context the reviewer wishes to correct two misunderstandings. Kennedy notes (p. 86) that in 1897 Cesare Burali-Forti discovered the paradox of the largest ordinal, often known as Burali-Forti's paradox. "This result", Kennedy adds, "went almost unnoticed until Bertrand Russell published a similar antinomy in 1903." What Kennedy does *not* remark, as the reviewer has done elsewhere, is that Burali-Forti never regarded his result as a paradox, and that no one else did either, until Russell turned it into one in *The Principles of Mathematics* (1903) by combining Burali-Forti's argument with a result from Cantor.[2]

The second misunderstanding concerns Peano and the axiom of choice, a subject that greatly interested Russell. Kennedy claims (p. 33) that Peano gave the first explicit statement of this axiom in his article of 1890 on

[1] See Gregory H. Moore and Alejandro R. Garciadiego, "Burali-Forti's Paradox: A Reappraisal of its Origins", *Historia Mathematica*, forthcoming August 1981.

[2] "The Origins of Zermelo's Axiomatization of Set Theory", *Journal of Philosophical Logic*, 7 (1978), 307-29 (at 308-9); "Beyond First-Order Logic: The Historical Interplay between Mathematical Logic and Axiomatic Set Theory", *History and Philosophy of Logic*, 1 (1980), 95-137 (at 104-5).

differential equations. What Peano actually did was quite different. In the course of a proof, Peano noted that it was not permissible to make infinitely many arbitrary choices, and hence he provided a rule for making the choices required in the proof. The axiom of choice, on the other hand, asserts that under very general conditions a function exists which justifies infinitely many arbitrary choices. Thus, in effect, Peano ruled out the grounds for believing such an axiom to be true. Except that the axiom of choice was first formulated by Zermelo only in 1904, one could perhaps say that Peano expressed the *negation* of this axiom. One can no more credit Peano with the axiom of choice than one can ascribe non-Euclidean geometry to Euclid.

In sum, Kennedy has written a biography that, while not definitive, will remain a fundamental source for anyone wishing to do historical research on Peano. Particularly useful are the bibliography of Peano's publications (whose entries Kennedy increased by twenty per cent beyond those in Cassina's standard bibliography), the list of papers by other authors which Peano presented to the Academy of Sciences at Turin, the brief biographies of Peano's professors, and especially the detailed discussion of Peano's school of logicians.

As concerns Russell, Kennedy does not go appreciably beyond his two earlier articles.[3] Indeed, the reviewer can supplement Kennedy's research on one detail by noting that, in a letter of 9 October 1899 to Couturat, Russell largely agreed with Couturat's criticisms of Peano.[4] One of those criticisms cast doubt on the need to distinguish between ϵ and $\supset$ in logic. Thus Russell, who later regarded this distinction as a major contribution of Peano, originally was little concerned with it and may have questioned its usefulness. Undoubtedly there is additional material in the Russell Archives which can further illuminate the Russell–Peano connection.

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[3] "What Russell Learned from Peano", *Notre Dame Journal of Formal Logic*, 14 (1973), 367-72; "Nine Letters from Giuseppe Peano to Bertrand Russell", *Journal of the History of Philosophy*, 13 (1975), 205-20.

[4] Couturat published those criticisms in "La Logique mathématique de M. Peano", *Revue de Métaphysique et de Morale*, 7 (1899), 616-46; see especially p. 628.