New work on Russell's early philosophy

by Nicholas Griffin

Bertrand Russell's Early Philosophy, Part II. Edited by Jaakko Hintikka. *Synthese*, vol. 46, no. 2 (Feb. 1981). Pp. 136. US$23.00.

THE SECOND PART of *Synthese's* special "early Russell" number contains seven papers: by Pears ("The Function of Acquaintance in Russell's Philosophy"), Hintikka ("On Denoting What?"), Cappio ("Russell's Philosophical Development"), Lycan ("Logical Atomism and Ontological Atoms"), Clark ("Acquaintance"), Coffa ("Russell and Kant") and Grandy ("Forms of Belief"), together with a reply to Coffa's paper by Hintikka. The papers in this volume all belong to the field of general philosophy (metaphysics, epistemology), compared with those published in Part I, which tend to concentrate on philosophy of logic and mathematics. The concept of acquaintance figures prominently—two of the papers are explicitly on the topic and in four more it is treated *inter alia*. The only real exception to this wave of interest in acquaintance is Coffa's paper which deals with the nature of Russell's logicism and its alleged refutation of Kant's philosophy of mathematics. The volume is both useful and extremely interesting. Most of the papers are very well done, though some of the interpretations stray far from the texts. In this review I shall concentrate on some issues concerning semantics and acquaintance, and offer some criticisms of Coffa's account of Russell's early (1903) logicism. Needless to say, much of interest will have to be left unnoticed.

I. *Semantics and ontology*

The title of Cappio's paper is seriously misleading since the paper deals essentially with the durability in Russell's thought of the semantical theory of *The Principles of Mathematics*. This semantical theory, called the "Naive Theory of Meaning" by Cappio (p. 193), consists of two principles:

(I). Some meaningful entities are meaningful only because there is something that they mean.
(II). Propositions are complexes of which their subjects are constituents. (P. 192)

It is certainly true that both of these sentences could have been written by Russell either before or after the theory of descriptions. The trouble is that Russell's surrounding philosophical doctrines changed so much over his career that in assenting to these sentences at different times Russell would in fact have been assenting to quite different propositions. For example, (II) is true for Russell when propositions are subsistent complexes (as in *The Principles of Mathematics*) or when they are merely linguistic (as in "The Philosophy of Logical Atomism"). The claims made in each case are quite different, and it would be hard for anyone to dispute (II) under the second (linguistic) interpretation. Similarly, in the *Principles* (I) is true of denoting complexes, in "On Denoting" it is true of proper names, and in "The Philosophy of Logical Atomism" of logically proper names. Nor can the Naive Theory (in any form) be identified, as Cappio seems to suppose (p. 193), with the Super-Naïve Augustinian theory that Wittgenstein (*1958*, §1) criticizes. Russell did in fact hold something approaching the Augustinian theory in the *Principles*, so there is a semantical shift in Russell from nearly super-naïveté to mere naïveté over the period 1903–05.

Lycan's paper deals with the question of why Russell thought that his method of logical analysis (involving the reparsing of sentences) vindicated his various ontological reduction programmes. Whether this belief of Russell's was justified is a question of major importance for all reductive brands of analytic philosophy which have, for the most part, adopted the method of logical analysis in the hope of making good their reduction programmes. Lycan argues that Russell maintained (a) that all and only logical fictions were epistemological fictions, and (b) that all and only epistemological fictions were ontological fictions.

Lycan (pp. 215–17) justifies (a) in Russell's system by means of the Russellian principle that only objects of acquaintance can be named (*1914*, 167; *1918*, 201). Then if *e* is an epistemological fiction (i.e. if *e* and its properties are known only by inference), then *e* is not an object of our acquaintance and thus we have no name for *e*, so that putative references to *e* disappear on analysis. Thus *e* is a logical fiction. Conversely, if *e* is an epistemological atom, then we are acquainted with *e* and we can name *e*. But if *e* has a name, references to it do not disappear on analysis. Thus *e* is a logical atom. Thus all epistemological atoms are logical atoms; equivalently, all logical fictions are epistemological fictions.

The only rigorous argument for (b) that Lycan finds rests on an easily rejected verifiability principle: "that if observing the truth of a proposition about *F*'s is our way of telling that *X* is present or that *X* is *G*, then for that proposition about *F*'s to be true is *what it is* for *X* to be present or *X* to be *G*" (p. 217). Lycan (pp. 218–19) suggests that Russell maybe didn't want a rigorous argument for (b). For if our knowledge of *X* can be inferred from our knowledge of *F*'s then Russell wants to claim that the question of whether there really is an *X* "does not concern us in any way" because *X* "cannot be a thing that comes into science in any way" (*1918*, 277). Thus what Russell seems to be claiming is that we may as well (for all scientific purposes) regard all epistemological fictions as ontological fictions (thereby saving metaphysical perplexities and avoiding ontological commitments). However, if (as I'll suggest in §2 below) Russell maintained (i) that quantifiers range only over items with which we are acquainted *and* (ii) that quantifiers carry ontological commitment, then we can construct *part* of a rigorous argument for (b) that Russell may have had in mind. What are needed to complete the argument are the converses of (i) and (ii)—for which, unfortunately, there is less textual evidence. Suppose that *o* is an ontological fiction: then by (ii) it is not the value of a bound variable, and thus, by the converse of (i), not an object of acquaintance. Hence, *o* is an epistemological fiction. Conversely, suppose that *e* is an epistemological fiction: then *e* is not an object of acquaintance, and thus by (i) does not lie within the domain of a quantifier. Therefore, by the converse of (ii), *e* is an ontological fiction. It seems to me very likely that Russell held (i) and (ii) and not unlikely that he held the converse of (i) as well. The converse of (ii), however, he seems definitely to have rejected, since in his system logically proper names also carry ontological commitment. But if logically proper names are the only other source of ontological commitment (and they seem to be), then the argument for (b) still goes through, for if *e* is an epistemological fiction then "*e*" is not a logically proper name (since we can only name that with which we are acquainted). I shall return to these matters in the next section.

2. *Acquaintance*

Russell first announced his principle of acquaintance at the end of "On Denoting" as an "interesting result" of the theory of descriptions (*1905*, 55). The principle received its definitive formulation a few years later in the form: "Every proposition which we can understand must be composed wholly of constituents with which we are acquainted" (*1910*, 159). Among the many problems which result are the following:

(a) Why did Russell think the principle of acquaintance was a "result" of the theory of descriptions?
(b) Since sentences containing expressions for definite descriptions are, on the theory of descriptions, paraphrased into canonical sentences

involving only quantifiers, bound variables, proper names, logical constants, and predicate expressions, what are the constituents of the proposition expressed by the canonical sentence? In particular, what constituents (if any) correspond to the quantifiers and bound variables?

Hintikka tackles the first question and argues that Russell ought to have replied that the principle of acquaintance follows from the theory of descriptions because the theory of descriptions removed putative references to non-existent items with which we could not be acquainted in favour of bound variables ranging over only items with which we are acquainted. It is easy to see why Russell ought to have taken this line; it is less easy to see that he didn't in fact do so. Hintikka argues that he didn't because (i) he wasn't "operating with a fully developed modern concept of a quantifier" (p. 181); and thus (ii) he failed to realize that ontological commitment is carried exclusively by the quantifiers because (iii) he thought that "[v]ariables, especially bound variables, were ... merely a notational device" (p. 181). Now we can easily admit point (i) without admitting (ii) and (iii). Russell, it is true, did not have a fully developed modern (i.e. classical) concept of the quantifier. There is in fact evidence that he had what we might with tongue in cheek call a "post-modern" concept of the quantifier, since there are hints of substitutional quantification in *Principia Mathematica*, I: 15, 127. Claim (ii) is, however, immediately suspect since it was the entire point of Russell's theory of descriptions (as opposed to Meinongian or noneist theories of descriptions) that sentences whose ordinary language form was "$\psi[(\iota x)(\phi x)]$" carried ontological commitments. But when such sentences were put in canonical form, viz.

$$“(\exists x)\ (\phi x\ \&\ (\forall y)\ (\phi y \supset x=y)\ \&\ \psi x)”,$$

what could possibly carry the ontological commitment if not the quantifiers and bound variables? In (ii) Hintikka merely echoes Quinean dogma about Russell's view of quantification (see Quine *1941*, 22; *1967*, 308; *1970*, 66). In fact, Quine's famous adage "to be is to be the value of a [bound] variable" (Quine *1939*, 22; *1948*, 15) was anticipated by thirty years in Russell. In 1906, immediately after the discovery of the theory of descriptions, Russell wrote in an unpublished manuscript: "What can be an apparent [i.e. a bound] variable must have some kind of being" (*1906*, fol. 106; also fol. 67).[1] Russell's account in the unpublished manuscript

[1] Ironically enough Hintikka even cites the fact that Russell called bound variables "apparent variables" as evidence that Russell did not take their ontological commitments

differs from Quine's in that it does not imply that bound variables take over the entire task of expressing ontological commitment (something which could follow only when a "no-names" theory such as Quine's had been adopted). But it does ensure that bound variables do carry ontological commitment. And this is sufficient also to undermine Hintikka's (iii), the claim that for Russell bound variables were merely a notational device.

Moreover, there is I think some quite positive evidence that Russell did take bound variables to range over objects of acquaintance. For if we consider what, in Russell's system, might be instantiated for a bound variable, the only possible answer is a proper name. But proper names, as Russell makes clear, can be used to refer only to objects of acquaintance (*1914*, 167; *1918*, 201). It would seem natural, therefore, to conclude that bound variables range only over objects of acquaintance. It is not easy to reconcile this with the evidence from *Principia*, however. There, for example, Russell says, "In order to understand the judgment 'all men are mortal,' it is not necessary to know [by acquaintance?] what men there are" (*PM*, I: 45); and, more equivocally,

> [A] function can be apprehended without its being necessary to apprehend its values severally and individually. If this were not the case, no function could be apprehended at all, since the number of values (true and false) of a function is necessarily infinite and there are necessarily possible arguments with which we are unacquainted. What is necessary is not that the values should be given individually and extensionally, but that the totality of the values should be given intensionally, so that, concerning any assigned object, it is at least theoretically determinate whether or not the said object is a value of the function. (*PM*, I: 39–40)

It is, of course, possible that to give the domain of a function "intensionally" is to be acquainted (in the required sense) with its members, though not to be acquainted "severally and individually" with them; whereas to "know what men there are" involves knowing them "severally and individually". It is also possible that there is here a change in Russell's views, for *Principia* precedes the full development of the theory of acquaintance which occurred as intensions were gradually squeezed from Russell's system. In fairness to Hintikka, however, it must be remarked that Russell's treatment of the quantifiers is often extraordinarily obscure and any sort of definitive account is beyond the scope of a review. What I

seriously. In fact, Russell was merely taking over Peano's terminology without, so far as I can see, ulterior philosophical motives.

hope to have shown is that it is not so clear as Hintikka maintains that Russell would have rejected Hintikka's response to question (a).

Question (b) was raised by G. E. Moore in a letter to Russell almost immediately after the publication of "On Denoting". According to Hintikka (p. 182), after 1905 Russell "did not any longer think of quantifiers and bound variables as being genuine constituents of propositions". But we have adduced sufficient evidence to show that this allegation is suspect. On the other hand, if quantifiers and bound variables are genuine constituents of propositions, what sort of constituents are they? In *The Principles of Mathematics* Russell advocated a highly idiosyncratic theory of the quantifiers in which quantifier phrases denoted multifarious objects (as distinct from unitary terms), each of the five distinct types of quantifier recognized in the *Principles* denoting a distinct object, and each object being distinct not by virtue of having different terms as components but by virtue of having different relations combining the terms together. (See Russell *1903*, Chap. 5.) This highly realist account is certainly lost after 1905. If Russell did, as I've suggested, restrict the range of variables to objects of acquaintance, then at least one problem associated with (b) is solved. For variables can now be introduced as constituents of propositions through their value-ranges. Quantified propositions contain as constituents all objects of acquaintance. But this doesn't solve the problem entirely, for we still have to distinguish existentially from universally quantified propositions.

An answer to this problem is suggested, I believe, by Romane Clark's valuable paper "Acquaintance" (pp. 231–46). Clark discusses the types of simplicity that are sometimes alleged to be involved in Russell's concept of acquaintance. Russell, himself, maintained that the actual relation of acquaintance was a simple (i.e. unanalyzable) relation between a mind and an object (e.g. in *1913*). He did not maintain that the objects of acquaintance are simple; indeed he maintained that they were in general complex (*1910*, 153). Moreover, elsewhere Russell makes it plain that he did not want to assume that there were any simple objects at all (*1918*, 202; *1924*, 337). So much has already been pointed out by Pears (*1967*, Chaps. 8, 9), Eames (*1969*, Chap. 4) and others, as well as by Clark (pp. 235–7), though it is still missed by some. Clark goes on to point out that the unanalyzability of the relation of acquaintance itself does not entail that "acts of acquaintance are conceptually simple acts, with no constituent operations helping to determine their references" (p. 238) and that the account of acquaintance that Russell gives is incompatible with acts of acquaintance being conceptually simple. This fact has not, to my knowledge, been previously noted. Indeed, the appealing analogy between acquaintance and an interpretation function which assigns elements and subsets of a domain to the syntactic elements of a formal language leaves no room for it. On the analogy the distinction between acquaintance and judgment is parallelled by the distinction between an interpretation function and evaluation functions which permit the computation of the semantic value of syntactically complex expressions from the assigned values of their syntactic elements.

That Russellian acts of acquaintance at any rate are not conceptually simple is shown by an analysis of some of the acts of acquaintance Russell admits. Russell, e.g., admits acquaintance with the self (e.g. in *1910*, *1913*) and with universals (e.g. in *1910*, *1912*, *1913*). Clark argues that such acts of acquaintance must be conceptually complex. To be acquainted with the self, is not to be acquainted with self "as though there were a cosmic pool of spiritual being" (p. 238); it is precisely to be acquainted with *the* self which one is. But then the expression of such an act of acquaintance involves description operators, and the determination of the object of such an act of acquaintance cannot be achieved without taking into account the separate role of the descriptor. Similar features occur in acts of acquaintance with universals. In such acts it is redness, not red, with which we are acquainted. Thus in this case also the object of the act can only be identified by means of an operator, this time an abstraction operator (expressed in English by the nominalizing suffix "ness"). Once we admit the conceptually distinct role of these operators in acts of acquaintance, our problems about the role of variables and their binders in propositions which we understand (and thus with the constituents of which we are acquainted) are solved. For quantifiers, like descriptors and abstraction operators, are variable binding operators. We may thus understand quantified propositions, despite the principle of acquaintance, without assuming that the quantifiers themselves are some esoteric sort of propositional constituent, provided (i) that Russellian quantifiers range over objects of acquaintance only, (ii) that the value-ranges of such quantifiers are constituents of the proposition, and (iii) the quantifications (more generally, the variable binding operations) are part of the act of acquaintance itself.[2] All this is not to say that Russell's semantic theory is easily defended, or even that it is consistent, but merely to point at least one way out of one fairly obvious difficulty.

None of this will work, however, if Pears is right in contending that Russell required all the constituents of a proposition to be simple (pp.

[2] Treating quantification in this way, essentially as a mental act, requires further changes in quantification theory. For an independent advocacy of such a position and some technical details, see van Fraassen *1982*. Presumably van Fraassen's programme for a subjectivist semantics includes similar treatments of other variable binding devices.

149, 151). It is strange to find Pears advocating this position since it conflicts with what he said in his book (*1967*, 143ff.). It is also inconsistent with the principle of acquaintance. For Russell writes, "When I speak of 'simples' I ought to explain that I am speaking of something not experienced as such, but known only inferentially as the limit of analysis" (*1924*, p. 337). It is easy to be confused by remarks like that in "The Philosophy of Logical Atomism" in which Russell says that "simples have a kind of reality not belonging to anything else" (*1918*, 270), which may suggest that he thinks that *only* simples are *properly speaking* real and thus can be genuine propositional constituents. But it seems to me that such passages should be understood in the way explained in "Logical Atomism" where Russell says "I do not believe that there are complexes or unities in the same sense in which there are simples" (*1924*, 336), but goes on immediately to explain that this is the result of the systematic ambiguity of existential expressions occasioned by type theory.

Pears does offer an argument for the claim that all propositional constituents are simple, but not a very good one:

> The name of a complex particular would have a meaning even if it were vacuous, and so a theory that identified its meaning with the complex particular itself would be insufficiently general. But if its meaning is not the complex particular, it does not introduce it as a constituent of a proposition. (P. 151)

But there are no vacuous names on Russell's theory, for names can only be bestowed on objects of acquaintance, and objects of acquaintance all exist (see above, also Russell *1959*, 64). Any vacuous referring expressions are, for Russell, definite descriptions. But definite descriptions, whether they denote or not, have no meaning. Thus if the complex particular is named it must exist—in which case Pears gives no objection to identifying the meaning with the complex particular. If it doesn't exist it can be referred to only by definite description, and definite descriptions have no meaning—in which case the question of the identification of the meaning with the complex particular does not arise. *No* definite description ever introduces a complex particular into a proposition; but *any* proper name may do so, provided the object of acquaintance on which the name is bestowed is complex and this requires, what Russell admits, that acquaintance can be with complex particulars.

3. *Logicism(s)*

Coffa, following a suggestion of Putnam's (*1967*, 20), distinguishes two types of logicism: *categorical logicism*, "the thesis that every mathematical theorem can be stated in terms of purely logical concepts and proved on the basis of purely logical premisses and rules of inference" (p. 249); and *conditional logicism*, the thesis that "logic suffices to formulate and prove all propositions of *pure* mathematics" where the propositions of pure mathematics are understood to be conditional in form with the axioms of the various branches of mathematics as their antecedents and the theorems of those branches as their consequents (p. 151). Coffa claims correctly that categorical logicism is the doctrine that Russell attempted to prove in *Principia Mathematica*; but Coffa is incorrect, in my view, when he claims that it was conditional logicism which Russell advocated in *The Principles of Mathematics*. There is indeed clear evidence in the *Principles* that Russell thought that logic sufficed to formulate (§3) and prove (§4) all the propositions of pure mathematics, and that these propositions were conditional in form (§1). Where Coffa goes wrong, I believe, is in claiming that these conditionals had axioms as their antecedents and theorems as their consequents. Rather the propositions of pure mathematics were, for Russell, formal (i.e. quantified) conditionals the consequents of which asserted some condition of *every* value of an untyped variable ranging absolutely without restriction over the domain of terms, while the antecedent imposed some categorical condition on the variable, thereby ensuring that the whole proposition remained true (by failure of antecedent, if necessary) for every value of the variable. Call this *single-sorted* categorical logicism[3] to distinguish it from the *many-sorted* (or type-theoretic) categorical logicism of *Principia*. Coffa (p. 250) does note this explanation of the conditional form of the propositions of pure mathematics in the *Principles*, but he plainly regards it as subsidiary to the explanation which is offered by the view that Russell was espousing conditional logicism in the *Principles*.[4] It would seem, indeed, that on Coffa's interpretation the propositions of pure mathematics would have to be doubly conditionalized: the first antecedent imposing category conditions on the variables, the second antecedent asserting the axioms, and the consequent asserting a theorem. Otherwise, the axioms of a branch of mathematics (e.g. topology) would

[3] Coffa's terminology (p. 250) suggests the less explicit but more attractive name "converse-Leninist logicism".

[4] It is very strange that Coffa (p. 261) notes Peano's use of unrestricted variables and his likely influence on Russell, but asserts that Peano's "conditional interpretation of mathematics differs from Russell's in one essential respect: the antecedents of Peano's conditionals are, in effect, intended to determine the range of all variables in the corresponding consequents." This was exactly Russell's view.

not be of the conditional form which Russell required of *all* mathematical propositions.[5] Similarly, without double conditionalization, arithmetic propositions, which Coffa specifically excludes (p. 251), would not be of the required conditional form.

Now it is indeed the case that as regards geometry Russell did adopt conditional logicism. The problem here was the competing axiom sets of Euclidean and non-Euclidean geometries, all of which had somehow to be included within the scope of mathematics (see the evidence cited by Coffa: Russell *1903*, 5, 8, 372–3, 429–30, 441–2). But Coffa's claim (p. 251) that Russell generalized this approach to the whole of mathematics is mistaken. None of the passages he cites supports this claim. The most promising passage is that in which Russell says that pure mathematics is "a subject in which the assertions are that such and such consequences follow from such and such premisses; not that entities such as the premisses described actually exist" (*1903*, 373). But this is only firm evidence if "premisses" can here be identified with "axioms", and this seems doubtful. At best the passage is ambiguous. Coffa's strongest textual evidence dates from a much later period when Russell wrote the introduction to the second edition of the *Principles* (*1937*, vii), where he explicitly states that his earlier position was conditional logicism. But there is good reason to doubt whether, in this instance, he was treating his earlier views with the exegetic care they deserved. Nor do we need to suppose that Russell was a conditional logicist to explain what he meant by his famous definition of mathematics as "the subject in which we never know what we are talking about [since content-loading expressions have been replaced by variables] nor whether what we are saying is true" (*1901*, 59–60), since, in Coffa's words, "we do not care about the truth values of either axioms or theorems, only about that of the implication" (p. 251). Rather, mathematical propositions do not require that such things as numbers exist, but merely assert propositions of the form "if x is a number, ϕx". Pure mathematics never asserts that there are numbers nor that ϕx is true of anything.

Apart from the strong textual evidence, Coffa's interpretation runs into two severe theoretical difficulties. If Russell was a conditional logicist, and not single-sorted categorical logicist, then not only pure mathematics but any theory capable of rigorous axiomatic formulation or indeed any theory[6] is logicizable. Geography as Coffa notes (p. 260)

[5] Russell, it seems, would admit non-conditional propositions as mathematical so long as the propositional function involved was true for all values of its variable(s), although he says he can't think of any examples (*1903*, 20). "$\hat{x}$ is a term" would be a likely candidate.

[6] Since any theory can be trivially axiomatized by treating all theses as axioms.

would be reducible to logic. But Russell makes a clear distinction between pure mathematics which is logicizable and applied mathematics which is not. The distinction is not, I think, tenable for conditional logicism, and this constitutes strong *prima facie* evidence that Russell was not a conditional logicist. The second theoretical problem is that Coffa's interpretation offers no way of explaining why conditional logicism did not appear in *Principia*. If Russell is interpreted as a single-sorted categorical logicist in the *Principles*, it becomes immediately obvious that such a position would have to have been abandoned once type theory was adopted.

Coffa's distinction between the two types of logicism is intended as a prelude to a re-evaluation of Russell's rejection of Kantian intuitions in mathematics. Russell left plenty of evidence that he regarded the successful completion of the logicist programme as a definitive refutation of Kant's view that mathematical reasoning involes intuition (e.g. *1903*, 4, 457–8; *1959*, 9, 56, 57), and indeed, Russell at one point said that at the time he regarded logicism as "a parenthesis in the refutation of" Kant (*1959*, 57). Hintikka (*1965a*, *1965b*, *1967*, *1969*, *1972*) has argued that there is in fact no conflict between Kant's philosophy of mathematics (correctly interpreted) and logicism; that Russell thought there was shows merely that Russell didn't understand Kant correctly. Coffa, in the second half of his article, defends Russell on this point. This defence, it seems to me, is largely independent of his claim that Russell was a conditional logicist, for any conflict between Kant's doctrine of intuitions and conditional logicism would hold *a fortiori* for categorical logicism as well. The exegetic questions in this issue are somewhat more complex than can be disposed of in a review: they include what Kant said about intuition in mathematics; what Russell, Hintikka and Coffa say about Kant; what Hintikka and Coffa say about Russell; and finally what Hintikka in his reply says about Coffa.

Hintikka detects two levels in Kant's philosophy of mathematics each with its own concept of intuition. At the first level intuition amounts to little more than instantiation, and is thus admitted by logicized mathematics. The synthetic character of mathematical inference, admitted by both Kant and Russell (before Wittgenstein), amounts to the claim that the conclusion of mathematical arguments may introduce more individual objects than the premisses (in a sense defined by Hintikka *1965b*). In this first sense of "intuition", which Hintikka (*1967*, 354) calls "basic", whatever Russell (*1903*, 43) admits as a term might be admitted by Kant as an intuition. In particular, there is no connection between these first-level intuitions and sensibility. Sensibility is introduced only at the second level, and the two types of intuition might accordingly be characterized as "*I*-intuitions" and "*S*-intuitions".

As Hintikka admits (*1967*, 355), it is the second level that makes "intuitions intuitive", and when Kant's full, two-stage doctrine is applied "[t]here is ... no room left in mathematics for intuitions that are not connected with sensibility" (*1967*, 366). He also admits that the use of *S*-intuitions in logic, and thus in logicized mathematics, leads to psychologism (*1969*, 62). In view of this, Coffa's suggestion that Russell had in mind the entire Kantian doctrine in which *I*-intuitions are reduced to *S*-intuitions, is eminently attractive. Russell was indeed quite right to reject, as Hintikka does, the full Kantian doctrine—his worst fault, on this account, is that he overlooked the virtues of Kant's first-level account taken on its own. Historically, as well as logically, Coffa's interpretation makes good sense. For Russell's early work on geometry (*1896a*, *1896b*, *1897*) took its Kantianism primarily from the Transcendental Aesthetic where *S*-intuitions figure most prominently.

Against this contention of Coffa's, Hintikka's reply is largely unavailing. Indeed, a good part of it is concerned to try and show that Kant employed *I*-intuitions at all, a claim Coffa doesn't challenge. Even on this point, however, Hintikka's case is far from conclusive and most of the evidence he cites is ambiguous. For example, he cites *The Critique of Pure Reason*, A713=B741, to show that "Kant's doctrine that mathematical arguments turn on the use of intuitions ... means merely that a mathematician considers his or her general concepts by means of particular representatives. The introduction of such particular representatives is what Kant defines construction to mean" (p. 266). What Kant says, however, is that "To *construct* a concept means to exhibit *a priori* the [non-empirical] intuition which corresponds to the concept." But by "non-empirical intuition" he means "representing the object ... by imagination alone" and his claim that such an intuition "must, as intuition, be a *single* object" need amount to no more than a rephrasing of the empiricist doctrine that ideas are always of particulars. So-called "pure intuitions", as much as empirical intuitions, are *S*-intuitions in Hintikka's account.

The key issue involved, however, is the *degree* of connection Kant requires between *S*-intuitions and sensibility. Both Coffa and Hintikka are in agreement that Kant (in the full system) leaves no room in mathematics "for intuitions that are not connected with sensibility" (Hintikka *1967*, 366). And both, it seems, agree that Kant required a stronger connection than the mere fact that sensibility was always involved in learning mathematics (e.g., that it took time to understand a mathematical proof, or that counting was a temporal process). It is true that in the *Prolegomena*, §10, Kant says "Arithmetic produces its concepts of number through successive addition of units of time." But even this (isolated) strong claim could, one suspects, be interpreted as no more than a claim as to the learnability conditions of arithmetic. Hintikka's claim (p. 267) is that Kant's second level theory which requires *S*-intuitions does so in order "to show the applicability of the whole system of mathematical truths and arguments to one's actual knowledge of the world." But this is not at all the impression one gets, e.g., in that notorious passage in the *Critique* about the syntheticity of 5+7=12, where he says that we cannot arrive at 12 by analyzing the concepts of 5 and 7 but "have to go outside these concepts, and call in the aid of the intuition which corresponds to one of them, our five fingers, for instance.... For starting with the number 7, and for the concept of five calling in the aid of the fingers of my hand as intuition, I now add one by one to the number 7 the units which I previously took together to form the number 5, and with the aid of that figure [*sc.* the hand] see the number 12 come into being" (B15–16). The hand is not here used to show the applicability of arithmetic to anatomy.

There is no denying that Kant's remarks, especially when taken as a whole, are often bafflingly obscure. In the "Transcendental Doctrine of Method", e.g., he gives (A716=B744) a relatively clear account of the use of figures (imagined or drawn) in geometric proof. It was exactly the sort of use that Russell objected to, and sought, with some difficulty, to avoid in preparing the French edition of his *Essay on the Foundations of Geometry*. But the subsequent account Kant gives of the use of intuition in algebra (A717=B745) defies rational reconstruction. However, it seems extremely doubtful whether Hintikka has successfully shown that Russell's interpretation of Kant was the wrong one, or that his own is the correct one. A full account of the issues would likely require a monograph.

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