Reply to Demopoulos

by Nicholas Griffin

IN HIS REVIEW[1] of the first of the *Synthese* special issues on Russell's early philosophy, William Demopoulos raises three objections to my contribution[2] to that volume. He complains, first, that I misunderstand Russell's account of mathematical propositions in *The Principles of Mathematics* (1903); second, that I am mistaken in supposing that between the *Principles* and *Principia Mathematica* (1910) Russell switched from a metaphysical to an epistemological account of the apodicticity of mathematical propositions, coming to emphasize their certainty rather than their necessity; and, third, that, contrary to what I said in my paper, Russell's type theory is not context-sensitive.

Only the second of Demopoulos's complaints is just, however, and that only partly. It is indeed true that the evidence Demopoulos quotes (*Principia*, 2nd ed., I: 12–13, 59) refutes my injudicious remark that Russell sought to demonstrate the certainty of mathematics by deduction from axioms which are certain (Gp. 118), as does Russell's transcendental procedure for arriving at an appropriate axiom set for mathematics. However, it does not refute, as Demopoulos seems to think, my wider claim that Russell was more concerned in *Principia* and later writings with the certainty of mathematics rather than its generality, a claim which is, I think, well documented in my paper. For one thing, with the introduction of type theory, the concept of generality was not available to Russell in the form in which he had used it in the *Principles*. For another, it is not unreasonable to suppose that Russell was attempting to establish the certainty of mathematics through a transcendental deduction of its axioms.

Demopoulos's other charges, however, seem to me completely without foundation. He charges me with "a rather unusual account of Russell's 'if-thenism'" (Dp. 164), namely, that mathematical propositions are conditionals with antecedents designed to protect the consequent against falsifying assignments of values to its variables. But to describe this as an account of Russell's "if-thenism" is already to beg the question, for it was precisely my point that Russell was not an if-thenist, at least in

[1] "New Work on Russell's Early Philosophy", *Russell*, n.s. 1, no. 2 (1981–82): 163–70. Page references to this work are preceded by D.

[2] "Russell on the Nature of Logic (1903–1913)", *Synthese*, 45 (1980): 117–88. Page references to this work are preceded by G.

the sense in which Putnam[3] introduced the term. It further begs the question, in Russell's type-free system of 1903, to talk as Demopoulos does (Dp. 164), of "improper substitutions" for variables, for in such a system *no* substitutions are improper. *Some* substitutions, however, will be falsifying unless the proposition is conditionalized and can be kept true by failure of antecedent if necessary. How else, without typing devices of some kind, can a proposition be true for all values of its variables, as Russell asserted the propositions of mathematics were?

Demopoulos "fail[s] to see why this account is either required or to be preferred over such traditional and straightforward formulations ... as ... that given by Quine in 'Truth by Convention'" (Dp. 165). But I didn't claim that this account of mathematical propositions was required or to be preferred, merely that it was Russell's—as Demopoulos would have seen had he read Russell instead of Quine, who seems (once again) to have been the source of an enduring misunderstanding about Russell. The textual evidence is, in fact, quite overwhelming. For example:

> It is customary in mathematics to regard our variables as restricted to certain classes: in Arithmetic, for instance, they are supposed to stand for numbers. But this only means that *if* they stand for numbers, they satisfy some formula, *i.e.* the hypothesis that they are numbers implies the formula. This, then, is what is really asserted, and in this proposition it is no longer necessary that our variables should be numbers: the implication holds equally when they are not so. Thus, for example, the proposition "x and y are numbers implies $(x+y)^2 = x^2+2xy+y^2$" holds equally if for x and y we substitute Socrates and Plato: both hypothesis and consequent, in this case, will be false, but the implication will still be true. Thus in every proposition of pure mathematics, when fully stated, the variables have an absolutely unrestricted field: any conceivable entity may be substituted for any one of our variables without impairing the truth of our proposition. (*Principles*, art. 7 *in toto*. See also pp. 8, 13, 36, 37–8, 86–7.)

Demopoulos thinks that one of the attractions of Quine's interpretation is that, on it, "the relation of logic to mathematics is the same as that which holds between logic and any other articulated body of knowledge" (Dp. 165). It seems to me that this fact alone should have been enough to cast doubt on Quine's interpretation, for Russell was a logicist about mathematics, not about (e.g.) geography. It is a relatively well-known fact about Russell that he thought the connection between logic and mathematics was considerably more intimate than that between logic and any other articulated body of knowledge: not all articulated bodies of knowledge are apodictic. The account I gave is exhibited in Russell's axiomatization of the propositional calculus (*Principles*, art. 18);[4] it was systemically necessary to remove difficulties in Russell's account of logical truth caused by his deployment of untyped variables; Russell adopted it from Peano;[5] it was recognized (and criticized) by Wittgenstein (*Tractatus*, 5.5351); and I have dealt with putative counter-evidence elsewhere.[6] I am not sure what else I can do to be convincing—except to urge that in understanding Russell a familiarity with Quine is insufficient.

Demopoulos's third point seems hardly more effective. He refuses to discuss my major claim that Russell, in 1913, sought an epistemic justification for logic, but believes it to be "definitely wrong" "in so far as it underlies" my view that type theory is context-sensitive (Dp. 166). In fact, it was never part of my intention to argue that Russell's attempt to develop the ramified hierarchy of orders out of the theory of judgment *entailed* (without further assumptions) the context-sensitivity of the hierarchy. Thus Demopoulos's simple argument against the former via a denial of the latter is not an effective critical strategy. Moreover, the argument that Demopoulos criticizes at length—namely that since Socrates alone was acquainted with Socrates, Socrates is a value of the function "$\hat{x}$ is an individual" only for Socrates, thus value ranges for functions vary from person to person—was not my main reason for claiming that the Russellian type hierarchy was context-sensitive. My main reason was that this is what Russell *says*. I cited (Gp. 138) three passages[7] but, since Demopoulos deemed none of these to be worth his attention, here are two more:

> It is unnecessary, in practice, to know what objects belong to the lowest type, or even whether the lowest type of variable occurring in a given context is that of individuals or some other. For in practice only the *relative* types of variables are relevant; *thus the lowest type occurring in a given context may be called that of individuals, so far as that context is concerned.* Accordingly the above account of

[3] "The Thesis That Mathematics Is Logic" in H. Putnam, *Philosophical Papers*, 2nd ed. (Cambridge: Cambridge University Press, 1979), 1: 20.

[4] On Putnam's if-thenism, the axioms would not be conditional in form, and thus would refute Russell's contention (*Principles*, art. 1) that all propositions of mathematics are conditional.

[5] See, e.g., "The Principles of Arithmetic, Presented by a New Method" (1889), in *Selected Works of Giuseppe Peano*, ed. H. C. Kennedy (Toronto: University of Toronto Press, 1973), pp. 101–34.

[6] "New Work on Russell's Early Philosophy", *Russell* (forthcoming), §3.

[7] *Principia*, 1: 42, 50; "Mathematical Logic as Based on the Theory of Types" in *Logic and Knowledge*, ed. R. C. Marsh (London: Allen and Unwin, 1956), p. 76.

individuals is not essential to the truth of what follows; all that is essential is the way in which other types are generated from individuals, however the type of individuals may be constituted. (*Principia*, I: 161–2; long italics mine)

If, as may be the case, whatever *seems* to be an "individual" is really capable of further analysis, we shall have to content ourselves with what may be termed "relative individuals", which will be terms that, *throughout the context in question*, are never analysed and never occur otherwise than as subjects. And in that case we shall have correspondingly to content ourselves with "relative names". (*Introduction to Mathematical Philosophy* [1919], pp. 173–4; long italics mine)

Ironically, on this point, Demopoulos would not have been led astray by Quine, who notes the context-sensitivity of Russell's type theory in *Set Theory and its Logic*, 2nd ed., p. 247.

My argument that Socrates is an individual only relative to context was intended to show that context-sensitivity was not a capricious feature of Russell's theory of types. Nothing much hangs on the particular argument, since there are many more like it. Demopoulos thinks that I am claiming that the argument is "a good plausibility argument against 'pseudo-functions' like '*x* [*sic*] is an individual'" (Dp. 166–7). But this was not my claim at all; "pseudo-function" is Demopoulos's term, not mine. Nor, so far as I know, is there any evidence to suggest that Russell, unlike Wittgenstein, thought "$\hat{x}$ is an individual" was a pseudo-function or in any way defective as a propositional function. My claim was rather that such functions could not be treated as propositional functions of logic, in the sense (modified from Russell's account in the *Principles* to allow for type theory) of yielding true propositions for every argument in their (type-restricted) range of values. Demopoulos's subsequent remark that by this criterion even "'$p \supset p$' would not be a propositional function of logic, since different people are acquainted with different propositions" (Dp. 167) shows that at least he's got the right idea—though he seems not to have noticed it stated either side of the passage he quotes from my paper (Gp. 138). It was for this very reason that I claimed that the simple and initially attractive revision of Russell's account of logical truth mentioned above led to "insuperable difficulties" (Gp. 138).

Moreover, the argument is not defective in the way Demopoulos claims. Consider the sentence (S) "Socrates is wise" in two different contexts, the first (C_1) as uttered by Socrates, and the second (C_2) as uttered by Demopoulos for whom "Socrates" means the teacher of Plato (Dp. 167). It is quite clear on Russellian grounds that S in C_1 has a different meaning from S in C_2. If we take the Russellian analysis of S in C_2 we get:

(1) $(\exists x)[x \text{ taught Plato } \& (\forall y)(y \text{ taught Plato} \supset x=y) \,\&\, x \text{ is wise}]$.

Whereas the Russellian analysis of S in C_1 is simply "wise (Socrates)", a substitution instance of "$\phi(a)$". Two Russellian points can be made about (1): First, if (1) is the properly canonical form of S in C_2, then anyone who understands it (e.g. Demopoulos) must be acquainted with all its constituents. Second, that if (1) is the full analysis of S in C_2, the substitution values of the variable "x" must be logically proper names, for as Russell notes (*Principia*, I: 67) definite descriptions are not substitution values of individual variables. Given that Demopoulos is not acquainted with Socrates, these two points lead to the same conclusion: that "Socrates" is not a legitimate substitution value for "x" in (1). For firstly, Socrates cannot be a constitutent of (1) if Demopoulos understands (1) and is not acquainted with Socrates. (Since quantifier expressions are to be interpreted semantically in accordance with the principle of acquaintance through their value ranges, members of which are objects of acquaintance, Socrates cannot be a member of the value range of "x" in (1).[8]) And, secondly, "Socrates" can only be a logically proper name for someone acquainted with Socrates, i.e. only for Socrates himself. Thus "Socrates" is not an admissible substitution value for "x" in (1). What this shows is that (1), with its quantification over medium-sized specimens of dry goods (such as Socrates), is not a full analysis of S in C_2. The principle of acquaintance requires that more be done, namely an analysis which yields quantification only over items with which Demopoulos is acquainted. How this was to be achieved is, of course, the sad and unfinished (indeed, barely started) story of how Socrates was to be constructed out of sense-data. But for Socrates, himself, none of these problems arise (at least in the days before Russell tried to construct the self out of neutral events). The upshot is that the analysis of S in C_1 stops with Socrates, whereas the analysis of S in C_2 must continue right down to sense-data if the claims of the principle of acquaintance are to be met. The result is that in the two contexts, we get individuals at different levels.

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[8] See Griffin, "New Work on Russell's Early Philosophy", *Russell*, n.s. 2, no. 2 (1982–83): 69–83.