Textual studies

Part 1 of *The Principles of Mathematics*

by Kenneth Blackwell

My book keeps on going through the Press: I do wish books could be published without one's having to see any more of them—it is really disgusting returning to one's own vomit. (Russell to G. Lowes Dickinson, 11 July 1902)

This study concerns the composition of the final version of Part 1 of *The Principles of Mathematics*. First I describe the available documents and then the plans for uncovering the hidden layer of revision in proof. The results of collating the final manuscript and the printed text lead to a reconstruction of the penultimate, pre-paradox draft of Part 1. Russell had the greatest difficulty in rewriting that Part and was stuck for a year. When he became unstuck in May 1902 he narrated his progress to his wife and other correspondents. There was much work to come, however, in revising the proofs over a period of nine months. At the beginning of this period he contacts Frege, and I provide the bibliographical and textual foundation for the study of Frege’s immediate impact while Russell continued trying to resolve the Contradiction. Some conclusions follow about scholarly requirements for the text of the *Principles*.

The documents. To be precise, there is not one text of the *Principles*, but several similar texts. The discovery of their differences is the subject of this study. A draft has been available in the Russell Archives since that time, and many researchers have made use of it in beginning their own investigations of the text of the *Principles*. In particular I. Grattan-Guinness has encouraged the final preparation of the draft for publication. The only other published study involving the *Principles*’ text is J. Alberto Coffa, “The Humble Origins of Russell’s Paradox”, *Russell*, nos. 33–34 (Spring–Summer 1979): 31–7.

The subject of the manuscripts of the *Principles* was first discussed in my brief description of them in Barry Feinberg, ed., *A Detailed Catalogue of the Archives of Bertrand Russell* (London: Continuum 1, 1967), p. 98. Alejandro García-Acero’s doctoral dissertation, “Bertrand Russell and the Origin of the Set-Theoretic Paradoxes” (University of Toronto, 1983), has a lengthy appendix devoted to “A Biobibliographical Note on *The Principles of Mathematics*”. Griffin has been subjecting these manuscripts to extensive scrutiny for *The Col-
of this paper. The final manuscript is available in the Russell Archives (file 230.030350.F1-20). It comprises over 900 leaves, mostly handwritten, and lacks only a small number, notably some chapters and sections in Parts II and VI. As printers' markings appear frequently on the manuscript, there is no doubt that the typesetting of the book was done from this manuscript, and not from any intervening typescript. The *Catalogue* of the first Archives mentions (pp. 98–9) twenty leaves of an earlier draft of Part I, and several longer attempts to write on the subject—the latter all predating Russell's encounter with the work of Peano in August 1900. The printed text in its first and only typesetting has had nine printings to date. The first edition may be described as follows.

**Title:** THE PRINCIPLES OF MATHEMATICS | BY | BERTRAND RUSSELL M.A., LATE FELLOW OF TRINITY COLLEGE, CAMBRIDGE | VOL. I. | CAMBRIDGE: at the University Press | 1903


Type, paper and binding: Set in Modern type. Printed on medium weight, cream-coloured, wove paper, endpapers same and overfolded and sewn in the first and last gatherings; edges untrimmed. Bound in dark blue, fine grain cloth. Double rule box blind-stamped close to edges on both boards. Stamped in gilt across spine: THE PRINCIPLES OF MATHEMATICS | RUSSELL | VOL. I. | [3-point publisher's crest] | CAMBRIDGE | UNIVERSITY PRESS. Full-width double rules blind-stamped across top and bottom of spine.

Published: May 1903 at 12s. 6d., and distributed from 27 June 1903 in U.S. by Macmillan at $3.50; number of copies, 1,000.

For the second “edition” (i.e. impression) of 1937, besides adding a new introduction, Russell made several corrections, and more were made in at least one later printing. Two of the original corrections involved a cancelled leaf in a second state of the second impression; in correcting them, further errors were introduced. Russell’s own copy (formerly Wittgenstein’s) of the first edition has two corrections in his hand. In an article Russell corrected a misprint, which, however, has never been corrected in the book. Finally, there is the American issue (1938) of the second impression and its reprintings; but as the sheets were imported from the British publisher, they need not be taken into account. No proofs are known to survive.

To those interested in the genesis of one of Russell’s most outstanding works, these various versions of the text provide the material to work on. From their comparison not only can the changes Russell made be examined, but a few thousand words of his thought on the philosophy of mathematics can be recovered. The long passages that Russell inserted or corrected in the book may help to explain how he came to his published views.

**Methodology.** I collated Part I, “The Indefinables of Mathematics”, of the printer’s copy with Part I of the ninth printing (1972) of the Principles. The variants are noted on the appended list. I have also reconstructed and recovered much of the previous draft: of Part I—one of the Parts Russell tells us were rewritten after 31 December 1900, when he finished the first draft. I have not tried to relate the earlier manuscripts of a book on the same subject, which textually are very different. (As Garciadiego and King point out, there are exceptions to this generalization.) The ninth printing was chosen because it contains any revisions...
made in 1937 and any since then during Russell's lifetime. The list of revisions therefore includes any 1937 revisions, which it would not if the 1903 edition had been collated. However, comparison with the first printing reveals that none of the revisions were introduced in 1937 or later. Any 1937 corrections of first edition misprints back to the manuscript could not be discovered in this way, but that is hardly a loss; and the alternative would be a separate collation of the first and ninth impressions.  

The list ignores alterations on the final manuscript itself, and alterations between it and earlier manuscripts. The former are readily apparent from glancing at the manuscript, and can even be differentiated according to the ink employed in making them. For this paper I wished to uncover what was not obvious, i.e. the revisions made in the typesetting and proofreading stages of the first edition and subsequently. Although both kinds are required for the study of how Russell wrote the Principles, the revisions made after he finished the manuscript and handed it over to the Cambridge University Press can be usefully regarded separately. After all, during the typesetting and proofreading stages, Russell was probably concerned to keep changes to a minimum because of the waste, expense and delay involved in resetting lines of type already (as we shall see) in pages, and we know he had kept the manuscript by him for seventeen months after finishing the first draft.  

The main reason for this delay was the discovery of the Contradiction in June 1901.  

To collate the manuscript with the printed text, I twice read the former phrase by phrase against the latter. In the terminology of textual criticism, there are two kinds of variants, substantives and accidentals. Accidentals are changes in spelling, capitalization, hyphenation, abbreviation, punctuation, paragraphing and italicization—changes that rarely affect the sense. Substantives are the words and their order. Some of these do not affect the philosophical sense of the passages changed, but the division of substantives into those of possible philosophical interest and those not is a very subjective enterprise, and there is a stylistic interest in Russell's non-philosophical rewording. I have not thought it worthwhile to report even the non-mechanical variants among the accidentals.  

This can be done with the Hinman Collator, which superimposes the image of a page from one copy of a book upon the image of the same page from another copy of the same typesetting of the book. The Russell Editorial Project possesses a Hinman. However, the task is extremely tedious, if not impossible, when a photo-offset impression of the original has resulted in the contraction or expansion of the lines of type. This situation obtains, unfortunately, when the ninth impression of the Principles is machined against the first.  

The last leaf of the manuscript is dated 23 May 1902. There are, however, three leaves otherleaves, 137-8, formerly 48-9 and before that 4-5, have "N." in the corner, indicating they are taken from Part II, all of whose pages bear that symbol (Part II in the final manuscript is dated June 1901). Two other leaves, 17-8, formerly 48-9 and before that 4-5, have "N." in the
corner. Four others are the first leaves of chapters, and the chapter titles match those listed in this table of contents. Most of the missing leaves can be located in the final manuscript, and one (fol. 105) has the title of another chapter as listed on this table of contents. Eight leaves were transcribed, with some changes, for the final manuscript. In the following reconstruction, the first two columns list the chapter titles of the penultimate manuscript and their folio numbers. Their location, if any, in the final manuscript is then noted, rewritten leaves appearing in both columns. The last column gives the printed page and chapter references.

**Chapter Title on Penultimate Manuscript & Contents Table**

<table>
<thead>
<tr>
<th>Chapter Title on Penultimate Manuscript &amp; Contents Table</th>
<th>Folio Nos.</th>
<th>Folio Nos. of Final MS.</th>
<th>Final P. &amp; Chap. Nos.</th>
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<tbody>
<tr>
<td>I. The Definition of Pure Mathematics</td>
<td>1-3</td>
<td>10-14 (rewritten)</td>
<td>7-9 (I)</td>
</tr>
<tr>
<td>II. Terms and Concepts</td>
<td>4-7</td>
<td>84 (rewritten)</td>
<td>43 (IV)</td>
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<td>8</td>
<td>85</td>
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<td>13-15</td>
<td>69-91</td>
<td>45-7 (IV)</td>
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<td>16-17</td>
<td>105-6</td>
<td>54-5 (V)</td>
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<td>18-23</td>
<td>185-90</td>
<td>95-8 (IX)</td>
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<tr>
<td>III. Classes and Relations</td>
<td>24-5</td>
<td>107 (rewritten)</td>
<td>55-7 (V)</td>
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<td>26-34</td>
<td>111-19</td>
<td>57-61 (V)</td>
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<tr>
<td>IV. Conjunction and Disjunction</td>
<td>35</td>
<td>172-3 (rewritten)</td>
<td>90 (VII)</td>
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<td>36</td>
<td>174</td>
<td>90-1 (VIII)</td>
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<td>37</td>
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<tr>
<td>VI. [&quot;Implication&quot; begins about here]</td>
<td>38-43 [missing]</td>
<td>135-8</td>
<td>70-2 (VI)</td>
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<td>44-9</td>
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<td>50-7 [missing]</td>
<td>53-8</td>
<td>26-9 (II)</td>
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<tr>
<td>VII. [&quot;Peano's Symbolic Logic&quot; begins about here]</td>
<td>64 [missing]</td>
<td>59-62</td>
<td>29-31 (II)</td>
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<td>65-8</td>
<td></td>
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</tr>
<tr>
<td>VIII. [&quot;Summary of Part I&quot; begins about here]</td>
<td>69-7 [missing]</td>
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Russell's use, in the final manuscript, of pages from a previous manuscript is typical of his method of composition and revision.

The penultimate manuscript was written before the Contradiction was discovered. There is no separate chapter on the topic, indicating at the least that this manuscript predates Russell's realization of the Contradiction's importance. Moreover, the penultimate draft is dated May 1901 and Russell states that he discovered the Contradiction in June 1901. It should be noted that there is no trace of the draft of Part I written in the fall of 1900, and that there exists a detailed "Plan for Book I: The Variable", dated April 1902 (in RA 230.030350), which the final manuscript follows so closely that only one chapter title is altered: VII, instead of being called "Assertions", becomes "Propositional Functions".

**Final composition.** The composition of the final version of Part I is another example of the terrific bouts of work of which Russell was capable; compare the writing of Theory of Knowledge eleven years later. We do not know how much Russell had worked on the Principles since discovering the Contradiction the previous spring. Grattan-Guinness points out that manuscript material is totally lacking in this area. Over the winter Russell gave two terms of lectures on mathematical philosophy and had accumulated so much written material that he could tell Jourdain on 16 March 1902 that he "intend[ed] publishing my lectures together with other material in a joint book with Whitehead, but probably not for two years or so. I also intend publishing a book (for which I have done the bulk of the work) on the philosophical aspects of the subject, and on controversial points." It is ironic that Russell should give prominence here to a work of which almost nothing is extant—namely the work that was intended to be Volume II of the Principles—rather than the Principles itself, towards which he had laboured since 1897. Barring the appearance of other evidence, we must assume that this is how matters stood in the early spring of 1902. Not only could Russell not see his way to completing Volume I, but on the personal side he has ceased to love Alys and can see no way of saving his marriage. By late April Alys's doctor has ordered her to live separately from Russell for a month, and he himself, as he confesses in almost daily postcards and then letters to her, has been very near a breakdown. The permanent result was the ethic of "The Free Man's Worship".

Russell at length recovers from the brink (cycling and emotional rest are the cures) and turns to his pile of manuscripts. In the first real letter he is allowed to write Alys after she enters her rest-cure, on 30 April from The Mill House, home of the Whiteheads, he remarks that he feels much fitter and goes on:
The Principles of Mathematics

I have decided that as my big book does not need very much more work to be finished, I must screw myself up to get it done while we are separated, as that will leave me freer to devote myself to thee, and will make me feel that it doesn’t matter if I do get a bit tired. I cannot, in the time and in my present condition, finish it in style, but I can patch up something that will do for publication.\footnote{Russell’s letters to Alys at this time are in the possession of Barbara Halpern, Oxford. Through her kindness copies are available in the Russell Archives.}

The next day he reports that he has already done some work (probably the detailed plan for Part I): “It will be tiring, but less so than the feeling that my book will never be finished.” On the 2nd of May he is back at their home at this time, Friday’s Hill. He writes a whole chapter that day, as he tells Alys on the 3rd. That must have been Chapter I, fifteen leaves on “The Definition of Pure Mathematics”, for II at forty-six leaves would have been too long. By the 4th he has established a daily work-schedule, and Beatrice and Sidney Webb have arrived to stay for two months. Russell continues to sleep well, reporting on the 5th that he has lately got through six or seven hours of work a day. The same day he tells Helen Thomas: “I am writing on any, every, a, some, all, the nature of inference, the essence of truth, and other light subjects. Except when I am fit, I cannot understand my own writings.” He reaches eighty leaves of manuscript on the 6th. That puts him at the end of III, “Implication and Formal Implication”. The letter of the 6th did not report on the complete day, for on the 7th he says that he worked the previous evening until midnight, “accomplishing a Chapter of 21 pages”. That was IV, “Proper Names, Adjectives, and Verbs”, which occupies folios 82–102 of the manuscript; in it Russell took over five leaves from his penultimate attempt. He must have disposed of v on the 7th (using thirteen old leaves). In preparing to visit his Uncle Rollo he notes: “Now I have to tackle the most difficult Chapter [VI] in the whole of my book—on the nature of Classes. I shall go to Dunrozel in a dream, with Classes all the time revolving in my head whatever may be happening outwardly” (8 May). His report on the day’s work there has been published.\footnote{See Carl Spadoni, “Philosophy in Russell’s Letters to Alys”, Russell, nos. 29–32 (1978): 29.} Suffice it to say that he wrote a chapter of thirty-one leaves. It is worth noting his mood: “I have been so long without real work, that I have come back to it with a kind of fever: everything else seems unreal and shadowy to me just now, and I work as if we were possessed.”

Russell is working very fast at this time, and there is no mention of Chapter VII. But nor was there a letter on the 10th. On the 11th, back at The Mill House, he is concerned with VIII, telling Alys: “… The Nature of the Variable dominates my thoughts, and must be decided today or tomorrow at an internal oecumenical Council.” But on the 11th he has a headache. The 12th is an extraordinary day of work, for he begins it still working on VIII:

I sat up till 1.30 last night to finish Part I of my Book, which was the task I had dreaded being never again equal to. I am not at all satisfied with it, but I fear it is the best I can do. I think of publishing as soon as possible, as I cannot rest till it is off my mind. This is not the true artistic conscience, but that is a luxury I can no longer afford for the present. (13 May)

Finishing Part I required not only completing VIII, but also writing IX, “Relations”, and X, “The Contradiction”. Half of IX (twelve leaves) was old, but all of X (fifteen leaves) was new. The handwriting shows Russell’s hurry—just like the end of Part II of Theory of Knowledge. He was so stimulated that he stayed awake until dawn before travelling to Bournemouth to visit Aunt Agatha. He had, after all, written 213 pages in just eleven days—or 168 when the previously written or transcribed pages are deducted.

We have long known from the Autobiography (1: 151) that Russell completed the Principles on 23 May. Yet with only a week to go he writes Alys from Friday’s Hill:

I expect to have my book quite finished in another two months, if only I can keep fit and go on working hard. It will not give me any feeling of elation, merely a kind of tired relief as at the end of a very long dusty railway journey. The book will be full of imperfections, and will raise innumerable questions that I don’t know how to answer. There is a great deal of good thinking in it, but the final product is not a work of art, as I had hoped it would be. I shall send it to the Press at once, as the load will not be off my mind until I cannot make any further corrections. (16 May)

Having finished Part I, he turns to the rest of the manuscript on the 16th and works eight hours on it. There is an interruption from visitors on the 18th and 19th. Alys evidently shoulders some of the responsibility for the book’s imperfections, but Russell tells her she need not feel that way. On the 21st, now at The Mill House, he is back to work. The next day he expects “very soon [to] reach the point when I shall have only more or less mechanical work to do”. He does not mention what he has been working on since the 14th. In a 1910 letter to Jourdain he was to claim that Parts I and II were “wholly later, May 1902” (Dear Russell …, p. 133). Yet he makes no reference to progress on individual chapters in
Possibly this is because they were not (or were no longer) a challenge in the way that Part I had been. He has, after all, dealt with the Contradiction. In this bout of work, he tells Alys, Russell has “learnt to pull myself together and make efforts which formerly I should have thought impossible; but I suppose there are limits to the process.” He imagines the appropriate dedication for the Principles: “To Moloch this Altar is dedicated by a Sacred Victim.” He is working now on matter and motion, i.e. Part VII. Still there is no premonition of how soon he will finish. The next day he is more interested in the effort with which “I flog my poor intellect into activity.” He adds: “I have been going over parts of my book which I thought would have to be re-written, and have persuaded myself that they require very little alteration, so that I shall finish it very soon indeed” (23 May). He notes that since their separation he has not read 100 pages of print. He does finish the manuscript that day, writing then to Lucy Martin Donnelly (quoted in the Autobiography, 1:163–4) and the next day to Alys: “Thee will be surprised and amused, after all my talk of 2 months, to hear that I finished my book yesterday. I found that a pile of old MS, which I had expected to have to re-write, required only a few additions and corrections, so I arrived at a sudden termination. I have never known or even imagined such a relief as I have been feeling.” He hopes his book will be out in October.

Proofreading. Russell mentions that he wrote the previous day to R. T. Wright. Wright was the renowned Secretary of the Cambridge University Press. Although no correspondence between Russell and Wright before 1909 survives, that is no indication of the frequency of their contact. As John G. Slater has suggested to me, Russell—being so often in Cambridge—probably carried out the bulk of his dealings with the Press in person. Wright responded rapidly to Russell’s overture regarding his manuscript. On the 25th Russell thinks he will take it to the Press the next Tuesday or Wednesday (the 27th and 28th) for the Press tomorrow, but in fact he takes the bulk of it that day and the rest the next. Wright tells him on the 28th that “they would begin printing at once.” Thus Wright took exactly one day to evaluate and accept the work (though the Syndics probably had to give formal acceptance at a regular meeting later). On the 29th the Whiteheads dined at the Wrights’. Wright must have known all about the progress of Volume II as well. For some days Russell works on an unidentified paper by Whitehead, looking forward to “lots of proof-correcting soon—the pleasantest and easiest work there is” (1 June). It turned out to be not so easy in the case of the Principles.

The next stage was production of a specimen page. Russell has to go twice to the Press on Monday the 2nd of June, and he notes to Alys on the 3rd that “the Press have sent me a specimen page, but have not got beyond that yet.” The reason, he tells Beatrice Webb on the 4th, is that the Press “are full of exam papers just now.” Presumably Russell returned the page straightforwardly, for on the 5th he tells Alys, who wanted him to bring it to their first meeting in nearly two months, that the page is returned the page straightaway, for on the 5th he tells Alys, who wanted him to bring it to their first meeting in nearly two months, that the page is back with the Press. The page does not survive, but we know what portion of the text was on it: folios 3–4, or pages 4:5–5:4 (end of line), are marked “Spec. page”. As is characteristic of the rest of the manuscript, there is no copy-editing of Russell’s prose; on these leaves there is not even any marking up for the compositor. Cambridge at this time (and since) was outstanding for its production of mathematical texts, Russell had already published The Foundations of Geometry and The Philosophy of Leibniz through Cambridge, and my hunch is that he found nothing to improve in the specimen page. Further evidence is the fact that the text marked for this purpose occupies exactly the equivalent of a page in the final printed version.

There is no more news of his book until the 18th, when the first proofs arrive. He takes a ten-day holiday in Cambridge, talking shop with Moore and others, then returns to Friday’s Hill. He is full of plans for work: “I shall have to read a lot of casual shop first, then polish up my lectures so as to be ready for publication, and then I mean to undertake a systematic study of the great philosophers preparatory to my Logic, which is to begin by an exposition and criticism of all previous logics of any importance” (12 June). Again we have a reference to the lost text of the lectures which were to go into Volume II. These plans for future work were to be delayed, however. The next day he is reading Boole’s Laws of Thought and on the 15th reports: “I am working very hard, reading the table of contents. On the 27th he says he will take the manuscript to the Press tomorrow, but in fact he takes the bulk of it that day and the rest the next. The text of The Principles of Mathematics 281

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19 See M. H. Black, Cambridge University Press, 1584–1984 (Cambridge, U.K.: Cambridge U.P., 1984), Chap. 11, esp. pp. 174, 179 and 186 for Wright’s duties. Black describes the process of acceptance by the Syndics in Russell’s time in his “Evolution at Cambridge”, Scholarly Publishing, 16 (Oct. 1984): 52; reprinted from The Cambridge Review, May 1984. Russell acknowledges Wright in the Preface (p. xix)—a rare recognition of publisher’s staff on Russell’s part. In June 1909 Wright asked Russell if the book should be reprinted in its present form, the last fifty copies having been sent to the bindery; there were no stereotype plates. In April 1910 Russell requested and received a new copy: “I have only one copy at present, and it is too much annotated to be any use for that purpose [that of a second edition].” Neither copy is known to survive. Russell’s letters to C. K. Ogden in 1922 show him selling Ogden a copy of the Principles (and two of Principia); but because of the concern over one of the latter being marked up, it is unlikely an annotated copy of the Principles was involved.

20 The analytical table of contents occupies eighteen pages. The manuscript is lost.
literature of my subject, now that I have finished writing about it. I can insert learned footnotes in the Proofs.” That is exactly what he did—inserting thirteen footnotes in the proofs of Part I alone. His new work binge continues, confessing on the 17th from Friday’s Hill: “Life to me is wholly unemotional and dry at present: Formal Logic fills the crannies of my brain.” While deep in his reading of the subject—he is now “full of Frege”—the first proofs arrive; his reaction is unemotional: “I know, in a kind of impersonal way, that it is a good book, but it gives me no satisfaction at all” (18 June). Russell and Alys meet on the 21st. On the 22nd more proofs come, and he works five hours. On the 25th he tells Couturat that the volume will probably appear in the winter and amount to about 500 pages. But the typesetting does not proceed as rapidly as Russell thought it would. He tells Dickinson on 2 August that “The proofs come occasionally, and seem to me very worthless ...” (Autobiography, 1: 184); and on 29 September he notes to Couturat that while he is very much occupied by the book, it is being printed slowly. The slow progress may have been connected with Russell’s substantial revisions. There are no more references to proofs in these letters until 5 February 1903. It is, therefore, impossible to tell how quickly Cambridge sent him proofs—perhaps a sheet (or signature) at a time. The Principles being an octavo volume, that was sixteen pages. There are, however, several references in letters to working with Whitehead. That work would include the proofs, since Whitehead read them (Preface, p. xviii, and Russell to Helen Thomas, 16 Sept. 1902). It is probable Russell received page proofs from the start. He had done so with the Leibniz. The printer’s copy of the manuscript is marked not only for compositors’ stints and footnotes, but also running heads and signature lines. For example the beginning of Chapter VI, folio 127, has a draft for the running head starting on page 65. Yet VI does not begin in the printed book until page 66. This error can be explained by the fact that Russell made enough additions to the text of VI in page proof to enlarge it by one page (see the variants for pp. 49–64; p. 64 has only fifteen lines).

The Preface was not written until 2 December, according to Russell’s journal, and the Press received it on the 10th. On 5 February Russell tells Alys that there will be more proofs, but only one or two sheets. On the 10th he is indexing the volume, meaning that he has final page proofs by then. The Principles of Mathematics was published in May 1903, Jourdain acknowledging a copy on the 10th. After telling Helen Thomas on the 13th that his book is out at last, Russell deprecates the achievement: “It seems to me a foolish book, and I am ashamed to think that I have spent the best part of six years upon it.”

Frege’s impact. Frege’s initial impact on Russell can be determined from this analysis and the list of variants. The first mention to Alys of reading Frege was on 18 June. Russell has, in fact, written Frege on the 16th to ask for offprints of his articles and to relate the difficulty he has discovered, i.e. the Contradiction. The manuscript of the Principles has been with the Press since 27 May. Frege responds on the 22nd: “Your discovery of the contradiction has surprised me beyond words and, I should almost like to say, left me thunderstruck, because it has rocked the ground on which I meant to build arithmetic.” Russell tells Alys of this in a letter written early on the 25th: “I have heard from Frege, a most candid letter: he says that my conundrum makes not only his Arithmetic, but all possible Arithmetics, totter.” The same day he praises the Grundgesetze to Couturat for containing many ideas Russell thought he had discovered. These are not, however, quite the first responses by Russell. He not only replied to Frege the previous day (the 24th) but must then have sent the Press the three-page insertion for the end of §103 in Chapter X. (The Press date-stamped it the 25th and mails were very fast, but Russell is in Haslemere at the time.) The insertion contains what may be the earliest reference to the appendix on Frege. However, this reference was deleted in revising proofs of the chapter. To judge the full text of the insertion, the reader must first find page 104, lines 6–31 in the text. He must realize that the preceding seven lines in the paragraph replaced in proof a much longer passage now found in the list of variants at 103: 44–104: 5. Finally, he must add to part of the insertion retained in the book the long passage deleted in proof at 104: 31. The end of this deletion contains the footnote: “The above discussion is largely influenced by Frege: see Appendix.” Russell’s letter to Frege of 24 June should be read in conjunction with these passages. The effect of the proof revisions was to reduce greatly the discussion of “logical imaginaries”.

We do not know when Russell received proof of sheet (or signature) 7, which included Chapter x. Nor do we know if he made all of the proof revisions on the first page proofs. We can, however, be fairly certain that plans for the appendix on Frege predated the receipt of Frege’s letter. Russell had been deeply impressed by Frege’s work in the previous

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The text of The Principles of Mathematics

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21 The only other author mentioned in the correspondence is Meinong (in a letter to Couturat). The Principles has no bibliography, but Albert C. Lewis of the Russell Editorial Project has compiled a checklist of works referred to in the book. Expansion to make the checklist an index and collation with the final manuscript would show what other references were added in proof.


24 The reference to the Grundgesetze at 10n. was not added in proof but rather is in the final manuscript. (So is the unindexed reference to Frege at 35n.) Russell provides an explanation in the Preface, p. xvi.
couple of weeks and in his first letter he asked for copies of Frege's minor works, because "I should like to discuss your work in [the Principles] in great detail." On 7 July he tells Frege that he will discuss his work in an appendix "because it is now too late to talk about it in detail in the text." The appendix, however, takes Russell a long time to finish; the Press does not receive it until 15 November, and Russell does not read proofs until February or so. Because of the latter hiatus, and the important correspondence on the Contradiction that he continues with Frege over the winter, there may have been substantial revisions to both this appendix and the one on the theory of types. They await collation.

Conclusion. The levels of revision in the Principles are many and various. The minimum tool needed by scholars is a complete list of the variants, and others are invited to contribute to the compilation. Secondly, a list of the alterations in the final manuscript is desirable. Because of the reliability of the printed text, so far as it has been examined here, a reset, variorum edition is not an immediate desideratum for textual reasons alone. Instead, we need an edition of the penultimate draft of Part I complete with its prior alterations in the manuscript. Some way should be devised of illustrating the alterations a year later on the leaves transferred to the final manuscript. This should not become beyond editorial ingenuity. When these tools are to hand, then scholars will be equipped to trace the effect (say) on Russell's early theory of denoting of his efforts to resolve the Contradiction.

The Bertrand Russell Archives

VARIANTS BETWEEN The Principles of Mathematics AND ITS MS.

PREFACE
xvi: 13 as an
xvi: 32 Appendices, which Appendices. These
xvi: 39 began an investigation into a form of the intention of writing on
xvii: 31 is one which is
xvii: 32 mathematics, and therefore to the present work.] mathematics
xvi: 7 are] are, at least in part,
  Georg Cantor] Professor Georg Cantor
xix: 24–34 vague. [For ... doctrine.]
  vague.
  xiv: 37 volume.] volume, and still more for undertaking the necessity troublesome task of printing the second volume.

CHAP. I DEFINITION OF PURE MATHEMATICS

7: 5 Plato] Plato: (Added fn. indicators are not noted any further.)
  7: 43–5 (fn. added)
  8: 29 S [If S (Cf. MS., fol. 20, l. 4.)
  9: 8 mathematics (§1.) mathematics.
  10: 13 attained] achieved
  11: 45 xxxvi]
  12: 6 duality] duality

CHAP. II SYMBOLIC LOGIC
12: 39 On] On the two-fold interpretation of logical formulae, Cf. Whitehead, Universal Algebra, Cambridge, 1898, Book II, Chaps. IV and V, where the limitations appear to be unperceived; on
  13: 28–30 hypothesis and ... to] hypothesis
  14: 12 if the fourth] if the former
  14: 13 fifth ... fifth ... fourth] latter ... latter ... former

15: 44 state] point out
  15: 45 exclusive unless ... so.] exclusive.
  16: 43–5 (fn. added)
  17: then 'p implies q' implies 'p' implies 'p' and 'p implies q' implies 'p', then 'p' is true.
  17: 29] or
  18: 24 BJ II.
  19: 31–2 as fundamental the notion of 'a class, a class as a fundamental notion,
  19: 42 verb or adjective] verb or adjective]
  19: 44 Halle, 1879] 1879
  20: 32 on] in (Missprint introduced in 1937.)
  20: 37 prime and] prime and (Missprint introduced in 1937.)
  20: 43–5 (fn. added)
  21: 5 classes, in some form] classes
  21: 17 classes (a or b) classes
  21: 17 class (not-a.) classes
  21: 32–3 is true ... x.] implied by "x is an " whatever x may be is true.
  23: 22 C. J. III.
  23: 35 C. S. Peirce] Peirce (Also at 23: 43.)
  23: 41 p. 104 ... 50.] p. 104.
  24: 23 primitive (i.e. indemonstrable])
  24: 41 R, i.e. ... relations:] R;
  26: 5 D. J. III.
  26: 42–3 (fn. added)
  28: 4 propositions (see §34, (3).) propositions
  28: 7–9 the ... indefinable] a highly artificial definition of the logical definition of the propositional function of two propositions is possible; but this definition is almost worthless
  28: 10 property] property of the logical product
  28: 10 the definition.] it.
  29: 15 for each propositional function] in fact
  29: 16 propositions and] a class of propositions
  29: 16 entities] a class of entities
  29: 18 entities] terms of the class of entities
  29: 32 various propositions of given form] a class of propositions
  29: 33 them ... variable] the propositions of the class
  30: 6 identity] contradiction
  30: 16 BJ II
  30: 36 or of propositions or of] or propositions
  31: 1 c] (Missprint: deleted in MS.)
  32: 19 as] as really

CHAP. III IMPLICATION AND FORMAL IMPLICATION
34: 24 C. J. III
34: 44 those rules of inference] the rules of inference, all of
36: 11–12 "for ... x."] "x is a k' implies x for all values of x.'
36: 14 equivalent, if q be a proposition,] equivalent
37: 11 this is not] this, we said, is
37: 12 For as] As
37: 13 meant, since] meant. For
37: 14–15 to propositions, ... x.] at least to propositions.
37: 20 proposition] propositional function
37: 27 noticed] observed
37: 42–3 "if x implies x, then 'x is a k' implies x'] 'x implies x' implies that 'x is a k implies x' (no closing single quote)

CHAP. IV PROPER NAMES, ADJECTIVES, AND VERBS
42: 30 concept] idea (Also at 42: 31.)
43: 10–11 proposition or ... some subordinate proposition] proposition
47: 45 (fn. added)

CHAP. V DENOTING
54: 17–18 classes, but ... class-concept.] classes.
54: 42–55 I differs little, if at all, from] is
54: 43 (fn. added)
54: 44–8 § 55: 36–9 (fn. added)
55: 40–2 (fn. added)
56: 3–4 class-concept, though the distinction is perhaps only verbal.] class-concept.
56: 4–5 class-concept, as distinguished from terms in general.] class-concept
56: 5 § 4 and so-on-and-so (Also at 56: 6 and 56: 7.)
56: 6 propositional function] proposition for all values of x
56: 8 all, whatever value we may give to x.] at all.
56: 13 proposition] propositions (Error in MS.)
56: 13 a] the term so-and-so
58: 24 combined in the specified manner.] in various ways.
60: 4 some (or an) ] (Also at 61: 3, 61: 5 and 61: 7.)
60: 35 twenty] sixteen
61: 16–18 have a common part.] (Thus cases (12)–(17) have been renumbered from (11)–(16) in MS.)
The text of The Principles of Mathematics

CHAP. VI CLASSES
16: 14 former] latter
16: 7 is unambiguously] is
17: 18 phraseology] language
17: 37 manifold, any, aggregate] manifold,
68: 39 object] entity
68: 44 (fn. added]
69: 43–4 (fn. added]
70: 39 held] held, I think,
70: 43 Leipzig, 1854 (2nd ed., Berlin,
1889), §3:] §3:
72: 10 are] is
72: 11 are] (bis]
76: 19–35 *There ... race.]* As a first step,
it is desirable to distinguish, in this con-
nection, a logical and an arithmetical
sense of one. Logically, one applies to
every logical subject, i.e., to every term,
and it must be beld that a class is a term as
well as a combination of terms. Whatever
can occur in a proposition is one in the
logical sense, and in this sense one seems
to be not opposed to many. But the
arithmetical one, is, at least apparently,
quite different: it is to be regarded as a
property of any class-concept a which is
such that, if a is a, and y differs from x,
then, for all values of y, x is not a. It is
necessary here to introduce the class-
concept rather than the class, in order to
explain how the class can be also
arithmetically one in a certain sense:
A class-concept which denotes all the
terms of a class is different from a class-concept
which denotes the class itself. Does this
raise logical obstacles to the above theory
of classes? The point is curious and im-
portant, and must be examined fully. It
appears, from what was said above, that
a class considered as a single term must be
a different entity from the same class con-
sidered as many. But this view will lead,
one might suppose, to the same con-
tradiction which resulted from the at-
tempt to distinguish concepts used as
such from concepts used as terms, e.g., is
and Being. A combination of terms, of
whatever kind, we are tempted to say,
must be itself a term, if it is in any way
possible for it to occur in propositions.
Yet if the class as one term is identical
with the class as many, it would seem that
a class-concept which denotes the one
denotes the other. But man is a dif-
ferent concept from class of all rational
animals; "x is a man" is a propositional
function satisfied by each individual
man, while "x is a class of all rational
animals" is a propositional function
satisfied by only one value, namely the
class men. Substituting the concepts of
classes for the class-concepts, men
denotes the class taken term by term, i.e.
the class as many, while classes of all ra-
tional animals denotes precisely the same
class taken as a single term. Hence, it
would seem, the distinction drawn by
Peano, between a term and a class of
which the term in question is the only
member, must be maintained, at least
when the term in question is a class. Or
rather, we must make an immediate distinc-
tion between a class as many and a
class as one, we must hold that the many
are only many, and are not also one. The
class as one may be identified with the
whole composed of the terms of the class,
I.e., in the case of men, the class as one
will be the human race. In the class sene-
of one, the logical and the arithmetical,
though they remain distinct, will now go
hand in hand, at least if, in the case of the
arithmetical one, we restore classes where
class-concepts are mentioned.
76: 36 always to be] which
76: 37 when there is] the contradiction of
76: 43–4 (fn. added]
80: 5 is evident] in evident (Error in MS.)
80: 30–81:5 points, etc. Starting ... many.] points, etc.
81: 6 class-concepts. class-concepts. We also
found that classes, except when they
consist of single terms, are essentially
many, and that a class considered as one
entity is something different from the
same class as many, being in fact the
whole composed of all the terms of the
class together. We discussed finally vari-
ous relations connected with that of a
term to a class of which it is a member,
and with inclusion.

CHAP. VII PROPOSITIONAL FUNCTIONS
86: 44–57: 14 In ... this] This
87: 15 aRb] the other
87: 21 which] by the way, which
The above case, where it is the concept in a
proposition, and not the term, that is to be
varied, seems finally to dispose of the
view that propositional functions can be
analyzed into variable terms and fixed
assertions; for "a ... b" is certainly not an
assertion about R in the proposition aRb.
Of course it may be said that, before vary-
ing R, we ought to substitute for aRb
the equivalent proposition "R is a relation
holding from a to b". But there is no gain
in this, for "holding from a to b" intro-
duces precisely that assertion "a ... b"
which was to be avoided.
87: 43–5 (fn. added]
88: 1 "R ... b"] the above

CHAP. VIII THE VARIABLE
89: 22 is any] is any (Misprint for in any?)

CHAP. IX RELATIONS
95: 31 XXX, §203] II
95: 32 description] definition
98: 8 as of which it is predictable.] predictable of it.
98: 24 of which] such that
98: 44–5 ways of combination] ways
99: 20 terms, §§46, 54] terms
99: 33 III] (Blank left in MS.)
100: 3 formerly] (§54] formerly

CHAP. X THE CONTRADICTION
101: 9 very plausible supposition] obvious
fact
101: 11 members] terms
101: 14 Then] (a if) If "a" is new style Greek for "a".
101: 14 v, then,
101: 17 (β] if
101: 19 (y] if
101: 20 members of] classes contained in
101: 21 contained in itself] not a u
101: 22 none ... is] therefore not predicable
of themselves
101: 23–5 itself, Thus ... not] itself.
101: 25 (β] if
101: 29 (those class-concepts] a class-concept
102: 32 proved in (β] proved
103: 34 XXIII, §§34 ff.] XII.

102: 38 them and no other terms] them
103: 4 either] it.
104: 5 the former] it
105: 7–23 begin. § Any ... types] begin.
106: 4–104: § 1] The proposi-
tional function which leads to the
difficulty may be expressed in several
equivalent forms. But the above form
suggests a possibility of solution on the
same lines as the solution of an analogous
contradiction discussed in Chapter VII.
When e is replaced by a variable proposi-
tional function, we have to consider all
propositional functions not satisfied by
the class of their own roots—meaning by
the roots of a propositional function, as in
the particular case of an equation, the
values satisfying it. It may be suggested
that we are here introducing a proposi-
tional function which involves treating
assertions as separable and analyzable
entities. If \( \phi \) be a variable propositional
function, the propositional function
suggested is the denial of \( \phi(k_\alpha) \) where \( k_\alpha \)
denotes, for the moment, the class of
roots of \( \phi \) considered as a single term.
Now in \( \phi(k_\alpha) \) the variable is \( \phi \); thus we
necessarily treat the assertion \( \phi \) as separable.
This we saw before to be improper,
and thus the contradiction would appear
to be resolved. We may say generally that
a variable assertion, or a variable proposi-
tional function, is not to be admitted.
Without caution, the class of any
function is not in itself an entity, but is either
a class of propositions, or any member of
this class. When it is itself varied, it be-
comes necessary to treat it as an entity
distinct from either of these, and this is
illegitimate. Or again, when \( \phi \) in \( \phi \) is varied,
this becomes necessary to treat \( \phi \) as variable:
it will not do to vary \( \phi \) as a whole, for that would give us all proposi-
tions, not all propositional functions.
Thus it would seem that "x is not a x" is
a proposition for every value of x, but is
not a propositional function when x is
variable. This result is only intelligible
by adhering strictly to the dependence of
classes upon propositional functions. It
has the result that, if any term which
occurs in a proposition is essentially a
class, we must not vary that term in the
proposition as it stands, but must first
add some hypothesis which removes the
necessity for regarding our term as a
class. It has also the result that $\varepsilon$ is not to
be taken as a fundamental notion, but is

to be strictly regarded as derivative from
propositional functions by the help of the
notion of such that.
104: 6–31 Moreover ... degree.] These
lines plus the passage subsequently omitted
at 104: 31 were received by the Press on 25
June 1902, with the note at the top of fol. 1
of the MS.: “[To be added at the end of
§103].”

104: 6 Moreover] At the same time,
104: 13 two kinds of] legitimate from il-
legitimate.
104: 18 we] that give rise to contradictions
of the type we
104: 25 \( \phi(f(\phi)) \) “\( x \) is an \( x \)” and “\( x \) is not
an \( x \)”
104: 26 it is] they are
104: 26 proposition] propositions
104: 26–7 is not a function] are not func-
tions
104: 31 degree.] degree; and it would seem
that logical imaginaries arise from logi-
cally quadratic forms in a way which
heightens the analogy. If such forms are,
as would appear to be the case, illegiti-
mate, we may conclude as follows: A
propositional function occurring in an
expression may be regarded as variable
provided, when a constant value which is
a propositional function is assigned to the
variable, the expression in question be-
comes a propositional function, or be-
comes a proposition concerning a con-
tant term which is independent of the
value assigned to the variable proposi-
tional function”. (“This case arises, for
example, in considering \( \phi(2) \), the class of
all propositions that can be made con-
cerning the number 2.) But if both asser-
tion and argument become different
when a different value is assigned to the
variable propositional function, then we
have a quadratic form, and contradic-
tions are liable to arise. We may, in a
word, consider various assertions about a
given subject, or the same assertion about
various subjects; but in a quadratic form,
we try to consider a variable assertion
about what is asserted, and here no
sufficient definiteness remains. [The
above discussion is largely influenced by
Frege: see Appendix.]
104: 37 in the original propositional func-
tion.] and thus treated as a single term.