Textual studies

Part I of *The Principles of Mathematics*

by Kenneth Blackwell

My book keeps on going through the Press: I do wish books could be published without one's having to see any more of them—it is really disgusting returning to one's own vomit. (Russell to G. Lowes Dickinson, 11 July 1902)

THIS STUDY CONCERNS the composition of the final version of Part I of *The Principles of Mathematics*.[1] First I describe the available documents and then the plans for uncovering the hidden layer of revision in proof. The results of collating the final manuscript and the printed text lead to a reconstruction of the penultimate, pre-paradox draft of Part I. Russell had the greatest difficulty in rewriting that Part and was stuck for a year. When he became unstuck in May 1902 he narrated his progress to his wife and other correspondents. There was much work to come, however, in revising the proofs over a period of nine months. At the beginning of this period he contacts Frege, and I provide the bibliographical and textual foundation for the study of Frege's immediate impact while Russell continued trying to resolve the Contradiction. Some conclusions follow about scholarly requirements for the text of the *Principles*.

The documents. To be precise, there is not one text of the *Principles*, but several similar texts. The discovery of their differences is the subject

[1] The publication of this study fulfils a hope expressed in the first issue of *Russell* (Spring 1971): 12. A draft has been available in the Russell Archives since that time, and many researchers have made use of it in beginning their own investigations of the text of the *Principles*. In particular I. Grattan-Guinness has encouraged the final preparation of the draft for publication. The only other published study involving the *Principles*' text is J. Alberto Coffa, "The Humble Origins of Russell's Paradox", *Russell*, nos. 33–34 (Spring–Summer 1979): 31–7. A recent exegetical paper on a portion of Part I is Nicholas Griffin and Gad Zak, "Russell on Specific and Universal Relations: *The Principles of Mathematics*, §55", *History and Philosophy of Logic*, 3 (1982): 55–67. The subject of the manuscripts of the *Principles* was first discussed in my brief description of them in Barry Feinberg, ed., *A Detailed Catalogue of the Archives of Bertrand Russell* (London: Continuum I, 1967), p. 98. Alejandro Garciadiego's doctoral dissertation, "Bertrand Russell and the Origin of the Set-Theoretic Paradoxes" (University of Toronto, 1983), has a lengthy appendix devoted to "A Biobibliographical Note on *The Principles of Mathematics*". Griffin has been subjecting these manuscripts to extensive scrutiny for *The Col-*

of this paper. The final manuscript is available in the Russell Archives (file 230.030350 FI–20). It comprises over 900 leaves, mostly handwritten, and lacks only a small number, notably some chapters and sections in Parts II and VI. As printers' markings appear frequently on the manuscript, there is no doubt that the typesetting of the book was done from this manuscript, and not from any intervening typescript. The *Catalogue* of the first Archives mentions (pp. 98–9) twenty leaves of an earlier draft of Part I, and several longer attempts to write on the subject—the latter all predating Russell's encounter with the work of Peano in August 1900. The printed text in its first and only typesetting has had nine printings to date. The first edition may be described as follows.

Title: THE PRINCIPLES | OF | MATHEMATICS | BY | BERTRAND RUSSELL M.A., | LATE FELLOW OF TRINITY COLLEGE, CAMBRIDGE | VOL I. | CAMBRIDGE: | at the University Press | 1903

Size, collation and pagination: 239 × 162 mm. $a^8b^8$1–33^{8}34^4 [$2 signed (−34$_2$)]; 284 leaves. [*2*], *i–v* vi–ix *x–xi* xii–xxix *xxx*, *1–3 10 33 42 53 66 82 89 95 101 108–111 117 121 124 129 137 143 149 154–157 170 176 184 188 197–199 207 218 227 234 239 245 252 257–259 270 276 287 296 304 312 325 331 338 346 355 369–371 381 393 404 419 429 437 445 456 462–465 469 474 480 482 489 494 499–501 523 529 535–536*. [Because of space, only inferred arabic page numbers are reported.]

Bibliographical contents: [*1–2*] blank *i* half-title *ii* publisher's addresses *iii* title *iv* Cambridge: | PRINTED BY J. AND C. F. CLAY, | AT THE UNIVERSITY PRESS. *v*–ix PREFACE. [dated London, Dec. 1902] *x* blank *xi*–xxix TABLE OF CONTENTS *xxx* blank *1*–528 text *529–534* INDEX *534* printer's notice below rule *535–536* blank.

Type, paper and binding: Set in Modern type. Printed on medium weight, cream-coloured, wove paper, endpapers same and overfolded and sewn in the first and last gatherings; edges untrimmed. Bound in dark blue, fine grain cloth. Double rule box blind-stamped close to edges on both boards. Stamped in gilt across spine: THE | PRINCIPLES | OF | MATHEMATICS | [17 mm. rule] | *RUSSELL* | VOL. I | [3-point publisher's crest] | CAMBRIDGE | UNIVERSITY PRESS . Full-width double rules blind-stamped across top and bottom of spine.

Published: May 1903 at 12*s.* 6*d.*, and distributed from 27 June 1903 in U.S. by Macmillan at $3.50; number of copies, 1,000.

For the second "edition" (i.e. impression) of 1937, besides adding a new introduction, Russell made several corrections, and more were made in at least one later printing.[2] Two of the original corrections involved a

lected Papers of Bertrand Russell, where they are to appear in Volumes 2 and 3. To this end John King has compiled a detailed account of the migration of leaves from one draft to another in "A Report on the Manuscripts for 'An Analysis of Mathematical Reasoning', 'The Fundamental Ideas and Axioms of Mathematics' and 'The Principles of Mathematics [1899–1900]'" (typescript, Russell Editorial Project, 1984). I wish to thank Bernd Frohmann, Nick Griffin, John King, Albert C. Lewis, John G. Slater, Carl Spadoni and Sheila Turcon for reading this study in revised form.

[2] See his correspondence with George Allen & Unwin during 1937; also the note in *Russell*, no. 1 (Spring 1971): 12. Later corrections first appear in the fourth impresion (1948) in the "Introduction to the Second Edition", following Russell's letter to Unwin of 17 July 1947. There are still misprints in the Introduction (at x: 44 and xii: 32).

cancelled leaf in a second state of the second impression; in correcting them, further errors were introduced.[3] Russell's own copy (formerly Wittgenstein's) of the first edition has two corrections in his hand.[4] In an article Russell corrected a misprint, which, however, has never been corrected in the book.[5] Finally, there is the American issue (1938) of the second impression and its reprintings; but as the sheets were imported from the British publisher, they need not be taken into account. No proofs are known to survive.

To those interested in the genesis of one of Russell's most outstanding works, these various versions of the text provide the material to work on. From their comparison not only can the changes Russell made be examined, but a few thousand words of his thought on the philosophy of mathematics can be recovered. The long passages that Russell inserted or replaced in proofreading the *Principles* could be as valuable as a newly discovered manuscript. Some provide evidence of Frege's immediate impact on Russell. Others are the correction of mistakes or inconsistencies. Yet the mistakes of a great thinker can be studied with profit and may help to explain how he came to his published views.

Methodology. I collated Part I, "The Indefinables of Mathematics", of the printer's copy with Part I of the ninth printing (1972) of the *Principles*. The variants are noted on the appended list. I have also reconstructed and recovered much of the previous draft of Part I—one of the Parts Russell tells us were rewritten after 31 December 1900, when he finished the first draft.[6] I have not tried to relate the earlier manuscripts of a book on the same subject, which textually are very different. (As Garciadiego and King point out, there are exceptions to this generalization.) The ninth printing was chosen because it contains any revisions

[3] The sixth leaf (pp. 19–20) of the fourth signature was cancelled in some copies, including that in Russell's library. Page 20 was reset. The purpose was to correct the "*e*" in line 21 to "ϵ" and the "*v*" in line 32 to "*u*". On the latter line, "in" became "on"; "class-concepts" lost its customary hyphen three lines below; and "and" in "*prime* and *integer*" was italicized two lines below that. Possibly Russell was not given proof of the cancellans before it was tipped in.

[4] The corrections concern "ξ" for "ζ" at 422: 26 and "Jermelo" in the Index. They were made in 1937. There are also five small blue feathers between pp. 274–5. It is not known whether Wittgenstein or Russell placed them there!

[5] "P. 379, Ligne 27, 'real' est une faute d'impression, qui doit être remplacée par 'complex'." From "Sur la Relation des mathématiques à la logistique", *Revue de métaphysique et de morale*, 13 (Nov. 1905): 913n.; trans. in Russell, *Essays in Analysis*, ed. Douglas Lackey (London: Allen & Unwin, 1973), p. 267n.

[6] *My Philosophical Development* (London: Allen & Unwin, 1959), p. 73. See also his letter to Helen Thomas (later Flexner) of 31 Dec. 1900, where he boasts of writing 200,000 words recently, and n. 13 below.

made in 1937 and any since then during Russell's lifetime. The list of revisions therefore includes any 1937 revisions, which it would not if the 1903 edition had been collated. However, comparison with the first printing reveals that none of the revisions were introduced in 1937 or later. Any 1937 corrections of first edition misprints back to the manuscript could not be discovered in this way, but that is hardly a loss; and the alternative would be a separate collation of the first and ninth impressions.[7]

The list ignores alterations on the final manuscript itself, and alterations between it and earlier manuscripts. The former are readily apparent from glancing at the manuscript, and can even be differentiated according to the ink employed in making them. For this paper I wished to uncover what was not obvious, i.e. the revisions made in the typesetting and proofreading stages of the first edition and subsequently. Although both kinds are required for the study of how Russell wrote the *Principles*, the revisions made after he finished the manuscript and handed it over to the Cambridge University Press can be usefully regarded separately. After all, during the typesetting and proofreading stages, Russell was probably concerned to keep changes to a minimum because of the waste, expense and delay involved in resetting lines of type already (as we shall see) in pages, and we know he had kept the manuscript by him for seventeen months after finishing the first draft.[8] The main reason for this delay was the discovery of the Contradiction in June 1901.[9]

To collate the manuscript with the printed text, I twice read the former phrase by phrase against the latter. In the terminology of textual criticism, there are two kinds of variants, substantives and accidentals. Accidentals are changes in spelling, capitalization, hyphenation, abbreviation, punctuation, paragraphing and italicization—changes that rarely affect the sense. Substantives are the words and their order. Some of these do not affect the philosophical sense of the passages changed, but the division of substantives into those of possible philosophical interest and those not is a very subjective enterprise, and there is a stylistic interest in Russell's non-philosophical rewording. I have not thought it worthwhile to report even the non-mechanical variants among the accidentals.[10] The list is available for consultation in the Archives.

[7] This can be done with the Hinman Collator, which superimposes the image of a page from one copy of a book upon the image of the same page from another copy of the same typesetting of the book. The Russell Editorial Project possesses a Hinman. However, the task is extremely tedious, if not impossible, when a photo-offset impression of the original has resulted in the contraction or expansion of the lines of type. This situation obtains, unfortunately, when the ninth impression of the *Principles* is machined against the first.

[8] The last leaf of the manuscript is dated 23 May 1902. There are, however, three leaves following it date-stamped 27 Jan. 1903 and containing §474. This shows that Russell increased by four the number of sections he had had in May.

[9] "Whitehead and *Principia Mathematica*", *Mind*, 57 (April 1948): 137.

The list is read as follows. At the left is a number such as 13: 28–30. This means page 13, lines 28–30 from the top. To the right there is first the reading from the printed text as it appears in the ninth impression, followed by a square bracket. The words after the bracket are the manuscript reading—i.e. the passage that was replaced by the words before the bracket. Editorial brackets enclose my comments. The reader should now take up his copy of the *Principles* and locate some of these changes. In two hours all the substantives in Part I can be located.

Results of the collation. The most emended chapters were found to be Chapter II, "Symbolic Logic", V, "Denoting", and X, "The Contradiction". The least emended were I, IV, VIII and IX. The variants comprise about 1,900 words in passages which do not appear in the printed text, some of them several hundred words long. The list also reveals the considerable passages that were inserted at the proof stage. The majority of the words which were replaced occur in Chapter X—after Russell began to correspond with Frege on the Contradiction. Although the corrected proofs are not extant, it would be madness to maintain that anyone but the author made the vast bulk of these revisions.

The manuscript of Part I is a combination of freshly written material and material taken over from the last time Russell had attempted to write the crucial—and most philosophical—opening chapters. The first folio is dated "[April [May 1902]" with "[April" lined through. Thirty-six of the 217 leaves bear a marking allowing them to be dated before that time. It was Russell's habit to jot in the upper left-hand corner an initial representing the Part to which the leaf belonged. Whereas all the leaves are initialled "I.M.", standing for "The Indefinables of Mathematics", thirty-six are also initialled "V." That stands for "The Variable", the title of Part I in a table of contents from the previous year.

Most of the penultimate draft of Part I can be reconstructed. The twenty leaves mentioned in the *Catalogue* include the table of contents and fifteen leaves of text foliated between 1 and 36. The first leaf is dated May 1901. All but four have the symbol "V." in the upper left-hand corner. Folios 135–6 (formerly 44–5) of the final manuscript have "Nc" in the corner, indicating they are taken from Part II, all of whose pages bear that symbol (Part II in the final manuscript is dated June 1901). Two other leaves, 137–8, formerly 48–9 and before that 4–5, have "N." in the

[10] For the distinction between mechanical and non-mechanical accidentals, see "Textual Principles and Methods" in *Cambridge Essays, 1888–99, The Collected Papers of Bertrand Russell*, Vol. 1 (London and Boston: Allen & Unwin, 1983): 447. See also my "'Perhaps You Will Think Me Fussy': Three Myths in Editing Russell's *Collected Papers*", in H. J. Jackson, ed., *Editing Polymaths: Erasmus to Russell* (Toronto: Committee for the Conference on Editorial Problems, 1983).

corner.[11] Four others are the first leaves of chapters, and the chapter titles match those listed in this table of contents. Most of the missing leaves can be located in the final manuscript, and one (fol. 105) has the title of another chapter as listed on this table of contents. Eight leaves were transcribed, with some changes, for the final manuscript. In the following reconstruction, the first two columns list the chapter titles of the penultimate manuscript and their folio numbers. Their location, if any, in the final manuscript is then noted, rewritten leaves appearing in both columns. The last column gives the printed page and chapter references.

Chapter Title on Penultimate Manuscript & Contents Table	*Folio Nos.*	*Folio Nos. of Final MS.*	*Final P. & Chap. Nos.*
I. The Definition of Pure Mathematics	1–3		
	4–7	10–14 (rewritten)	7–9 (I)
II. Terms and Concepts	8	84 (rewritten)	43 (IV)
	9	85	43–4 (IV)
	10–12		
	13–15	89–91	45–7 (IV)
III. Classes and Relations	16–17	105–6	54–5 (V)
	18–23	185–90	95–8 (IX)
IV. Conjunction and Disjunction	24–5	107 (rewritten)	55–7 (V)
	26–34	111–19	57–61 (V)
V. The Variable	35		
	36	172–3 (rewritten)	90 (VIII)
	37	174	90–1 (VIII)
VI. ["Implication" begins about here]	38–43 [missing]		
	44–9	135–8	70–2 (VI)
	50–7 [missing]		
VII. ["Peano's Symbolic Logic" begins about here]	58–63	53–8	26–9 (II)
	64 [missing]		
	65–8	59–62	29–31 (II)
VIII. ["Summary of Part I" begins about here]	69–? [missing]		

Russell's use, in the final manuscript, of pages from a previous manuscript is typical of his method of composition and revision.

The penultimate manuscript was written before the Contradiction was discovered. There is no separate chapter on the topic, indicating at the least that this manuscript predates Russell's realization of the Contradiction's importance. Moreover, the penultimate draft is dated May 1901 and Russell states that he discovered the Contradiction in June 1901.[12] It should be noted that there is no trace of the draft of Part I written in the fall of 1900,[13] and that there exists a detailed "Plan for Book I: The

[11] This puts their origin in the draft of 1899–1900, Book I, Chap. I. See King, p. 9.
[12] "Whitehead and *Principia Mathematica*", p. 137.
[13] Garciadiego claims that Russell did not write Parts I, II and VII that fall. My chief reservation in this hypothesis is how Russell could have started writing his book in the middle, given the revolution in the fundamentals of his thought since encountering Peano. There is also the question of defective memory, of which Russell cannot be lightly accused (*cf. Autobiography*, Vol. I [London: Allen & Unwin, 1967]: 145).

Variable", dated April 1902 (in RA 230.030350), which the final manuscript follows so closely that only one chapter title is altered: VII, instead of being called "Assertions", becomes "Propositional Functions".

Final composition. The composition of the final version of Part I is another example of the terrific bouts of work of which Russell was capable; compare the writing of *Theory of Knowledge* eleven years later.[14] We do not know how much Russell had worked on the *Principles* since discovering the Contradiction the previous spring. Grattan-Guinness points out that manuscript material is totally lacking in this area.[15] Over the winter Russell gave two terms of lectures on mathematical philosophy and had accumulated so much written material that he could tell Jourdain on 16 March 1902 that he "intend[ed] publishing my lectures together with other material in a joint book with Whitehead, but probably not for two years or so. I also intend publishing a book (for which I have done the bulk of the work) on the philosophical aspects of the subject, and on controversial points."[16] It is ironic that Russell should give prominence here to a work of which almost nothing is extant—namely the work that was intended to be Volume II of the *Principles*—rather than the *Principles* itself, towards which he had laboured since 1897. Barring the appearance of other evidence, we must assume that this is how matters stood in the early spring of 1902. Not only could Russell not see his way to completing Volume I, but on the personal side he has ceased to love Alys and can see no way of saving his marriage. By late April Alys's doctor has ordered her to live separately from Russell for a month, and he himself, as he confesses in almost daily postcards and then letters to her, has been very near a breakdown. The permanent result was the ethic of "The Free Man's Worship".

Russell at length recovers from the brink (cycling and emotional rest are the cures) and turns to his pile of manuscripts. In the first real letter he is allowed to write Alys after she enters her rest-cure, on 30 April from The Mill House, home of the Whiteheads, he remarks that he feels much fitter and goes on:

[14] See the Introduction to *Theory of Knowledge: The 1913 Manuscript*, ed. Elizabeth Ramsden Eames in collaboration with myself, *Collected Papers*, Vol. 7 (London and Boston: Allen & Unwin, 1984): xxii–xxviii.
[15] "How Bertrand Russell Discovered His Paradox", *Historia Mathematica*, 5 (1978): 130.
[16] Grattan-Guinness, *Dear Russell—Dear Jourdain* (London: Duckworth, 1977), p. 16. For more on the lectures at Trinity College and Volume II, see Russell's letters to Louis Couturat, especially that of 2 Oct. 1901, in which he appears to give the title of the volume.

> I have decided that as my big book does not need very much more work to be finished, I must screw myself up to get it done while we are separated, as that will leave me freer to devote myself to thee, and will make me feel that it doesn't matter if I do get a bit tired. I cannot, in the time and in my present condition, finish it in style, but I can patch up something that will do for publication.[17]

The next day he reports that he has already done some work (probably the detailed plan for Part I): "It will be tiring, but less so than the feeling that my book will never be finished." On the 2nd of May he is back at their home at this time, Friday's Hill. He writes a whole chapter that day, as he tells Alys on the 3rd. That must have been Chapter I, fifteen leaves on "The Definition of Pure Mathematics", for II at forty-six leaves would have been too long. By the 4th he has established a daily work-schedule, and Beatrice and Sidney Webb have arrived to stay for two months. Russell continues to sleep well, reporting on the 5th that he has lately got through six or seven hours of work a day. The same day he tells Helen Thomas: "I am writing on *any*, *every*, *a*, *some*, *all*, the nature of inference, the essence of truth, and other light subjects. Except when I am fit, I cannot understand my own writings." He reaches eighty leaves of manuscript on the 6th. That puts him at the end of III, "Implication and Formal Implication". The letter of the 6th did not report on the complete day, for on the 7th he says that he worked the previous evening until midnight, "accomplishing a Chapter of 21 pages". That was IV, "Proper Names, Adjectives, and Verbs", which occupies folios 82–102 of the manuscript; in it Russell took over five leaves from his penultimate attempt. He must have disposed of V on the 7th (using thirteen old leaves). In preparing to visit his Uncle Rollo he notes: "Now I have to tackle the most difficult Chapter [VI] in the whole of my book—on the nature of Classes. I shall go to Dunrozel in a dream, with Classes all the time revolving in my head whatever may be happening outwardly" (8 May). His report on the day's work there has been published.[18] Suffice it to say that he wrote a chapter of thirty-one leaves. It is worth noting his mood: "I have been so long without real work, that I have come back to it with a kind of fever: *everything* else seems unreal and shadowy to me just now, and I work as if I were possessed."

Russell is working very fast at this time, and there is no mention of Chapter VII. But nor was there a letter on the 10th. On the 11th, back at The Mill House, he is concerned with VIII, telling Alys: "... The Nature

[17] Russell's letters to Alys at this time are in the possession of Barbara Halpern, Oxford. Through her kindness copies are available in the Russell Archives.

[18] See Carl Spadoni, "Philosophy in Russell's Letters to Alys", *Russell*, nos. 29–32 (1978): 29.

of the Variable dominates my thoughts, and must be decided today or tomorrow at an internal oecumenical Council." But on the 11th he has a headache. The 12th is an extraordinary day of work, for he begins it still working on VIII:

> I sat up till 1.30 last night to finish Part I of my Book, which was the task I had dreaded being never again equal to. I am not at all satisfied with it, but I fear it is the best I can do. I think of publishing as soon as possible, as I cannot rest till it is off my mind. This is not the true artistic conscience, but that is a luxury I can no longer afford for the present. (13 May)

Finishing Part I required not only completing VIII, but also writing IX, "Relations", and X, "The Contradiction". Half of IX (twelve leaves) was old, but all of X (fifteen leaves) was new. The handwriting shows Russell's hurry—just like the end of Part II of *Theory of Knowledge*. He was so stimulated that he stayed awake until dawn before travelling to Bournemouth to visit Aunt Agatha. He had, after all, written 213 pages in just eleven days—or 168 when the previously written or transcribed pages are deducted.

We have long known from the *Autobiography* (I: 151) that Russell completed the *Principles* on 23 May. Yet with only a week to go he writes Alys from Friday's Hill:

> I expect to have my book quite finished in another two months, if only I can keep fit and go on working hard. It will not give me any feeling of elation, merely a kind of tired relief as at the end of a very long dusty railway journey. The book will be full of imperfections, and will raise innumerable questions that I don't know how to answer. There is a great deal of good thinking in it, but the final product is not a work of art, as I had hoped it would be. I shall send it to the Press at once, as the load will not be off my mind until I cannot make any further corrections. (16 May)

Having finished Part I, he turns to the rest of the manuscript on the 16th and works eight hours on it. There is an interruption from visitors on the 18th and 19th. Alys evidently shoulders some of the responsibility for the book's imperfections, but Russell tells her she need not feel that way. On the 21st, now at The Mill House, he is back to work. The next day he expects "very soon [to] reach the point when I shall have only more or less mechanical work to do". He does not mention what he has been working on since the 14th. In a 1910 letter to Jourdain he was to claim that Parts I and II were "wholly later, May 1902" (*Dear Russell ...*, p. 133). Yet he makes no reference to progress on individual chapters in

Part II. Possibly this is because they were not (or were no longer) a challenge in the way that Part I had been. He has, after all, dealt with the Contradiction. In this bout of work, he tells Alys, Russell has "learnt to pull myself together and make efforts which formerly I should have thought impossible; but I suppose there are limits to the process." He imagines the appropriate dedication for the *Principles*: "To Moloch this Altar is dedicated by a Sacred Victim." He is working now on matter and motion, i.e. Part VII. Still there is no premonition of how soon he will finish. The next day he is more interested in the effort with which "I flog my poor intellect into activity." He adds: "I have been going over parts of my book which I thought would have to be re-written, and have persuaded myself that they require very little alteration, so that I shall finish it very soon indeed" (23 May). He notes that since their separation he has not read 100 pages of print. He does finish the manuscript that day, writing then to Lucy Martin Donnelly (quoted in the *Autobiography*, I: 163–4) and the next day to Alys: "Thee will be surprised and amused, after all my talk of 2 months, to hear that I finished my book yesterday. I found that a pile of old MS, which I had expected to have to re-write, required only a few additions and corrections, so I arrived at a sudden termination. I have never known or even imagined such a relief as I have been feeling." He hopes his book will be out in October.

Proofreading. Russell mentions that he wrote the previous day to R. T. Wright. Wright was the renowned Secretary of the Cambridge University Press.[19] Although no correspondence between Russell and Wright before 1909 survives, that is no indication of the frequency of their contact. As John G. Slater has suggested to me, Russell—being so often in Cambridge—probably carried out the bulk of his dealings with the Press in person. Wright responded rapidly to Russell's overture regarding his manuscript. On the 25th Russell thinks he will take it to the Press the next Tuesday or Wednesday (the 27th and 28th) for the Syndics' consideration. He is still making final corrections and compiling

[19] See M. H. Black, *Cambridge University Press, 1584–1984* (Cambridge, U.K.: Cambridge U.P., 1984), Chap. 11, esp. pp. 174, 179 and 186 for Wright's duties. Black describes the process of acceptance by the Syndics in Russell's time in his "Evolution at Cambridge", *Scholarly Publishing*, 16 (Oct. 1984): 52; reprinted from *The Cambridge Review*, May 1984. Russell acknowledges Wright in the Preface (p. xix)—a rare recognition of publisher's staff on Russell's part. In June 1909 Wright asked Russell if the book should be reprinted in its present form, the last fifty copies having been sent to the bindery; there were no stereotype plates. In April 1910 Russell requested and received a new copy: "I have only one copy at present, and it is too much annotated to be any use for that purpose [that of a second edition]." Neither copy is known to survive. Russell's letters to C. K. Ogden in 1922 show him selling Ogden a copy of the *Principles* (and two of *Principia*); but because of the concern over one of the latter being marked up, it is unlikely an annotated copy of the *Principles* was involved.

the table of contents.[20] On the 27th he says he will take the manuscript to the Press tomorrow, but in fact he takes the bulk of it that day and the rest the next. Wright tells him on the 28th that "they would begin printing at once." Thus Wright took exactly one day to evaluate and accept the work (though the Syndics probably had to give formal acceptance at a regular meeting later). On the 29th the Whiteheads dined at the Wrights'. Wright must have known all about the progress of Volume II as well. For some days Russell works on an unidentified paper by Whitehead, looking forward to "lots of proof-correcting soon—the pleasantest and easiest work there is" (1 June). It turned out to be not so easy in the case of the *Principles*.

The next stage was production of a specimen page. Russell has to go twice to the Press on Monday the 2nd of June, and he notes to Alys on the 3rd that "the Press have sent me a specimen page, but have not got beyond that yet." The reason, he tells Beatrice Webb on the 4th, is that the Press "are full of exam papers just now." Presumably Russell returned the page straightaway, for on the 5th he tells Alys, who wanted him to bring it to their first meeting in nearly two months, that the page is back with the Press. The page does not survive, but we know what portion of the text was on it: folios 3–4, or pages 4: 5–5: 4 [end of line]), are marked "Spec. page". As is characteristic of the rest of the manuscript, there is no copy-editing of Russell's prose; on these leaves there is not even any marking up for the compositor. Cambridge at this time (and since) was outstanding for its production of mathematical texts, Russell had already published *The Foundations of Geometry* and *The Philosophy of Leibniz* through Cambridge, and my hunch is that he found nothing to improve in the specimen page. Further evidence is the fact that the text marked for this purpose occupies exactly the equivalent of a page in the final printed version.

There is no more news of his book until the 18th, when the first proofs arrive. He takes a ten-day holiday in Cambridge, talking shop with Moore and others, then returns to Friday's Hill. He is full of plans for work: "I shall have to read a lot of casual shop first, then polish up my lectures so as to be ready for publication, and then I mean to undertake a systematic study of the great philosophers preparatory to my Logic, which is to begin by an exposition and criticism of all previous logics of any importance" (12 June). Again we have a reference to the lost text of the lectures which were to go into Volume II. These plans for future work were to be delayed, however. The next day he is reading Boole's *Laws of Thought* and on the 15th reports: "I am working very hard, reading the

[20] The analytical table of contents occupies eighteen pages. The manuscript is lost.

literature of my subject, now that I have finished writing about it. I can insert learned footnotes in the Proofs."[21] That is exactly what he did—inserting thirteen footnotes in the proofs of Part I alone. His new work binge continues, confessing on the 17th from Friday's Hill: "Life to me is wholly unemotional and dry at present: Formal Logic fills the crannies of my brain." While deep in his reading of the subject—he is now "full of Frege"—the first proofs arrive; his reaction is unemotional: "I know, in a kind of impersonal way, that it is a good book, but it gives me no satisfaction at all" (18 June). Russell and Alys meet on the 21st. On the 22nd more proofs come, and he works five hours. On the 25th he tells Couturat that the volume will probably appear in the winter and amount to about 500 pages. But the typesetting does not proceed as rapidly as Russell thought it would. He tells Dickinson on 2 August that "The proofs come occasionally, and seem to me very worthless ..." (*Autobiography*, I: 184); and on 29 September he notes to Couturat that while he is very much occupied by the book, it is being printed slowly. The slow progress may have been connected with Russell's substantial revisions.

There are no more references to proofs in these letters until 5 February 1903. It is, therefore, impossible to tell how quickly Cambridge sent him proofs—perhaps a sheet (or signature) at a time. The *Principles* being an octavo volume, that was sixteen pages. There are, however, several references in letters to working with Whitehead. That work would include the proofs, since Whitehead read them (Preface, p. xviii, and Russell to Helen Thomas, 16 Sept. 1902). It is probable Russell received page proofs from the start. He had done so with the *Leibniz*.[22] The printer's copy of the manuscript is marked not only for compositors' stints and footnotes, but also running heads and signature lines. For example the beginning of Chapter VI, folio 127, has a draft for the running head starting on page 65. Yet VI does not begin in the printed book until page 66. This error can be explained by the fact that Russell made enough additions to the text of V in page proof to enlarge it by one page (see the variants for pp. 49–64; p. 64 has only fifteen lines).

The Preface was not written until 2 December, according to Russell's journal, and the Press received it on the 10th. On 5 February Russell tells Alys that there will be more proofs, but only one or two sheets. On the 10th he is indexing the volume, meaning that he has final page proofs by then. *The Principles of Mathematics* was published in May 1903, Jourdain

[21] The only other author mentioned in the correspondence is Meinong (in a letter to Couturat). The *Principles* has no bibliography, but Albert C. Lewis of the Russell Editorial Project has compiled a checklist of works referred to in the book. Expansion to make the checklist an index and collation with the final manuscript would show what other references were added in proof.

[22] See Walter H. O'Briant, "Russell on Leibniz", *Studia Leibnitiana*, 11 (1979): 179. See also Russell's letter to Alys of 25 June 1900.

acknowledging a copy on the 10th. After telling Helen Thomas on the 13th that his book is out at last, Russell deprecates the achievement: "It seems to me a foolish book, and I am ashamed to think that I have spent the best part of six years upon it."

Frege's impact. Frege's initial impact on Russell can be determined from this analysis and the list of variants. The first mention to Alys of reading Frege was on 18 June. Russell has, in fact, written Frege on the 16th to ask for offprints of his articles and to relate the difficulty he has discovered, i.e. the Contradiction.[23] The manuscript of the *Principles* has been with the Press since 27 May. Frege responds on the 22nd: "Your discovery of the contradiction has surprised me beyond words and, I should almost like to say, left me thunderstruck, because it has rocked the ground on which I meant to build arithmetic." Russell tells Alys of this in a letter written early on the 25th: "I have heard from Frege, a most candid letter: he says that my conundrum makes not only his Arithmetic, but all possible Arithmetics, totter." The same day he praises the *Grundgesetze*[24] to Couturat for containing many ideas Russell thought he had discovered. These are not, however, quite the first responses by Russell. He not only replied to Frege the previous day (the 24th) but must then have sent the Press the three-page insertion for the end of §103 in Chapter X. (The Press date-stamped it the 25th and mails were very fast, but Russell is in Haslemere at the time.) The insertion contains what may be the earliest reference to the appendix on Frege. However, this reference was deleted in revising proofs of the chapter. To judge the full text of the insertion, the reader must first find page 104, lines 6–31 in the text. He must realize that the preceding seven lines in the paragraph replaced in proof a much longer passage now found in the list of variants at 103: 44–104: 5. Finally, he must add to part of the insertion retained in the book the long passage deleted in proof at 104: 31. The end of this deletion contains the footnote: "The above discussion is largely influenced by Frege: see Appendix." Russell's letter to Frege of 24 June should be read in conjunction with these passages. The effect of the proof revisions was to reduce greatly the discussion of "logical imaginaries".

We do not know when Russell received proof of sheet (or signature) 7, which included Chapter X. Nor do we know if he made all of the proof revisions on the first page proofs. We can, however, be fairly certain that plans for the appendix on Frege predated the receipt of Frege's letter. Russell had been deeply impressed by Frege's work in the previous

[23] Gottlob Frege, *Philosophical and Mathematical Correspondence*, ed. G. Gabriel *et al.*, abridged by B. McGuinness, trans. H. Kaal (Oxford: Blackwell, 1980), p. 130.

[24] The reference to the *Grundgesetze* at 19n. was not added in proof but rather is in the final manuscript. (So is the unindexed reference to Frege at 35n.) Russell provides an explanation in the Preface, p. xvi.

couple of weeks and in his first letter he asked for copies of Frege's minor works, because "I should like to discuss your work in [the *Principles*] in great detail." On 7 July he tells Frege that he will discuss his work in an appendix "because it is now too late to talk about it in detail in the text." The appendix, however, takes Russell a long time to finish; the Press does not receive it until 15 November, and Russell does not read proofs until February or so. Because of the latter hiatus, and the important correspondence on the Contradiction that he continues with Frege over the winter, there may have been substantial revisions to both this appendix and the one on the theory of types. They await collation.

Conclusion. The levels of revision in the *Principles* are many and various. The minimum tool needed by scholars is a complete list of the variants, and others are invited to contribute to the compilation. Secondly, a list of the alterations in the final manuscript is desirable. Because of the reliability of the printed text, so far as it has been examined here, a reset, variorum edition is not an immediate desideratum for textual reasons alone. Instead, we need an edition of the penultimate draft of Part I complete with its prior alterations in the manuscript. Some way should be devised of illustrating the alterations a year later on the leaves transferred to the final manuscript. This should not be beyond editorial ingenuity. When these tools are to hand, then scholars will be equipped to trace the effect (say) on Russell's early theory of denoting of his efforts to resolve the Contradiction.

The Bertrand Russell Archives

VARIANTS BETWEEN *The Principles of Mathematics* AND ITS MS.

PREFACE
xvi: 13 as an] as
xvi: 22 Appendices, which] Appendices. These
xvi: 39 began an investigation into] formed the intention of writing on
xvii: 31 is] is one which is
xvii: 32 mathematics, and therefore to the present work,] mathematics
xviii: 7 are] are, at least in part,
xviii: 21 Georg Cantor] Professor Georg Cantor
xix: 24–34 vague. ¶For ... doctrine.] vague.
xix: 37 volume.] volume, and still more for undertaking the necessarily troublesome task of printing the second volume.

CHAP. I DEFINITION OF PURE MATHEMATICS
7: 5 Plato*:] Plato: ⟨*Added fn. indicators are not noted any further.*⟩
7: 43–5 ⟨*fn. added*⟩
8: 29 *S*] If S ⟨*Cf. MS., fol. 20, l. 4.*⟩
9: 8 mathematics (§1).] mathematics.
10: 13 attained to] achieved
11: 45 XXXVI] V
12: 6 duality] dualism

CHAP. II SYMBOLIC LOGIC
12: 39 On] On the two-fold interpretation of logical formulae, cf. Whitehead, Universal Algebra, Cambridge, 1898, Book II, Chaps. IV and V, where the limitations appear to be unperceived; on
13: 26 A.] I.
13: 28–30 hypothesis and ... to] hypothesis
14: 12 if the fourth] if the former
14: 13 fifth ... fifth ... fourth] latter ... latter ... former

15: 44 state] point out
15: 45 exclusive unless ... so.] exclusive.
16: 43–5 ⟨*fn. added*⟩
17: 3 then "'*p* implies *q*' implies *p*" implies *p*.] and "*p* implies *q*" implies *p*, then *p* is true.
17: 29 and] or
18: 24 B.] II.
18: 31–2 as fundamental the notion of *class*,] *class* as a fundamental notion,
19: 42 verb or adjective] verb
19: 44 Halle, 1879] 1879
19: 44–5 1893, p. 2.] 1893.
20: 32 on] in ⟨*Misprint introduced in 1937.*⟩
20: 37 *prime and*] *prime* and ⟨*Misprint introduced in 1937.*⟩
20: 43–5 ⟨*fn. added*⟩
21: 5 classes, in some form] classes
21: 17 classes (*a* or *b*)] classes
21: 17 class (not-*a*).] class.
21: 32–3 is true ... *x*.] implied by "*x* is an *a*" whatever *x* may be is true.
23: 32 C.] III.
23: 35 C. S. Peirce] Peirce ⟨*Also at 23: 43.*⟩
23: 41 p. 104; ... 50.] p. 104.
24: 23 primitive (*i.e.* indemonstrable)] primitive
24: 41 *R*, *i.e.* ... relations;] R;
26: 35 D.] III.
26: 42–3 ⟨*fn. added*⟩
28: 4 propositions (see §34, (3)).] propositions.
28: 7–9 the ... indefinable] a highly artificial definition of the logical product of two propositions is possible; but this definition is almost worthless
28: 10 property] property of the logical product
28: 10 the definition.] it.
29: 13–15 angles. But ... 62).] angles.
29: 15 for each propositional function] in fact
29: 16 propositions and] a class of propositions and
29: 16 entities] a class of entities
29: 18 entities] terms of the class of entities
29: 32 various propositions of given form] a class of propositions
29: 33 them ... variable] the propositions of the class
30: 6 identity] contradiction
30: 16 B] II
30: 36 or of propositions or of] or propositions or
31: 1 *c*] ⟨*Misprint: deleted in MS.*⟩
32: 19 as] as really

CHAP. III IMPLICATION AND FORMAL IMPLICATION
34: 24 C] III
34: 44 those rules of inference] the rules of inference, all of
36: 11–12 "for ... *x*,"] "'*x* is a *k*' implies *x* for all values of *x*"
36: 14 equivalent, if *q* be a proposition,] equivalent
37: 11 this is not] this, we said, is
37: 12 For as] As
37: 13 meant, since] meant. For
37: 14–15 to propositions, ... *x*."] at least to propositions.
37: 20 proposition] propositional function
37: 27 noticed] observed
37: 42–3 "*if x* implies *x*, then '*x* is a *k*' implies *x*"] "'*x* implies *x*' implies that '*x* is a *k* implies *x*" ⟨*no closing single quote*⟩

CHAP. IV PROPER NAMES, ADJECTIVES, AND VERBS
42: 30 concept] idea ⟨*Also at 42: 31.*⟩
43: 10–11 proposition or ... or some subordinate proposition] proposition
47: 45 ⟨*fn. added*⟩

CHAP. V DENOTING
54: 17–18 classes, but ... *class-concept.*] classes.
54: 42—55: 1 differs little, if at all, from] is
54: 43 ⟨*fn. added*⟩
54: 44–5 & 55: 36–9 ⟨*fn. added*⟩
55: 40–2 ⟨*fn. added*⟩
56: 3–4 class-concept, though the distinction is perhaps only verbal.] class-concept.
56: 4–5 class-concept, as distinguished from terms in general,] class-concept
56: 5 *u*] so-and-so ⟨*Also at 56: 6 and 56: 7.*⟩
56: 5–6 propositional function] proposition for all values of *x*
56: 8 all, whatever value we may give to *x*.] all.
56: 13 proposition] propositions ⟨*Error in MS.*⟩
56: 13 *u*] the term so-and-so
58: 24 combined in the specified manner.] in various ways.
60: 4 some (or an)] an ⟨*Also at 61: 3, 61: 5 and 61: 7.*⟩
60: 35 twenty] sixteen
61: 16–18 have a common part. ¶(11) ... part.] have a common part. ⟨*Thus cases (12)–(17) have been renumbered from (11)–(16) in MS.*⟩

61: 27 A (or some)] A ⟨*Also at 61: 29.*⟩
61: 27 every] any
61: 30–6 part. ¶(18) ... *a.*] part.
61: 38 implication (which has not always been stated)] implication
63: 9 large] great
63: 27 term or conjunction of terms] term
64: 41 a-man" (cf. p. 54, note)] a man"
64: 43 which, where it exists,] which
64: 45 me. On ... 50.] me.

CHAP. VI CLASSES

66: 14 former] latter
67: 7 is unambiguously] is
67: 19 phraseology] language
67: 37 manifold, aggregate,] manifold,
68: 39 object] entity
68: 44 ⟨*fn. added*⟩
69: 43–4 ⟨*fn. added*⟩
70: 39 held] held, I think,
70: 43 Leipzig, 1854 (2nd ed., Berlin, 1889), §3.] §3.
72: 10 *are*] is
72: 11 are ⟨*bis*⟩] is
76: 19–35 ¶There ... race.] ¶As a first step, it is desirable to distinguish, in this connection, a logical and an arithmetical sense of *one*. Logically, *one* applies to every logical subject, *i.e.* to every term; and it must be held that a class is a term as well as a combination of terms. Whatever can occur in a propositon is one in the logical sense, and in this sense *one* seems to be not opposed to *many*. But the arithmetical *one* is, at least apparently, quite different: it is to be regarded as a property of any class-concept u which is such that, if x is a u, and y differs from x, then, for all values of y, y is not a u. It is necessary here to introduce the class-concept rather than the class, in order to explain how the class can be also arithmetically one in a certain sense. A class-concept which denotes all the terms of a class is different from a class-concept which denotes the class itself. Does this raise logical obstacles to the above theory of classes? The point is curious and important, and must be examined fully. ¶It appears, from what was said above, that a class considered as a single term must be a different entity from the same class considered as many. But this view will lead, one might suppose, to the same contradiction which resulted from the attempt to distinguish concepts used as such from concepts used as terms, *e.g. is* and Being. A combination of terms, of whatever kind, we are tempted to say, must be itself a term, if it is in any way possible for it to occur in propositions. Yet if the class as one term is identical with the class as many, it would seem that a class-concept which denotes the one must denote the other. But *man* is a different concept from *class of all rational animals*; "x is a man" is a propositional function satisfied by each individual man, while "x is a class of all rational animals" is a propositional function satisfied by only one value, namely the class *men*. Substituting the concepts of classes for the class-concepts, *men* denotes the class taken term by term, *i.e.* the class as many, while *classes of all rational animals* denotes precisely the same class taken as a single term. Hence, it would seem, the distinction drawn by Peano, between a term and a class of which the term in question is the only member, must be maintained, at least when the term in question is a class. Or rather, we must admit an ultimate distinction between a class as many and a class as one, we must hold that the many are only many, and are not also one. The class as one may be identified with the whole composed of the terms of the class, *i.e.*, in the case of men, the class as one will be the human race. The two senses of *one*, the logical and the arithmetical, though they remain distinct, will now go hand in hand, at least if, in the case of the arithmetical *one*, we restore classes where class-concepts are mentioned.
76: 36 always to be] which we
76: 37 when there is] the contradiction of
76: 43–4 ⟨*fn. added*⟩
80: 5 is evident] in evident ⟨*Error in MS.*⟩
80: 30–81: 5 points, etc. Starting ... many.] points, etc.
81: 6 class-concepts.] class-concepts. We also found that classes, except when they consist of single terms, are essentially *many*, and that a class considered as one entity is something different from the same class as many, being in fact the whole composed of all the terms of the class together. We discussed finally various relations connected with that of a term to a class of which it is a member, and with inclusion.

CHAP. VII PROPOSITIONAL FUNCTIONS

86: 44–87: 14 In ... this] This
87: 15 aRb] the other
87: 24 which] , by the way, which
87: 28–38 ¶The ... relation.] ¶The above case, where it is the concept in a proposition, and not the terms, that is to be varied, seems finally to dispose of the view that propositional functions can be analyzed into variable terms and fixed assertions; for "a .. b" is certainly not an assertion about R in the proposition aRb. Of course it may be said that, before varying R, we ought to substitute for aRb the equivalent propositon "R is a relation holding from a to b". But there is no gain in this, for "holding from a to b" introduces precisely that assertion "a ... b" which was to be avoided.
87: 43–5 ⟨*fn. added*⟩
88: 1 "R ... b"] the above

CHAP. VIII THE VARIABLE

89: 22 is any] is any ⟨*Misprint for* in any?⟩

CHAP. IX RELATIONS

95: 31 XXV, §200] II
95: 32 description] definition
98: 8 as those of which it is predicable.] predicable of it.
98: 24 of which] such that
98: 44–5 ways of combination] ways
99: 20 terms (§§48, 54)] terms
99: 33 III] ⟨*Blank left in MS.*⟩
99: 38–9 occasion (§55)] occasion
100: 3 formerly (§54)] formerly

CHAP. X THE CONTRADICTION

101: 9 very plausible supposition] obvious fact
101: 11 members] terms
101: 14 Then (a) if] If ⟨*"α" is new style Greek for "α".*⟩
101: 14 v,] v, then,
101: 17 (β) if] if
101: 19 (γ) if] if
101: 20 members of] classes contained in
101: 21 contained in itself] not a u
101: 22 none ... is] is therefore not predicable of themselves
101: 23–5 itself. Thus ... not.] itself.
101: 25 (δ) if] if
101: 29 those class-concepts] a class-concept
101: 32 proved in (β)] proved
101: 34 XLIII, §344 ff.] XII.
102: 38 them and no other terms] them
103: 4 either.] it.
103: 4–5 the former] it
103: 7–23 begin. ¶Any ... types.] begin.
103: 44–104: 5 ¶It ... x.] ¶The propositional function which leads to the difficulty may be expressed in several equivalent forms. But the above form suggests a possibility of solution on the same lines as the solution of an analogous contradiction discussed in Chapter VII. When ϵ is replaced by a variable propositional function, we have to consider all propositional functions not satisfied by the class of their own roots—meaning by the roots of a propositional function, as in the particular case of an equation, the values satisfying it. It may be suggested that we are here introducing a propositional function which involves treating assertions as separable and analyzable entities. If ϕx be a variable propositional function, the propositional function suggested is the denial of $\phi(k_\phi)$ where k_ϕ denotes, for the moment, the class of roots of ϕx considered as a single term. Now in $\phi(k_\phi)$ the variable is ϕ; thus we necessarily treat the assertion ϕ as separable. This we saw before to be improper, and thus the contradiction would appear to be resolved. We may say generally that a variable assertion, or a variable propositional function, is not to be admitted without caution. A propositional function is not in itself an entity, but is either a class of propositions, or any member of this class. When it is itself varied, it becomes necessary to treat it as an entity distinct from either of these, and this is illegitimate. Or again, when ϕ in ϕx is varied, it becomes necessary to treat ϕ as variable: it will not do to vary ϕx as a whole, for that would give us all propositions, not all propositional functions. Thus it would seem that "x is not an x" is a proposition for every value of x, but is not a propositional function when x is variable. This result is only intelligible by adhering strictly to the dependence of classes upon propositional functions. It has the result that, if any term which occurs in a proposition is essentially a class, we must not vary that term in the proposition as it stands, but must first add some hypothesis which removes the necessity for regarding our term as a

class. It has also the result that ϵ is not to be taken as a fundamental notion, but is to be strictly regarded as derivative from propositional functions by the help of the notion of *such that*.

104: 6–31 Moreover ... degree.] ⟨*These lines plus the passage subsequently omitted at 104: 31 were received by the Press on 25 June 1902, with the note at the top of fol. 1 of the MS.*: "[To be added at the end of §103]".⟩

104: 6 Moreover] At the same time,

104: 13 two kinds of] legitimate from illegitimate

104: 18 we] that give rise to contradictions of the type we

104: 25 $\phi\{f(\phi)\}$] "x is an x" and "x is not an x"

104: 26 it is] they are

104: 26 proposition] propositions

104: 26–7 is not a function] are not functions

104: 31 degree.] degree; and it would seem that logical imaginaries arise from logically quadratic forms in a way which heightens the analogy. If such forms are, as would appear to be the case, illegitimate, we may conclude as follows: A propositional function occurring in an expression may be regarded as variable provided, when a constant value which is a propositional function is assigned to the variable, the expression in question becomes a propositional function, or becomes a proposition concerning a constant term which is independent of the value assigned to the variable propositional function*. [*This case arises, for example, in considering $\phi(2)$, the class of all propositions that can be made concerning the number 2.] But if both assertion and argument become different when a different value is assigned to the variable propositional function, then we have a quadratic form, and contradictions are liable to arise. We may, in a word, consider various assertions about a given subject, or the same assertion about various subjects; but in a quadratic form, we try to consider a variable assertion about what is asserted, and here no sufficient definiteness remains. [The above discussion is largely influenced by Frege: see Appendix.]

104: 37 in the original propositional function.] and thus treated as a single term.

104: 38 In ... one.] Thus if w be the class of all classes which can be made single subjects, but as such are not members of themselves, w will be only many, not one, and the question whether w as one is a member of w as many is a question concerning an entity which has no being, in fact a non-entity.

104: 42–105: 15 overcome. ¶A class ... mystery*.] overcome.

105: 25 it.] it. ¶It may be asked whether identity, as defined in Chapter II, does not involve a variable form of proposition. We said that x is identical with y when y belongs to any class to which x belongs, i.e. when "x is a u" implies "y is a u". As it stands, this definition involves the varying of classes; but this can be avoided by substituting "'u is a class' implies that 'x is a u' implies 'y is a u'". Here u has become an unrestricted variable. But I cannot find any such transformation for the case of "x is not an x".

105: 35 terms.] terms. ¶The view advocated above, that propositional functions are more fundamental than other classes, has the disadvantage that it *seems* to render the meaning of ϵ variable from class to class. For, if classes must not be variables, the reason is that this would involve the variation of assertions. This difficulty is avoided by the above observation that classes may be varied so long as they are not made into *subjects* of a propositional function. This is not required in ordinary propositions containing ϵ, and thus ϵ may be allowed to have a fixed meaning.

105: 45 ⟨*fn. added*⟩

106: 18 the *is* of] *is* in

106: 32–4 class-concepts. But ... admitted.] class-concepts.

106: 41 function of one variable] function

107: 15 IX] VIII

107: 24 apparent fact] fact

107: 27–37 it ... absent.] w is only many and not one, and that the same holds generally of classes which can only be defined by means of variable propositional functions. In such cases, the class must not be made the subject of propositions, the notion of *all* terms of the class (collectively) does not exist, and the class has no number of terms, because number (as we shall see) depends upon the collective *all*.

107: 38 ⟨*fn. added*⟩