Russell and the development of mathematics

by Gregory H. Moore

George Temple. *100 Years of Mathematics*. London: Duckworth, 1981. Pp. xvi, 316. £32.00.

AT THE RISK of antagonizing certain historians of mathematics, one might categorically assert that the most difficult period about which to write the history of mathematics is that of the last hundred years. This difficulty is due both to the extreme technical complexity of the historical material and to its enormous quantity. While a historian of Greek mathematics must make the most of fragments and a handful of surviving documents, the modern historian is confronted with a plethora of material, which, regrettably, often has gaps at the most inconvenient places and whose interconnections are frequently obscure at best. Thus the reviewer regards with considerable sympathy the task that George Temple has set himself.

Temple, a former Sedleian Professor of Mathematics at Oxford, is best known for his contributions to that part of mathematics known as analysis, particularly generalized functions, and to mathematical physics. It is as a mathematician rather than as a historian that he describes his purpose in the work under review:

> This book has no pretensions to give an encyclopaedic history of the mathematics of [the last hundred years].... Its object is to present the work of some of the mathematicians who have carried out this transformation of mathematics, to describe their ideals, their concepts, their methods and their achievements.... This book has been written therefore to appeal to those who desire a broad survey of the main currents of mathematical thought. (P. 1)

To carry out this programme, Temple divides his book, as Caesar divided Gaul, into three parts: number, space, and analysis. The topic of number receives four chapters—devoted respectively to real numbers, infinitesimals, transfinite numbers, and (somewhat anomalously, considering the previous chapter) finite and infinite numbers—while the topic of space is subdivided into chapters on vectors, measurement, algebraic geometry, and topology. Analysis, to which more than half of the book is devoted, is allotted seven chapters—on functions, the notions of derivative and integral, distributions, differential equations, the calculus of variations, potential theory, and mathematical logic. Russell's work is discussed at sporadic intervals, but it is pleasing to find mention of his logicism, his criticisms of definitions of real number, his work on the foundations of geometry, and his theory of types.

In the introduction Temple acknowledges two of his predecessors, the book *Eléments d'histoire des mathématiques* by Nicolas Bourbaki[1] and the essay "A Half-Century of Mathematics" by Hermann Weyl,[2] and says about them: "These essays set a high standard of exposition, but they do not pretend to cover ... all the main divisions of the subject. There is room for a history of mathematical ideas which will demand less mathematical expertise and offer a more detailed account of the motivation of research" (p. 1). Nevertheless, Temple's account demands almost as much mathematical expertise as Bourbaki's (and, indeed, much more than that of Weyl) but fails to provide a better account of what motivated research.

Temple has a tendency to conflate Russell's genuine accomplishments with his marginal contributions. In particular, when describing the construction of the real numbers put forward by the nineteenth-century mathematicians Weierstrass, Cantor, and Dedekind, Temple writes: "The task of submitting these theories to a minute logical analysis has been carried out with devastating results by Russell [in *The Principles of Mathematics*] and the best service which an historian can now render is to describe the fundamental principles of analysis which a benign interpretation can discover in the wrecks of these theories" (p. 13). On the contrary, Cantor's construction (using Cauchy sequences of rational numbers) and Dedekind's construction (using Dedekind cuts in the rationals) remain the standard constructions of the real numbers. The later mathematical improvements in these two constructions are of a secondary nature.

It is in the chapter on mathematical logic (which Temple places idiosyncratically in the part of the book devoted to analysis) that Russell's work is discussed most fully. Here one finds a brief treatment of Russell's paradox, logicism, the theory of types, and Russell's critique of the Peano postulates for the natural numbers. From a mathematical standpoint Temple has little to say on these topics that is not well known. From a philosophical perspective he stresses Russell's application of Ockham's razor to real numbers, i.e., Russell's claim that a real number *is* a Dedekind cut (or a Cauchy sequence of rational numbers). What

[1] Paris: Hermann, 1969.

[2] *American Mathematical Monthly*, 58 (1951): 523–53.

Temple does not mention, but which is very relevant here, is that later mathematicians identified a real number with an equivalence class of Cauchy sequences and in this way dispensed altogether with the process of abstraction used by Cantor and criticized by both Frege and Russell.

The most serious flaws in Temple's book are due to his fundamental ambivalence as to whether he is writing a history of mathematics or merely an exposition of various mathematical ideas. His obligations are different in the two cases. If, on the one hand, he is writing a history, then his primary concern is to analyze the development of mathematical ideas in the period under consideration and to explain why they developed as they did; here the chronological order of related ideas is of fundamental importance. If, on the other hand, he is writing an exposition, his chief concern is to expound the ideas themselves and hence to arrange his material thematically with relatively little concern for chronology. Temple's arrangement of material strongly suggests that he intends to present an exposition rather than a history. Yet he repeatedly emphasizes that, while his selection of material is somewhat personal, he is contributing a *history* of mathematics. Unfortunately, it seems that his history is unhistorical and that his exposition lacks clarity.

Part of Temple's difficulty appears to stem from his lack of familiarity with the works of those historians of mathematics who have written about the nineteenth and twentieth centuries. His bibliography is notable for its lack of references to the works of such historians.[3] It is of questionable value to discuss, for example, the development of the Lebesgue integral without using the historical writings of Thomas Hawkins. A greater familiarity with the works of professional historians of mathematics would have increased the sophistication of Temple's book considerably, and might have suggested that he not take literally his witticism that "it is one of the chief embarassments of a historian that mathematical discoveries are not made in the right chronological order ..." (p. 67).

Temple has made a number of errors, concerning both historical interpretation and historical fact, of which the reader should be made aware. It is a dubious claim that the study of trigonometric series "forced" Cantor to propose a theory of irrational numbers (p. 25); indeed, it had not forced Riemann to do so. More plausibly, Temple

[3] The reader will find reference to the older works of such historians in Kenneth O. May, *Bibliography and Research Manual of the History of Mathematics* (Toronto: University of Toronto Press, 1972) and to more recent works in Gregory H. Moore and Lynn A. Steen, "Bibliography on the History of Twentieth Century Mathematics", *The History of Mathematics from Antiquity to the Present: A Selective Bibliography*, ed. Joseph W. Dauben (New York: Garland, forthcoming in 1984).

might have noted that Cantor's theory likely reflects the influence of Weierstrass, under whom Cantor studied in Berlin. Furthermore, it is misleading to say that the algebraic numbers can be well-ordered with order-type η (p. 30); for this order-type is *not* well-ordered, being the order-type of the rational numbers. It is simply mistaken to say (p. 39) that Lebesgue considered the axiom of choice[4] to be "a valuable instrument for suggesting lines of research" and it is equally erroneous to claim (p. 109) that Cantor introduced the notion of open set. This notion was introduced by Lebesgue, although the term "open" (in the anachronistic and broader sense of "not closed") comes from W. H. Young. Again, von Neumann's version of set theory (based on the notions of function and argument) is by no means a "revolt" against set theory, contrary to Temple's claim (p. 138). All in all, Temple is often tempted to propose overly idiosyncratic interpretations on the basis of minimal evidence.

Finally, it seems useful to present a selection of the secondary errors which are unpleasantly widespread in the book. The Russellophile may be interested to know, in particular, that the index is rather inaccurate. Whereas Temple's index states that references to Russell can be found on pp. 2, 22–3, 74, 255 and 259, in fact such references do not occur on pp. 22–3 but rather on 2–3, 13, 32–3, 66, 74, 255, 256, 259–62, 266–7 and 274. Repeated misspellings of names are common: E. Huntingdon for E. Huntington (pp. 73, 76, 294, and 310), W. Luxembourg for W. Luxemburg (pp. 23, 24, 297, 310), B. Russel for B. Russell (p. 33), Thorald Skolem for Thoralf Skolem (p. 22), and J. Heinenoort for J. van Heijenoort (p. 32).

In conclusion, the gentle reader is advised to look elsewhere for an introduction to the mathematics of the last hundred years. The bibliographies mentioned in footnote 3 may aid the reader in this search.

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[4] Concerning the axiom of choice, and how Lesbesgue and Russell regarded it, see the reviewer's book *Zermelo's Axiom of Choice: Its Origins, Development, and Influence* (New York: Springer, 1982).