Russell's logicism versus Oxbridge logics, 1890–1925:

a contribution to the real history of twentieth-century English philosophy

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Stout took me to see Bradley—a black-bearded man with a very intellectual, very sensitive face, beautiful by the beauty of the mind that appears in it. His manners are very courteous and slightly shy. He has the spirituality of those who have worked in spite of great physical pain. I loved the man warmly. We discussed philosophy for some time. I vexed him very much (quite unintentionally) by saying that in philosophical discussion, so far as I could see, one arrives usually at an ultimate difference as to premisses, where argument is no longer possible. This seemed to him scepticism and an attack upon his life's work. He controlled himself completely, but with difficulty. I was very sorry I had vexed him. (Russell's journal, 1 Dec. 1902*)

1. INTRODUCTION: TWO LITTLE BOOKS

As this paper is largely an essay on contrasts and disagreements, it is

* Russell's journal is kept among the Ottoline Morrell papers, Harry Ransom Humanities Research Center, University of Texas, Austin. A copy is held at the Russell Archives, McMaster University, Hamilton, Canada; this archive is cited as "RA". It is published in Russell's *Collected Papers*, Vol. 12: *Contemplation and Action, 1902–14*, ed. R.A. Rempel, A. Brink and M. Moran (London: 1985), pp. 7–28 (p. 13).

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appropriate to set the scene by the example of two little *duodevicensimi* (18mo format) volumes published in the late nineteenth century. One of them represents (part of) the traditional view of logic then current, while the other describes the new discipline of mathematical logic. The contrast is very great: indeed, format is one of the few features which they have in common.

The *Logic* of W.S. Jevons (1835–1882) first appeared in 1876 in Macmillan's "science primers" series and then in later editions, in his lifetime and posthumously.[1] In 135 pages he ran through the basic ideas of names, terms, types of propositions, syllogistic reasoning and inference. He also presented induction as the foundation of scientific theories, with related topics such as variations and analogies, and finished off with a survey of fallacies in deductive and inductive reasoning. A selection of questions for each chapter completed the text. No symbolic methods were introduced, although a little use was made of Euler diagrams.[2]

The *Logica matematica* of C. Burali-Forti (1861–1931) was published by Hoepli in 1894 in their series of small manuals.[3] His 155 pages comprised a text of four chapters: in turn, a survey of "general notions", on numbers, propositions, and their connectives; "ratiocination", containing primitive propositions, "polysyllogisms" (known as "sorites" in England, where sets of propositions are treated together) and the exegesis of the propositional calculus with some applications to arithmetic; "classes", in which propositional functions were introduced and their corresponding classes considered, again often in connection with arithmetic; and finally "applications", including the relation between classes and individuals, mathematical functions and their inverses, four types of definition (nominal, under hypothesis, of "an entity in itself" and so constituting part of the theses of a theory, and by abstraction), and the (in)dependence of propositions in a theory. Although an introductory volume, as required by the series, it was far more specific in its purpose than Jevons's book (which, interestingly, appeared in the

[1] W.S. Jevons, *Logic* (London: 1876); I have used the 1883 edition. On Jevons's contributions to logic in general, see W. Mays and D.P. Henry, "Jevons and Logic", *Mind*, n.s. 62 (1953): 484–505.

[2] Jevons, Chap. 11. In those days, unlike today, the difference between Euler and Venn diagrams was usually understood; on it, see my "The Gergonne Relations and the Intuitive Use of Euler and Venn Diagrams", *International Journal of Mathematical Education in Science and Technology*, 8 (1977): 23–30.

[3] C. Burali-Forti, *Logica matematica* (Milan: 1894). It was based on his teaching at the University of Turin (p. vi).

series in translation[4]), exhibiting symbols from its first page: indeed, in his preface Burali-Forti contrasted "Aristotelian, or scholastic, logic" with the new "*mathematical logic*".

2. OXBRIDGE TRADITIONS IN LOGIC

Although an Italian work, Burali-Forti's manual is suitable for use here, since it was this mathematical logic of his mentor Peano that was to serve as the principal basis of Russell's contributions to logic. First, however, let us return to the traditional fields as represented by Jevons, for it was these which dominated English logic at the end of the century (and, as we shall note in section 7 below, much later also). In this section I shall present the principal figures of the time and highlight some general features of their systems. I have spread my brief to cover Oxford as well as Cambridge, since the differences between the thoughts of the two centres were much less than their respective divergencies from Russell's new view. Indeed, there is even a geographical appropriateness in this extension, for during the period 1896–1910 of his principal concern with logic Russell lived much of the time in Cambridge, and then in Oxford (although not to be near its logicians!).

Three figures of interest were based at Cambridge. J.N. Keynes (1852–1949) is today remembered best as the father of his famous economist son (whose funeral service at Westminster Abbey in 1946 both father and mother attended); but his *Studies and Exercises in Formal Logic* appeared first in 1884, and in several later editions. W.E. Johnson (1858–1931), whose association with Cambridge began with his birth, did not start to publish his *Logic* until 1921; but a few earlier papers had indicated his line, and his help was acknowledged in the prefaces of books by several other philosophers, including Russell (in fact, he looked at the manuscript of *Principia Mathematica* for Cambridge University Press). J.M.E. McTaggart (1866–1925) also passed his career at Cambridge, and began to publish his brand of logic in the 1890s; the most pertinent volume here is *A Commentary on Hegel's Logic* (1910). He shared with Russell the property of being elected to the Apostles (a secret debating society, largely based on undergraduates) on the recommendation of Whitehead,[5] but otherwise they came to have little philosophically in common; for while McTaggart came to

[4] W.S. Jevons, *Logica*, trans. C. Cantoni (Milan: 1878).

[5] See V. Lowe, *Alfred North Whitehead. The Man and His Work*, Vol. 1 (Baltimore: 1985), pp. 126–9. The membership of the Apostles to 1914 is given in P. Levy, *Moore: G.E. Moore and the Cambridge Apostles* (London: 1979), pp. 300–11.

his Hegelianism in rejection of a materialist standpoint, Russell's change was in the opposite direction, as we shall see in section 3 below.

At Oxford the principal logician was F.H. Bradley (1846–1924), educated there and Fellow of Merton College: he devoted himself entirely to research, and his principal work from our point of view is his *Principles of Logic* (1883). Closely allied in various ways was B. Bosanquet (1848–1923), who however only taught there for the decade between 1871 and 1881: his *Knowledge and Reality: A Criticism of F.H. Bradley's Principles of Logic* came out in 1885, when he was living in London, to be followed three years later by his *Logic, or Morphology of Knowledge*. A further Bradley disciple and critic was H.H. Joachim (1868–1939), Oxford trained and resident, taking the chair of logic in 1919; of chief concern here is his *The Nature of Truth* (1906). After graduation he spent three years at the University of St. Andrew's, where Bosanquet was to take a chair in 1903. Bosanquet moved there at the same time as the psychologist-philosopher G.F. Stout (1860–1944), whose role in the story is played mainly as the editor from 1891 to 1920 of *Mind*. Bosanquet stayed up north only for five years, but Stout remained until retirement in 1936.

To the same generation as Bradley and Bosanquet belongs J. Cook Wilson (1849–1915): like Johnson, his views did not appear until the 1920s, in the posthumous *Statement and Inference* (1926). His principal follower at Oxford was H.W.B. Joseph (1867–1943), whose *An Introduction to Logic* first appeared in 1906, with a revised edition a decade later. Joseph's contemporary F.C.S. Schiller (1864–1937) stayed mostly at Oxford until the 1920s, publishing his *Formal Logic* (1912) during that time; but in 1926 he moved to California and wrote *Logic for Use* (1929), sarcastically expressing his dislike for Oxford in the preface.

As this paper is centred around Russell and his position, a detailed account of these various philosophers is not appropriate here.[6] This is

[6] The potted biographies above have been drawn from various sources, including *The Encyclopedia of Philosophy*, the *Enciclopedia filosifica*, and the *Dictionary of National Biography*. Not surprisingly, the richest history of English philosophy for the period covered by this paper is by a German: R. Metz, *Die philosophische Strömungen in Grossbritannien*, 2 vols. (Leipzig: 1935). Vol. 1 contains accounts in its latter pages of all our Oxbridge philosophers, except Cook Wilson; he appears in Vol. 2, as do quite detailed surveys of the philosophy of Russell (and of Whitehead and Moore) and a summary of the rise of mathematical logic. This book appeared in English, as the single volume *A Hundred Years of British Philosophy* (London: 1938). The best text of English-speaking origin comes from Australia: J. Passmore, *A Hundred Years of Philosophy*, 2nd ed. (London: 1966), Chaps. 4–11 *passim*. See also n. 105.

I omit C.L. Dodgson (1832–1898), although his *Alice* books show him to be a philosophical logician of considerable penetration. Unfortunately he did not receive the attention that he deserved. See also n. 78 below.

a merciful relief, for by and large they are not the clearest or pithiest writers among their profession. Some principal features are outlined here; further details arise in section 4. Broadly speaking, most of them were idealists of an Hegelian orientation, though to varying degrees of adherence. Judgment and inference were the chief topics, although their character and content were disputed: whether a judgment was a psychological act, or the product of such an act, or the constituent of meaning; whether it dealt with reality (or Reality), or was part of Reality itself; whether an inference was a particular kind of judgment or not, and how each related to propositions. Other areas of study and dispute included categorical *vis-à-vis* hypothetical judgments, universals and particulars, and theories of truth (coherence, correspondence, and so on), the logic and character of relations, and logical and grammatical forms. Inductive or empirical logic was often included, as logic moved into the philosophy of science and thus to knowledge more generally.

This realm of concern was known as "philosophical logic"; normally it made no use of symbolism, apart perhaps from some modest use in expressing the various modes of syllogistic reasoning. When "formal" or "symbolic" logic was called for, the algebraic logic was drawn on, at least Boolean algebra (in the more "practical" version developed especially by Jevons); sometimes it included Schröder's logic of relations and/or Peirce's logic of relatives, together with the pertaining theory of quantification.[7]

The presiding genius over these traditions was not Boole or his successors, but Hegel. For Bradley, Bosanquet and especially McTaggart, Hegelian philosophy was a major influence on questions such as the status of logic *vis-à-vis* dialectic, the place of the Absolute, and the criticism of a psychologistic theory of judgment. But there were a variety of differences within the panorama. For example, in a variant version McTaggart replaced monism by a pluralist ontology admitting the

[7] One of the best Oxbridge presentations of the late nineteenth century is W.E. Johnson, "The Logical Calculus", *Mind*, n.s. 1 (1892): 3–30, 235–50, 340–57; it is especially interesting on quantification. As an example of the complications of this period, Johnson was positively influenced by Jevons, who was a strong critic of Mill; and none of these three subscribed to the various idealist traditions.

J.M. Baldwin's *Dictionary of Philosophy and Psychology*, Vol. 2 (New York and London: 1902) treated logic quite generously: L. Couturat and C. Ladd-Franklin covered "Symbolic Logic or Algebra of Logic" in some detail (pp. 640–51), supplementing C.S. Peirce's own account of "Logic" in general (pp. 23–7) and "Relatives (Logic of)" (pp. 447–50, where Russell and Whitehead's recent efforts were briefly noted and Schröder's contributions sharply criticized).

distinctness of individuals (while still remaining in harmony with the unity, of course). Again, Bosanquet criticized his fellow monist Bradley for maintaining a distinction between hypothetical and categorical judgments. Bradley himself took many of his logical ideas from other German sources, such as Lotze and Sigwart; as far as logic was concerned, he opposed both empiricist traditions such as Mill's and also psychologistic traditions, regarding a judgment as concerned with its meaning (its ideal content, one might say), not with an ensemble of physical examples, or with the mental act or event involved in forming it.

The empiricist tradition in England was most forcefully advocated by Cook Wilson, who broadly followed Mill, and also opposed ultimate explanations and the importance of judgments. However, inference was still a major concern. Similarly, Johnson and Stout adopted more eclectic positions, with varying (confused?) skeins of allegiance to idealism and to realism. Finally, Schiller also opposed Bradley and McTaggart, but followed W. James into a generally pragmatic standpoint, wondering if *all* types of logic may be empirical and claiming that all meanings were acquired only in use rather than being fixed in some Real or Ideal realm.

These authors did not intend their work to serve only at the "research" level: in addition, apart from Bradley they gave courses at their universities, sometimes assisted by more minor figures. This brings us naturally to the young Russell.

3. RUSSELL'S MATHEMATICAL LOGICISM

As an undergraduate, Russell acquired a reading list in philosophy in 1892 from Joachim. "I remember only two items in the list", he recalled later, "one was Bradley's *Logic* which, he said, was good but hard; the other was Bosanquet's *Logic* which, he said, was better but harder."[8] For the next five years he worked within Kantian and Hegelian traditions, the latter seemingly in a pluralist version; but when addressing the Apostles on the serious question "Seems, Madam? Nay, It Is", he found himself out of sympathy with both Bradley and McTaggart, and took as fundamental the distinction between appear-

[8] B. Russell, *My Philosophical Development* (London: 1959), p. 37. Russell was related to Joachim by marriage. The list is to be found in a letter probably written on 27 September 1892 (RA, file 710.110946).

ance and reality.[9] Then, in the following two years, two crucial experiences happened: firstly, with G.E. Moore (1873–1958) he broke completely with all idealist traditions in 1898 and took up a strongly realist and empiricist stance; secondly, he discovered Peano's mathematical logic at the International Congress of Philosophy in Paris in 1900.[10]

Under these influences Russell was able to reorientate his thought on the foundations of mathematics. After his fellowship essay, published as *An Essay on the Foundations of Geometry* (1897), he had been trying to ground "mathematical reasoning" on something or other. An effort of 1898 shows influence from Whitehead's *Universal Algebra* of that year, in the prominence of the manifold and the exploration of a logical calculus. Another draft of 1899–1900 was post-Moore but pre-Peano, and showed a much greater interest than he had hitherto exhibited in Cantor's set theory.[11] However, these drafts were largely set aside after the great discovery of Peano's work in 1900. To this new mathematical logic he rapidly added a logic of relations, and then advanced to the logicist thesis that it could all serve as the sole source and ground for "all" mathematics. He even gave a lecture course at Cambridge on "The Principles of Mathematics" in the academic year

[9] The manuscript is published in Russell's *Collected Papers*, Vol. 1: *Cambridge Essays, 1888–99*, ed. K. Blackwell, A. Brink, N. Griffin, R.A. Rempel and J.G. Slater (London: 1983), pp. 105–11. His notes of 1898 on McTaggart's lectures on Lotze are held in the Ottoline Morrell papers (n. *): I have not seen them. On this part of Russell's life, see N. Griffin, "The Tiergarten Programme", and on his concern with geometry at this time, which I do not discuss here, see J.L. Richards, "Bertrand Russell's *Essay on the Foundations of Geometry*, and the Cambridge Mathematical Tradition". Both these papers are to appear in the proceedings of the 1984 conference on Russell's early technical philosophy, held at Toronto.

[10] See *My Philosophical Development*, pp. 54–5, 65–8 (where the story on p. 65 of Peano's immediate delivery of his papers at the Congress is a slip of Russell's memory: they were posted on afterwards). Note the importance of *personal contact* with Peano: Russell had possessed since July 1898 at least one Peanesque essay, but its importance had not dawned on him (copy in RA of M. Pieri, "I Principii della geometria di posizione ...", *Memorie della Reale Accademia della Scienze di Torino*, (2), 48 [1899]: 1–62.

[11] These manuscripts, and various others, are due to appear in Russell's *Collected Papers*, Vol. 2. I shall not discuss his manuscripts in this paper, but refer for a survey to my "Bertrand Russell's Logical Manuscripts: an Apprehensive Brief", *History and Philosophy of Logic*, 6 (1985): 53–74 (here, pp. 57–9). They are deployed with great effect in N. Griffin, "Russell on the Nature of Logic (1903–1913)", *Synthese*, 45 (1980): 117–88. I also only cite Russell's unpublished correspondence, with little discussion of it.

1901–02, for the Mathematics Tripos, the first course of its kind offered in Britain;[12] but also in 1901 he found his paradox.[13]

Since this account brings out the difference between Russell's new approach and the rejected idealism, one similarity is worth stressing. While the metaphysics of neo-Hegelianism was now abandoned, its methodology was retained, to the important extent that he was sensitive to the detection of paradoxes in theories: indeed, he may at first have regarded this paradox as just another puzzle, to be resolved in some "higher" theory. However, he came to see that it was a much more fundamental setback than that, and—again in a neo-Hegelian vein—he collected as many paradoxes as he could, to gauge the extent of the difficulties. In this way he was lucky, as well as clever, to find his paradox at a relatively early stage—in contrast to Frege, for example, who had the first volume of his *Grundgesetze der Arithmetik* published and the second in proof before learning of it from Russell.

Despite this setback, Russell did not abandon the basic notion of logicism; he gave it extended prosodic treatment in his *The Principles of Mathematics* (1903), including a sketched attempt to solve the paradox. Stout found the book to be "fundamentally wrong";[14] but Russell pressed on, with the collaboration of his former teacher A.N. Whitehead (1861–1947), and they eventually produced three volumes of their *Principia Mathematica* (1910–13), in which the full symbolic story was revealed. In between a variety of papers appeared, the most substantial of those in English coming out in the *Proceedings of the London Mathematical Society* and in the *American Journal of Mathematics*, not in philosophical journals.

[12] See *Cambridge University Reporter*, (1901–02), p. 41. Russell lectured at 5.00 p.m. on Tuesdays and Thursdays for the first two terms. This was the only course that he gave: Whitehead's teaching is described in n. 15 below. Both men taught for the Special Board on Mathematics; meanwhile, over at Moral Sciences the usual courses were given by Keynes (backed by one Niven) in "Logic" for all three terms, and Johnson on "Advanced Logic" for two terms. This latter course was taught only on demand; see C.D. Broad, "William Ernest Johnson", *Proceedings of the British Academy*, 17 (1931): 491–514 (p. 502).

[13] See my "How Bertrand Russell Discovered His Paradox", *Historia Mathematica*, 5 (1978): 127–37; J.A. Coffa, "The Humble Origins of Russell's Paradox", *Russell*, nos. 33–34 (1979): 31–7; and also G.H. Moore, "The Roots of Russell's Discovery of Paradoxes in Logic", to appear in the proceedings of the 1984 conference described in n. 10.

[14] Letter to Russell, quoted in my "Bertrand Russell's Logical Manuscripts" (n. 11), p. 53. Moore wrote a long review for the *Archiv für Geschichte der Philosophie* (letter to Russell, 23 Oct. 1905, in RA), but it was never published. It is kept among the Moore papers, Cambridge University Library (not consulted); copy in RA.

Who were Russell's intellectual friends and supporters during this ime? At Cambridge there was Whitehead, who lectured in various spects of the foundations of mathematics and mathematical logic from 902;[15] and also the young G.H. Hardy (1877–1947).[16] Moore's backing ould only be of a general kind, and he even left the University for ome years in 1904. There was another supporter in P.E.B. Jourdain 1879–1919), an undergraduate participant in Russell's lecture course f 1901–02, who later wrote badly on the foundations of mathematics nd excellently on its history;[17] and a fellow mathematics student with ourdain at Trinity and also a member of the Apostles, R.G. Hawtrey 1879–1975), who made his name in economics (like yet another Apostle, Keynes *fils*) but who knew enough logic to read the manuscript of *Principia Mathematica*.[18] In Oxford, where Russell lived from 1905 and wrote much of *Principia Mathematica* and many of his philosophical papers, there was G.G. Berry, employee of the Bodleian Library and 'the only man I found in Oxford who knew any mathematical logic."[19] Elsewhere very few people could take an informed interest; the barister A.B. Kempe (1849–1922), who refereed one of his papers for the London Mathematical Society;[20] A.T. Shearman (1866–1937), student

[15] As Whitehead did not blossom as a philosopher largely until after this period, and in any case, unlike Russell, not as an offshoot of logicism, I shall not discuss his position in detail here. A welcome source on his mathematics is provided in Lowe's biography (n. 5), Chaps. 9–14 (here, pp. 208–12 on Whitehead's Cambridge teaching). According to the *Cambridge University Reporter* (see n. 12), Whitehead offered one or two terms' worth of lectures under varying titles. Since the times came to be arranged at mutual convenience, the numbers of students were presumably small (see Lowe, p. 311). In the late 1890s he had taught his universal algebra.

[16] Hardy's early mathematical interests included set theory, and the surviving letters (both sides) in RA show his lively interest in Russell's work. A study of the two men would be worth a paper.

[17] See my *Dear Russell—Dear Jourdain* (London: 1977) on their relationship and correspondence.

[18] See Hawtrey's letters to Russell in RA. Another undergraduate at Trinity at this time was Jevons's son, Herbert, who also became a well-known economist.

[19] Russell's note attached to the exceedingly interesting letters from Berry. It appears from this note that Berry invented the visiting-card paradox as well as the one which Russell named after him (see *Dear Russell—Dear Jourdain*, pp. 50–1, 175–6).

[20] On the refereeing, see my "The Russell Archives: Some New Light on Russell's Logicism", *Annals of Science*, 31 (1974): 387–406 (at p. 398). Kempe's "Memoir on the Theory of Mathematical Form", *Philosophical Transactions of the Royal Society of London*, 177 (1886): 1–70, is a good contribution to foundations: it has never exercised influence, and is quite forgotten. His *Nachlass* was sorted out by the Historical Manuscripts Commission, during the 1970s, and is now stored in the West Sussex Record Office, Chichester (file NRA 17595): it includes some letters from Russell (of which the replies are in RA), Peirce and various mathematicians.

and then Fellow of University College London, who was soon to write in the new vogue;[21] and H. MacColl (1837–1909), also a London graduate and then a retired Boulogne schoolmaster.[22] When compared with the Oxbridge group, it was not a prestigious gathering.[23]

4. SIMILARITIES AND DIFFERENCES BETWEEN THE OLD AND NEW LOGICS

So far the outline of these old and new traditions in logic has been sketchy; in the rest of the paper I probe a little more deeply, comparing in this section some aspects of the various traditions under a quintet of headings, as examples from the wide range of issues that could be raised. Then I provide an ensemble of reactions to, and treatments of, them during the 1910s and 1920s in the next two sections. However, the account of the various logics is still very selective, while hopefully indicating the character of the larger, unknown story. Four limitations are worth noting now: I have used Bradley's *Logic* (1883) and Russell's *Principles* as the main sources for the respective traditions; in general I cite authors' books more than their papers, and not all of them even then; among the periodical literature *Mind* is given preference, as the single most important venue for dispute; and philosophical issues outside logic are touched on only occasionally.

4.1 *Logic in "logical" order.* As is now well known, Russell's mathematical logic was laid out in a pretty clear order of topics. The *Principles*, for example, started out with the propositional and predicate calculi; ran through related topics such as implication, denoting and the status of classes and relations; then dealt with the logicistic definitions of finite cardinal and ordinal numbers, the real numbers and continuity, transfinite arithmetic; and then moved to the calculus, parts of geometry, space, matter, and elements of dynamics. The influence of mathematics on the imposition of order lies not only in the order adopted but also on the growth in importance during the nineteenth century of axiomatic methods, or at least systematic presentations. The

[21] Shearman's writings are described in subsection 5.1 below. He left money to finance an occasional series of lectures in logic, which are still held at University College London.

[22] MacColl's work is noted in subsection 4.5 below.

[23] Of course, Russell was gaining some appreciation abroad, where things always happen more quickly: not only Peano's circle but also Couturat in France, and Veblen, Huntington and Bôcher in the U.S.A. Letters from them survive in RA. A broader study of Britain than is attempted here would include other figures: R.B. Haldane, and E.E.C. Jones, for example.

calculus is a particularly important case; from the 1820s on Cauchy argued not only for an approach based on limits (which became dominant thereafter) but also for an ordered treatment of the material.

By contrast, the traditional logicians did not stress order so strongly; in particular, those of a (neo-)Hegelian persuasion rather deprecated it. Bradley's *Logic* begins with the words: "It is impossible, before we have started Logic, to know at what point our study should begin. And after we have studied it, our uncertainty may remain."[24] He chose to lead off with judgments, covering in his first Book (including an oddly titled "Chapter II (*continued*)") their "general character" and principal forms, followed by basic principles (identity, contradiction, excluded middle and double negation) and their "quantity" (principally, extension versus intension) and modality. The second Book covered inference, a first part on its general character and some special forms, the second dealing with association of ideas, and modes of inference, including Jevons's logical machine. The third and last Book also treated inference, with an ensemble of considerations in two ill-divided parts, including its general characteristics, analysis and synthesis, "the cause and the because", and validity. In addition, he began the terminal essays of the second edition (1922) with the meta-judgment: "In treating of inference, judgment and ideas, whatever order we adopt has its own disadvantage."[25]

4.2 *Inference or implication?* By contrast with Bradley, inference received little attention in Russell's *Principles*; the index lists only four entries. For him implication was to be preferred, although to the modern view inference was mixed up with it. His longest essay on implication was a paper of 1906, which stated early on that

> in order that one proposition may be inferred from another, it is necessary that the two should have that relation which makes one a consequence of

[24] F.H. Bradley, *The Principles of Logic*, 2nd ed. (Oxford: 1922), p. 1. I cite this version as it contains the text of the first (1883) edition unchanged; as is described in subsection 6.3 below, the new material was all additional. A valuable introduction to Bradley's philosophy, and that of some of his contemporaries, is furnished in A. Manser, *Bradley's Logic* (Oxford: 1983); here, see pp. 47–8. See also Manser and G. Stock, eds., *The Philosophy of F.H. Bradley* (Oxford: 1984).

Both sides of a quite extensive correspondence between Russell and Bradley are available in RA. Bradley's papers are conserved at Merton College, Oxford (not consulted).

[25] Bradley, *ibid.*, p. 597.

> the other. When a proposition q is a consequence of a proposition p, we say that *p implies q*. Thus deduction depends upon the relation of *implication*....[26]

A similar scan of the index of Bradley's *Logic* shows that implication received no treatment in the first edition. However, since like Russell the idealists did not distinguish logic from metalogic, the difference here should not be over-stressed. For example, in the *Principles*, Russell agreed with Bradley that "in a particular inference, the rule according to which the inference proceeds is not required as a premiss."[27]

Russell's index omitted "judgment" entirely; but he did use a somewhat similar notion, that of the *assertion* of a proposition. This was introduced in his account of implication, to solve "a very difficult problem". The difficulty was caused by his failures to distinguish implication from inference and use from mention; for it was caused by the (alleged) fact that the rule of *modus ponens*, which "eludes formal statement, and points to a certain failure of formalism in general", allowed the antecedent p, when asserted as true, to be dropped from $p \supset q$, so that q was asserted on its own, and thus "proved".[28] Again in the 1906 paper,

> If I say "Caesar died", I assert the proposition "Caesar died"; if I say "'Caesar died' is a proposition", I make a different assertion, and "Caesar died" is no longer asserted, but merely considered.[29]

This distinction is not the same as the traditional one between categorical and hypothetical judgments, at least for our idealists, where the issue hung on reference to facts or to ideas, on the status of universal affirmation, and related matters.[30]

The issue of implication and inference was aired between Bradley and Russell in 1910 in *Mind*. In response to Bradley's vaguely phrased view that implication means "that something is both itself and more than itself",[31] Russell held that in an inference, "the premiss and the

[26] B. Russell, "The Theory of Implication", *American Journal of Mathematics*, 28 (1902): 159–202 (p. 159).

[27] B. Russell, *The Principles of Mathematics* (Cambridge: 1903; repr. London: 1937), p. 41, in the chapter on "Implication and Formal Implication".

[28] *Ibid.*, pp. 34–5.

[29] Russell (n. 26), p. 161.

[30] Bradley (n. 24), pp. 41–90. For commentary, see Manser (n. 24), pp. 107–17.

[31] F.H. Bradley, "On Appearance, Error and Contradiction", *Mind*, n.s. 19 (1910): 153–84 (p. 180). Note also his solution to Russell's paradox by denying self-membership to classes (pp. 181–2; compare his *Logic* [n. 24], pp. 174–82), his association of zero with negation (pp. 183–4), and his great general praise for Russell (pp. 178–9, 185).

implication are known to us first, and are the means by which we come to know the conclusion."[32] Unfortunately his principal writings are less clear on the point. One reason, the failure to distinguish theory from meta-theory, was mentioned above. One might say that his revolt into pluralism did not go far enough, for his logic is monistic in the sense discussed here.

4.3 *For or against the copula*. One of the best known and recognized features of the mathematical logic of Peano and Russell (and also of Frege) was the replacement of subject–predicate analyses of propositions in terms of "is" or "are" by dissection into propositional function and its argument, with all the attendant consequences for quantification theory, the role of both membership and inclusion in set theory, the distinction between nothing and the empty set, new insights on intension and extension, and so on. The importance of this novelty was well understood at the time: for example, Burali-Forti's little manual *began* by saying that "we write the sign ε, first letter of the word εστί, in place of the affirmation *is a*."[33] Russell gave it similar prominence in his writings, initially in a manuscript apparently written for *Mind* around 1901 but for some regrettable reason not published,[34] and again in the *Principles*, especially in the chapters on classes and on his paradox.[35] The traditionalists were very slow (or even incapable) of spotting the importance of these issues; they stuck to the extensionalistic whole–part theory, a theory of collections inherited from Boolean algebra. Thus they continued to dissect with some crude instruments.[36]

This ignorance of the "new" set theory is well evident in the reactions to Russell's paradox itself. For example, Cook Wilson, writing to Bosanquet in 1904, found it to be a "rather foolish fallacy"; for him it was "a fallacious verbal mistake of Russell's in fact that a *class as many* could be distinguished from a *class as one*", since "a class as one" was

[32] B. Russell, "Some Explanations in Reply to Mr. Bradley", *Mind*, n.s. 19 (1910): 373–8 (p. 375); note also his claim now to eliminate classes (pp. 376–7), and his dismissal of Bradley's views on negation (pp. 377–8).

[33] Burali-Forti (n. 2), p. 1.

[34] B. Russell, "Recent Italian Work on the Foundations of Mathematics" (RA); to appear in *Collected Papers*, Vol. 3. I ascribe its destination as *Mind* from consideration of its length, level, and similarity of title to his "Recent Work on the Philosophy of Leibniz", *Mind*, n.s. 12 (1903): 177–201.

[35] *The Principles of Mathematics*, Chaps. 6 and 10; compare his letter to Jourdain in *Dear Russell—Dear Jourdain*, p. 133.

[36] However, the traditionalists did not necessarily only affirm subject/predicate dissections; in particular, Bradley rejected them for judgments. However, his motivation lay in his monism (see N. Griffin, "What's Wrong with Bradley's Theory of Judgment?", *Idealistic Studies*, 13 [1983]: 198–225).

"a unified manifold of elements" and so could not belong to itself.[37] Unfortunately, this form of an extensionalistic view of classes rendered whole areas of respectable mathematics impossible. The mathematical ignorance of the idealistic philosophers inured them from the discomfort of this knowledge;[38] but Wilson is a different case, for he had a mathematical training and indeed in 1905 published an interesting book on graph theory[39] (then indeed a little developed branch of mathematics). So his hostility seems a little hard to understand.

As has been mentioned in section 3, the difference in mathematical competence between the old and the new logicians is one of their chief points of difference. Now one of the novelties of mathematical logic was its *non-equational* character, in contrast with Boolean algebra and its extensions into the logic of relations and relatives, where "=" referred to the co-extensionality of classes and/or the equivalence of propositions. In addition, it paid little attention to duality, which, especially in Schröder's hands, had become a prominent feature of the post-Boolean algebraic logic.[40] Further, in being applied to (or, in Russell's view, embracing) arithmetic, it had bearing on quantitative mathematics, a possibility which traditional logic could not emulate.

4.4 *Pluralism, relations and non-existence.* Most of the idealists were monists, and as a consequence regarded relations as internal (that is, in some way adherent to the objects which they related). Bradley did not discuss the matter in his *Logic*, but gave it prominence in his succeeding *Appearance and Reality* (1893). "Every quality in relation has, in consequence, a diversity within its own nature", he wrote there, not very limpidly, "and this diversity cannot immediately be asserted of the quality. Hence the quality must exchange its unity for an internal relation."[41]

[37] Cook Wilson's letter is published in his posthumous *Statement and Inference*, ed. A.S.L. Farquharson, 2 vols. (Oxford: 1926), I: cix–x. He had passed on these views to Stout, who echoed them to Russell in letters of Russell of 1901 and 1903 (held in RA). This view is similar to Bradley's cited in n. 31.

[38] Take, for example, J.M.E. McTaggart, *A Commentary on Hegel's Logic* (Cambridge: 1910), Chap. 3, on "Intensive and Extensive Quantity". In his preface he thanked Russell for reading this chapter, but on its p. 43 he claimed that his treatment "would only be slightly affected" by Russell's recent claim that Hegelian philosophy was irrelevant to criticisms of mathematics ("Mr. Haldane on Infinity", *Mind*, n.s. 17 [1908]: 238–42 [p. 242]).

[39] J. Cook Wilson, *On the Traversing of Geometrical Figures* (Oxford: 1905).

[40] On algebraic logic see, for example, F. Barone, "Peirce e Schroeder", *Filosofia*, 17 (1966): 181–224. For a fairly extensive comparison of it with mathematical logic, see my "Wiener on the Logics of Russell and Schröder ...", *Annals of Science*, 32 (1975): 103–32.

[41] F.H. Bradley, *Appearance and Reality* (London: 1893), p. 31.

Russell had been interested in relations during his idealist phase,[42] and maintained his interest after his revolt against idealism; but on finding such a monism in his studies of Leibniz he turned to a pluralist position, in which the relation was held to be independent of its arguments.[43] In the *Principles* he quoted the above passage from Bradley, and argued against its position in his account of pluralism.[44] The matter came up again in their 1910 exchange in *Mind*, where Russell defended his view that a complex could be analyzed into its components and still retain a certain sense of unity in itself.[45] The differences over relations were a particularly important example of the disaccord.

Russell took a non-standard position on reference in 1905 when he published his paper "On Denoting". Famous today, it faced reluctance from editor Stout over its acceptance for publication.[46] This theory is now so well known that an account is not needed here: the main point to stress here is its strict stipulations on the reference of definite descriptions, in tune with his new realist and reductionist views, and strongly in contrast to the luxurious Hegelian world of real objects. "Wherever we predicate, we predicate about something which exists beyond the judgment", claimed Bradley in his *Logic*, "and which (of whatever kind it may be) is real, either inside our heads or outside them. And in this way we must say that 'is' never can stand for anything but 'exists'."[47]

[42] See Russell's letter of 13 September 1898 to Moore, and his manuscript of 1899 entitled "The Classification of Relations" (both in RA, the latter to appear in *Collected Papers*, Vol. 2). Moore entered tardily into this discussion with his paper "External and Internal Relations", *Proceedings of the Aristotelian Society*, n.s. 20 (1919–20): 40–62 (revised in his *Philosophical Studies* [London: 1922], Chap. 9).

[43] See Russell's account in *My Philosophical Development*, pp. 54–5.

[44] *The Principles of Mathematics*, pp. 221–6 (quotation on p. 224).

[45] Bradley, "On Appearance, Error and Contradiction" (n. 31), pp. 179–80; Russell, "Some Explanations" (n. 31), pp. 373–4. For commentary , see R. Jager, *The Development of Bertrand Russell's Philosophy* (London: 1972), pp. 53–99; and J. Vuillemin, *Leçons sur la première philosophie de Russell* (Paris: 1968), pp. 222–7.

[46] B. Russell, "On Denoting", *Mind*, n.s. 14 (1908): 479–93; various reprints, including in Russell's *Logic and Knowledge*, ed. R.C. Marsh (London: 1956), pp. 39–56. On Stout's unwillingness, see Russell, *My Philosophical Development*, p. 83.

Another interesting point of comparison between Russell and Oxbridge, not studied here, is their respective interpretations of Meinong.

[47] Bradley, *The Principles of Logic*, p. 42. In his paper Russell praised Bradley's *Logic* at one point (*ibid.*, p. 481); but this was in agreement with the view that universal affirmative propositions (for Russell, judgments for Bradley) were hypothetical rather than categorical. On the difference between these two positions, see D.S. Scarrow, "Bradley's Influence upon Modern Logic", *Philosophy and Phenomenological Research*, 22 (1961–62): 380–2.

Mathematics seems to have had an influence on Russell's theory, too, in that the interest in definite rather than indefinite descriptions surely arose from the need to express values of (single-valued) mathematical functions in his developing logicism. His interest in Peano had initially been sparked by Peano's insistence at the 1900 Paris congress that definitions of classes needed a symbol for "the".[48]

Along with Russell's rejection of monism came a dismissal of the absolute theory of truth, for it too drew on an all-embracing conception. One would have expected, therefore, Bradley to have advocated it strongly; but his *Logic* is rather quiet on truth, and the remarks he presented are far from happy. For example, he presented the "Principle of identity" as "Truth is at all times true", which he confessed to be

> not very clear, but perhaps it will find acceptance with most readers. What it means, however, is much more definite, and will be much less welcome. The real axiom of Identity is this: *What is true in one context is true in another.* Or, if any truth is stated so that a change of events will make it false, then it is not a genuine truth at all.
>
> To most readers this axiom, I have little doubt, will seem a false statement.[49]

Jourdain was to pick this up in 1909 in a letter to Russell, wondering if "Bradley thought that at least one of his readers will accept a principle whose meaning seems to the reader false." Russell, however, interpreted it as Bradley giving "2 statements of his principle, and expects his readers to think that they express *different* principles."[50] Around this time, in fact, Bradley was systematizing his thoughts on truth in the volume *Essays on Truth and Reality* (1914); but while he pinned his hopes on a coherence theory of truth, he found absolute truth to be unobtainable.[51]

A strong claim for the coherence theory of truth, with its attendant idealism and even monistic aspirations towards absolute truth (and also internal relations), came forth in 1906 in Joachim's book *The Nature of Truth*. Russell read the chapter on his and Moore's new realism,[52] and

[48] See Russell's report of this detail to Wiener in my "Wiener on the Logics of Russell and Schröder ..." (n. 40), p. 110.

[49] Bradley, *The Principles of Logic*, p. 143.

[50] See my *Dear Russell—Dear Jourdain*, pp. 117–18.

[51] See D. Holdcroft, "Bradley and the Impossibility of Absolute Truth", *History and Philosophy of Logic*, 2 (1981): 25–39. See also n. 98 below.

[52] H. Joachim, *The Nature of Truth. An Essay* (London: 1906); Russell's reading was acknowledged on p. 5.

wrote against Joachim in a paper of the following year delivered to the Aristotelian Society. He found the correspondence theory most congenial to his position: a true proposition corresponds to a fact, and the ensemble of truths is a piecemeal affair, *not* glued together as some sort of monistic intension.[53] However, his view left him with difficulties of its own, for he could not allow a *false* proposition to correspond to some "objective falsehood" (his phrase) without infringing his desire to eliminate abstract objects. This quandary helped him to abandon his currently favourite logicistic system, his "substitutional theory", for it assumed truth-values of propositions among its objects.[54] But he never sorted himself out on the matter: when reprinting the 1907 essay in 1910 he omitted the part on objective falsehoods,[55] and he both admitted and denied propositional quantification in *Principia Mathematica*.[56] Later Joachim rather mercilessly pointed this out, and claimed that Russell's abandonment of false propositions "betrays a deplorable weakening of his faith."[57]

4.5 *The feasibility of "possible"*. Another logician of this time who admitted a richer realm of existence than Russell's, without however ascending to the Hegelians' capacity for reification, was MacColl. He evolved some individual views of his own on logic from the 1870s, which at last gained some currency at the beginning of the new century, when he was in his sixties.

We take first the last two of a sequence of short papers placed by MacColl in *The Athenaeum*. In the first he presented his view of "possible statement" as one "not incompatible with our data, or definitions", contrasted it with Bradley's definition as a consequent of a set of conditions of which some actually obtained, and noted that Russell had used this form in his 1897 *Essay* on geometry. In the last paper he

[53] B. Russell, "On the Nature of Truth", *Proceedings of the Aristotelian Society*, n.s. 7 (1906–07): 28–49. Compare his paper of the same title in *Mind*, n.s. 15 (1906): 528–33; by contrast, see R.F.H. Hoernlé's largely enthusiastic review of Joachim's book on pp. 546–53.

[54] For details of this matter, see my "The Russell Archives" (n. 20), 398–401; certain other factors were involved in the demise of the substitutional theory. A selection of the pertinent manuscripts is planned for *Collected Papers*, Vol. 4; one was published posthumously in Russell's *Essays in Analysis*, ed. D.P. Lackey (London: 1973), pp. 165–89, though bereft of editorial explanation.

[55] B. Russell, *Philosophical Essays* (London: 1910), Chap. 6.

[56] In *Principia Mathematica*, Vol. 1 (Cambridge: 1910), compare pp. 41 and 129 (printing of the 2nd edition of 1925).

[57] H. Joachim, *Logical Studies*, ed. L.J. Beck (Oxford: 1948), pp. 257–60: see also pp. 230, 244–51 on aspects of Russell's thought. This book was posthumously edited by Joachim's son-in-law.

criticized as illegitimate the use of "possible" in advocating non-Euclidean geometries, including infinitesimals as existent objects.[58] In a short reply Russell modified Bradley's definition slightly, and exposed the unhappiness of MacColl's applications to geometry.[59]

A more extended statement appeared in *Mind* the following year, when MacColl took the view that the existential import of propositions had to extend to a "universe of unrealities" as well as to that of "realities"; however, rather than granting existence all round he admitted the categories of impossibility and certainty into predication.[60] In a short reply Russell quibbled over the need for more realities, appealing to his theory of definite descriptions (which was soon to appear in the journal) for a criterion of existence of realities and to (Peano's) definition of the non-emptiness of a class to define its existence as the sense "used in symbolic logic".[61] (As in fact Russell used the word "existence" in still other senses,[62] his explanation here was not complete!) MacColl held his ground, with the example of two disjoint classes of real members whose empty intersection class was apparently "logically equivalent to an unreal class made up of unreal members."[63]

In 1906 MacColl's *Symbolic Logic* appeared, a slim book of 140 pages. He presented this theory in considerable detail, admitting not only "true" and "false" but also "certain" (as always true), "impossible" (as always false) and "variable" (as possibly true sometimes and possibly false otherwise); he also developed a symbolic language to state these properties of predicative propositions.[64] *Mind* sent the book for review to Russell, who praised many of its features, but found the five-fold distinction just mentioned as arising "from the fact that he deals rather with verbal expressions than with what is meant by them" and

[58] H. MacColl, "Symbolic Logic. VI" and "VII", *The Athenaeum*, nos. 4,005 and 4,007 (30 July and 13 Aug. 1904): 149–51, 213–14, citing Bradley from Keynes's *Logic* and Russell from *An Essay on the Foundations of Geometry* (Cambridge: 1897), pp. 198, 199.

[59] B. Russell, "Non-Euclidean Geometry", *The Athenaeum*, no. 4,018 (29 Oct. 1904): 592–3.

[60] H. MacColl, "Symbolic Reasoning", *Mind*, n.s. 14 (1905): 74–81; also in Russell, *Essays in Analysis*, pp. 308–19.

[61] B. Russell, "The Existential Import of Propositions", *Mind*, n.s. 14 (1905): 398–401; also *Essays in Analysis*, pp. 98–102.

[62] See the gory details in *Dear Russell—Dear Jourdain*, pp. 98–102.

[63] MacColl, [Reply to Russell], *Mind*, n.s. 14 (1905): 401–2; also in *Essays in Analysis*, pp. 319–22.

[64] H. MacColl, *Symbolic Logic and Its Applications* (London: 1906), pp.1–23, for the outline. Much of his theory was developed in his earlier papers on logic, which are not discussed in this paper.

thus was "importing into logic the defects of common speech". Further, the three extra categories could apply to propositional functions, of which the values could vary with different values of the argument variable(s); but even then MacColl's unrealities should not be admitted into the ranges of these variables.[65]

The disagreement between the two men rumbled on in the pages of *Mind* over the following years, without adding any essentially new points.[66] MacColl's work is seen now as a pioneering contribution toward modal logics, whose aims include the encapsulation of some features of common speech against which Russell had demurred. In many ways MacColl's advocacy was not clear (especially the status of the unreal entities); but in his lonely isolation on the French coast he came to a new view as distant from the Oxbridge orthodoxies as from Russell's new views. Russell himself appreciated this novelty, for at the end of his review of the book he wrote:

> The present work is not quite in line with those of other current writers on symbolic logic; but it has merits which most of their works do not have, and it serves in any case to prevent the subject from getting into a groove. And since one never knows what will be the line of advance, it is always most rash to condemn what is not quite in the fashion of the moment.[67]

5. SOME APPRAISALS OF LOGIC OF THE 1910S

Between 1910 and 1913 Whitehead and Russell published the first three volumes of their projected *Principia Mathematica*,[68] and logicism received a definitive printed statement. But in this section I concern myself with a few other writings of this period on logic, to see how logicism was handled. The order of treatment is chronological in publication.

5.1 *Shearman on "symbolic logic"*. As was mentioned in section 2, Shearman taught logic at University College London, during the 1900s. In 1906 he published a volume on *The Development of Symbolic Logic*. In his preface he declared himself as starting out from the opinion

[65] B. Russell, [Review of *ibid.*], *Mind*, n.s. 15 (1906): 255–60.

[66] H. MacColl, "Symbolic Logic (a Reply)", *Mind*, n.s. 15 (1906): 470–3; "'If' and 'Imply'", 17 (1908): 151–2; B. Russell, "'If' and 'Imply', a Reply to Mr. MacColl", pp. 300–1; MacColl, "'If' and 'Imply'", pp. 453–5. There is also a substantial group of letters from MacColl of this period in RA.

[67] Russell, review of MacColl (n. 65), p. 260.

[68] Whitehead planned and partly wrote a fourth volume on geometry, but he abandoned it and had the manuscripts destroyed upon his death.

expressed in correspondence by Johnson, to the effect that it was an error to regard the various symbolic systems as radically distinct and that instead there was "the logical calculus" towards which all were working. Luckily Shearman did not base his text upon this false premiss but surveyed some of the practitioners of algebraic logic, MacColl's work (of which he was very critical) and the mathematical logic of Peano, Frege, Russell and Whitehead.[69] While no clear-cut impression comes over, the book would have given the uninformed reader some idea of the current interest in symbolic logic(s). His review of Russell's *Principles* in *Mind* for 1907 showed a good grasp of the book; he pointed to several non-trivial infelicities of phrase, proposed the extensionalistic solution to the paradox (but with a more careful discussion of the issue than Cook Wilson's noted in subsection 4.3 above) and gave some attention to the latter parts of the book, where dynamics and matter were treated. He also took in the recent discussion in *Mind* between Russell and MacColl.[70]

Shearman's other volume, *The Scope of Formal Logic*, came out in 1911; and instead of a panoramic survey he concentrated solely on mathematical logic. In 160 pages he covered not only the basic components of mathematical logic but also the construction of number and space.[71] Again, while his treatment was neither penetrating nor free from slips, it was a serviceable introduction; indeed, it was one of the first reasonably extended statements in English on Frege's work after Russell's appendix to the 1903 *Principles*.[72]

5.2 *The Encyclopaedia Britannica*. Normally one would not regard general encyclopaedias as appropriate sources in a context such as this; but the eminent eleventh edition of the *Encyclopaedia Britannica*, which appeared in twenty-nine volumes in 1910 and 1911, merits its inclusion here. Logic was dealt with almost entirely in a long article under that name, forty pages in length, in two roughly equal parts. The respective

[69] Shearman also included an extensive study of Castillon's proposed system of 1803 (*The Development of Symbolic Logic* [London: 1906], pp. 94–134). MacColl replied to his criticisms in a suite of writings which partly intersect with the exchange with Russell cited in nn. 63–6.

[70] A.T. Shearman, [Review of *The Principles of Mathematics*], *Mind*, n.s. 16 (1907): 254–65. Note that it appeared four years after Russell's book: presumably Shearman wrote it after Johnson failed to produce one (on this, see Broad [n. 12], p. 503).

[71] A.T. Shearman, *The Scope of Formal Logic* (London: 1911). He did not include *Principia Mathematica*, Vol. I in his account (p. xi), as it had only just been published. Some letters from him survive in RA.

[72] Note the sharp review of Shearman *ibid.* by A.E. Taylor in *Mind*, n.s. 21 (1912): 264–7, itself notable for the reviewer's deeper knowledge of Frege. Compare n. 94 below and text.

authors were Oxford men, though neither was an eminent logician of his day. In "The Problems of Logic" T. Case covered syllogisms, Aristotle's tradition, empirical logic, judgment and inference: Kant and Hegel, and their followers, were the preferred modern writers, and they dominated the bibliography. Then H.W. Blunt dealt with the history by running through the Greeks and the scholastic and Renaissance periods, followed by Hobbes, Locke, Kant and the Hegelians again: a short final section on "Logic from 1830–1910" discussed only Bradley and Bosanquet of recent English writing.[73] Mathematical logic appeared only in the four-and-a-half-page article on "Mathematics", and then because its author was Whitehead.[74]

Thus the new approach had only gained a modest place in this general survey of knowledge. Furthermore, it was seen as part of mathematics rather than of philosophy, like the teaching of Whitehead and Russell at Cambridge noted in section 3.

5.3 *Jourdain's contributions*. Between 1910 and 1913 Jourdain published a sequence of lengthy articles on the history of "mathematical logic and the principles of mathematics" in the *Quarterly Journal of Pure and Applied Mathematics*. His choice of subjects is interesting—in order, Leibniz, Boole, MacColl, Frege, Peano and Jevons—for it reflects his interest both in mathematical and algebraic logic. Russell was not a subject, but he read all the manuscripts;[75] in addition, MacColl, Frege and Peano read the drafts of the pieces on themselves, and Jourdain incorporated their comments into the final version. Further, the articles on Boole and Jevons drew on hitherto unknown manuscript sources, one group in fact now being lost.[76] For anyone capable of following the technicalities, they gave a good insight into the newer developments, and the differences between the various approaches: the survey of MacColl's contributions was especially judicious. Probably the Oxbridge logicians never even heard of the articles.

[73] "Logic" in *Encyclopaedia Britannica*, 11th ed., Vol. 16 (1911): 879–96 (Case), 896–919 (Blunt). For a list of the entries for philosophy, see Vol. 29 (1911): 939–40.

[74] A.N. Whitehead, "Mathematics", *ibid.*, Vol. 18 (1911): 879–82. There were many articles elsewhere on individual branches of mathematics, including reprints from the tenth edition of his piece on the axioms of geometry, and his revision of Russell's survey of non-Euclidean geometry (Vol. 11 [1910]: 730–6, 724–30).

[75] See *Dear Russell—Dear Jourdain*, pp. 118–19, 125, 128, 132.

[76] P.E.B. Jourdain, "The Development of Theories of Mathematical Logic and the Principles of Mathematics", *Quarterly Journal of Pure and Applied Mathematics*, 41 (1910): 324–52 (Leibniz, Boole); 43 (1912): 219–314 (MacColl, Frege and Peano); and 44 (1913): 113–28 (Jevons). The lost material is the bound volume of Booleana cited by Jourdain as "H." (p. 332).

Jourdain also publicized and discussed Russell's logic in various other writings. A pleasant version was a suite of satirical pieces which appeared occasionally between 1905 and 1907 in *The Granta*, the Cambridge University undergraduate journal,[77] and then were expanded into an article called "The Philosophy of Mr. B*rtr*nd R*ss*ll" in *The Monist*.[78] In their way they subtly pointed to many aspects of mathematical logic; and in a book review in *Mind* Jourdain showed his own insight very strikingly when he urged logicians to "follow Frege" and distinguish between Paris and "Paris".[79] Thus he recognized the need to distinguish a language from its metalanguage, and moreover that Frege had perceived it already. These are commonplaces today, but then they were painfully absent from logic, whether in its traditional or new forms.

5.4 *Schiller on logic.* So far we have not noticed F.C.S. Schiller; but his *Formal Logic* (1912) gives us an opportunity. As a pragmatist (he led an English version of American pragmatism, which he called "humanism") he was enchanted neither by the idealists nor by Russell—indeed, he was more of an outsider than anyone else. For him logic was strictly dependent on psychology, a function of emotion, intention, and other such personal factors; his book was sub-titled "A Scientific and Social Problem". Hence, as he stressed his preface, "*It is NOT possible to abstract from the actual use of the logical material and to consider 'forms of thought' in themselves, without incurring thereby a total loss, not only of truth but also of meaning.*" As one consequence, "A *Fallacy* is a reasoning which may be known to be 'bad' from an inspection of its form" (although elsewhere form was not to be distinguished essentially from matter). This definition was broad enough to earn Russell his one mention in the work, for having "delighted the philosophic world with many puzzles of the self-contradictory kind"; but he gave as an example a remark by Russell on multiplication sums which will never be thought of, a situation which only a pragmatist would find

[77] P.E.B. Jourdain, "The Wise Man's Problem", *Granta*, 18 (1904–05), May-week number, pp. 23–4; "Four Little Puzzles ...", 20 (1906–07): 123–5; "Essays on Logic. By the Hon. Bertrand Russell", *ibid.*, p. 158.

[78] P.E.B. Jourdain, "The Philosophy of Mr. B*rtr*nd R*ss*ll", *The Monist*, 21 (1911): 483–508. A succeeding article appeared in 26 (1916): 24–62; and the two were revised into a short book of this title (London and Chicago: 1918), followed by a selection from the writings of Dodgson to reveal his logical skill (compare n. 6 above). Russell contributed several of the chapters (see *Dear Russell—Dear Jourdain*, pp. 125–7). Among Jourdain's more serious writings on Russell see "Mr. Bertrand Russell's First Work on the Principles of Mathematics", *The Monist*, 22 (1912): 149–58.

[79] P.E.B. Jourdain, [Review of H. Berkeley, *Mysticism in Modern Mathematics* (Oxford: 1910)], *Mind*, n.s. 20 (1911): 88–97 (p. 91).

fallacious.[80] Naturally, he also adopted the pragmatic theory of truth, in which truth is defined in terms of usefulness.

Russell reviewed the book sarcastically for *The Nation*, deploring Schiller's ignorance of the development of logic since Boole and judging the pragmatic theory of truth to be the syllogistic conclusion drawn from the premisses "Everything in Aristotle is rubbish" and "All truth is in Aristotle."[81] A correspondence sprang up between the two as a result of this review, in which Schiller wanted to apply truth theory to propositional functions (like MacColl earlier) and tried to associate judgments with useful truths; so the discussion was not very profitable.

5.5 *The logic "encyclopaedia".* In 1913 appeared the first volume of the *Encyclopaedia of the Philosophical Sciences*, a 270-page work subtitled "Volume 1. Logic". The forms of words already quoted show its Hegelian leanings, as does its composition as the English edition of a German project; but the text following was not single-minded, and indeed shows well the various conflicting but active styles in logic. The Hegelian side was represented by enthusiastic pieces by the Germans A. Ruge and W. Windelbrand (the joint editors of this encyclopaedia) and a pompous contribution by the Italian B. Croce; but in addition a long essay by the American J. Royce, while affirming a version of pragmatism, made extensive use of Russell's logic of relations in developing a general theory of order, and Russell's French follower L. Couturat gave a dutiful (though superficial) account of mathematical logic, in praise echoed by the Italian F. Enriques. Finally, one N. Losskij, based at St. Petersburg, wrote on "The Transformation of the Concept in Modern Epistemology and Its Bearing on Logic", the latter being of the traditional kinds.[82]

[80] F.C.S. Schiller, *Formal Logic* (London: 1912), p. 373.

[81] B. Russell, "Pragmatism and Logic" [review of *ibid.*], *The Nation*, 11 (1912): 258–9. Compare the original entry for December 1902 on the visit of Bradley, following the text quoted at the beginning of this paper: "Afterwards I made a fierce onslaught on F.C.S. Schiller, with his William-James doctrine that the truth is what it pays to believe.... Stout asked if I felt any compunction for my behaviour" (copy at RA, where may also be seen both sides of the Russell–Schiller correspondence, about to be mentioned).

[82] *The Encyclopaedia of the Philosophical Sciences*, Vol. 1: *Logic*, trans. B.E. Meyer, English edition ed. H. Jones (London: 1913). The articles mentioned in the text are: Ruge (pp. 1–6), Windelband (pp. 7–66), Royce (pp. 67–135), Couturat (pp. 136–98), Croce (pp. 199–215), Enriques (pp. 216–39), and Losskij (pp. 240–67).

I refer here also to the unpublished PH.D. thesis of N. Wiener (1894–1964), which the author showed to Russell in 1913. It compared Schröder's and Russell's theories of relations, and Russell made important comments on it (see my "Wiener on the Logics of Russell and Schröder ..." [n. 40]).

Russell reviewed this book for *The Nation* in a piece entitled "Competitive Logic". He began by pointing out that the book was "not an encyclopaedia in any recognized sense", and doubted if philosophy was "ready for such treatment anyway". After giving each article a judiciously lukewarm welcome, he returned to this theme with one of his best touches of wit, illuminating the confused and contradictory panorama of logic: "The book in fact resembles a compendium on the British Constitution composed during the Civil War, with an introduction by King Charles and an epilogue by Oliver Cromwell."[83]

One point of controversy concerning mathematical logic was the range of its use. Joseph took a critical line in the second edition of his *An Introduction to Logic* (1916), where he stuck to (his version of) the usual things (judgment, inference, induction, and so on), and criticized Russell for "representing either all thinking as a kind of mathematics, or all thinking as class-thinking, and mathematics as merely a special sort of class-thinking".[84] The point was very pertinent, and was to remain so.

6. FURTHER APPRAISALS OF LOGIC, 1919–25

6.1 *Russell's philosophical progress.* Even while completing his part of *Principia Mathematica* Russell was developing his broader philosophical interests. His introductory volume *The Problems of Philosophy* (1912) showed the directions of their turn, beginning with a chapter with the Bradleian title "Appearance and Reality" but containing severe criticisms of idealism in all its main forms. The later chapters included rejections of monism and of internal relations, and an account of knowledge by acquaintance and knowledge by description.[85] Then came in 1913 an attempted book on epistemology; it was abandoned after criticisms by L. Wittgenstein (1889–1951), but not for its *general* thrust,[86]

[83] B. Russell, "Competitive Logic" [review of Ruge], *The Nation*, 14 (1914): 771–2; the piece is not signed, but it is known to be by Russell. Compare Broad's review in *Mind*, n.s. 23 (1914): 274–7, where Royce's interest in induction captured his attention.

[84] H.W.B. Joseph, *An Introduction to Logic*, 2nd ed. (Oxford: 1916), p. 228. See also pp. 543–53 on mathematical reasoning (pp. 503–12 of the first edition of 1906), which Russell reviewed with qualified warmth in *The Tribune*, 7 July 1906, p. 2.

[85] B. Russell, *The Problems of Philosophy* (London: 1912). Bosanquet's long and interesting review in *Mind*, n.s. 21 (1912): 546–64, is summed up in some ways on p. 560: "what we who have been trained in another school miss alike in Locke and in Mr. Russell, is the point of view of the whole."

[86] This manuscript is published as Russell's *Collected Papers*, Vol. 7 (London: 1984). For commentary, see T. Iglesias, "Russell's *Theory of Knowledge* and Wittgenstein's Earliest Writings", *Synthese*, 60 (1984): 285–332; and especially N. Griffin, "Russell's Multiple Relation Theory of Judgment", *Philosophical Studies*, 47 (1985): 213–47.

and was replaced by *Our Knowledge of the External World* (1914). From 1910 until 1916 Russell was in fact back at Trinity, with a college lectureship in mathematics secured at the instigation of Whitehead, and he even taught mathematical logic at times. But he was becoming more and more engaged in philosophy, sometimes conducting Sunday evening discussions with McTaggart.[87]

But the principal positive influence upon Russell's thought had come from Wittgenstein. At the end of the First World War he came to the position called "logical atomism", in which knowledge by acquaintance or description was set in a general reductionist epistemology of atomic and molecular propositions and the existence of inferred entities.[88]

This story belongs to Russell's philosophical career in general, and our concern lies with his logic. In the next subsection I shall note some of the new followers that he acquired during the 1910s, and then note the continuing contrast with the Oxbridge traditions.

6.2 *Russell and his supporters.* In 1918 Russell, then forty-six years old, seized the chance offered by a period of imprisonment to write a popular account of his logicism. His publisher, Stanley Unwin, suggested to the moral and political philosopher J.H. Muirhead (1855–1940) that he invite Russell to contribute the volume to the "Library

[87] This is a suitable occasion to put on record the following recollection sent to me in 1972 by a correspondent who wishes to remain anonymous. As a young mathematics student he attended one of these evenings during the winter of 1913–14:

> MacTaggart and Russell held periodic coffee parties on Sunday evenings in the latter's rooms attended by a dozen or so students from various colleges: the "audience" sat in silence in a half ellipse with McTaggart to the left and B.R. to the right. One of them—I think B.R.—would open the topic and the topic was argued to and fro while the audience listened in solemn and awed silence....
>
> On the dispersal and strolls back I remembered the usual view was that McTaggart was the sounder but Bertrand Russell won by sheer speed—the rapier beating the sabre.
>
> At the last conference ... Bertrand Russell called me aside as a mathematician I suppose and likely to appreciate the gravity of his statement—"I have just realised that I have failed—it is easy to establish the unit one but I have omitted to establish a second like unit"—(I won't guarantee the precise wording but it's not far off). He went on to say "I have finished".
>
> With this ringing in my mind and the dreadful distress of a great scholar I was glad I was alone.

The latter remark probably relates to Russell's reductionist assumption in *Principia Mathematica* of only one individual in the construction of arithmetic, which leads to several problems when cast in type theory. Further, Whitehead had forgotten to deploy this assumption, and some parts of the second volume had had to be reset in 1911. On this curious episode, see my "The Russell Archives" (n. 20), pp. 402–4.

[88] B. Russell, "The Philosophy of Logical Atomism", *The Monist*, 28 (1918): 495–527, 29 (1919): 32–63, 190–222, 345–80; also in *Logic and Knowledge*, pp. 178–281.

of Philosophy" series. The book, *Introduction to Mathematical Philosophy*, came out in the following year; and it must have been well received, for it was soon being reprinted. Indeed, it is still well known, and needs no summary here; but of note is Muirhead's "Editor's Note" at the front, addressed to those who might "think that this book is out of place in the present Library." Referring to Russell's preface, he added: "It is not necessary to agree with what he there suggests as to the readjustment of the field of philosophy by the transference from it to mathematics of such problems as those of class, continuity, infinity, in order to perceive the bearing of the definitions and discussions that follow on the work of the 'traditional philosophy'."[89] In this clause, and the rest of his note, Muirhead beautifully captured some of the issues involved in the current transition of interest in logic and in philosophy.

Other notable contributions to the new vogue of philosophy, including revisions to logicism itself, were soon to appear: Wittgenstein's *Tractatus* (1921) including Russell's introduction, with its throw(n)-away suggestion of a hierarchy of languages;[90] Russell's own new material for the second edition of *Principia Mathematica*, published in 1925;[91] and articles by his student, F.P. Ramsey (1903–1930), which came out the following year.[92] By and large, these changes distanced mathematical logic even further from the traditional forms than before, for there was a notable increase in extensionalistic interpretations of quantification, conjunction and disjunction, and propositional functions. Later, the Vienna Circle was to take this approach to an extreme.[93]

[89] [J.H. Muirhead], "Editor's Note", in B. Russell, *Introduction to Mathematical Philosophy* (London: 1919), p. vii. Later in this series appeared the English translation of Metz cited in n. 7.

[90] L. Wittgenstein, *Tractatus Logico-Philosophicus* (London: 1922), including Russell's introduction. The German text of both first appeared on its own in 1921.

[91] A.N. Whitehead and B. Russell, *Principia Mathematica*, 2nd ed., Vol. 1 (Cambridge: 1925).

[92] F.P. Ramsey, "The Foundations of Mathematics", *Proceedings of the London Mathematical Society*, (2), 25 (1926): 338–84, and "Mathematical Logic", *Mathematical Gazette*, 13 (1926): 185–94; also in *The Foundations of Mathematics*, ed. R.B. Braithwaite (London: 1931), pp. 1–81; and *Foundations* ..., ed. D.H. Mellor (London: 1978), pp. 152–232.

Mention might be made of "a Cambridge mathematician friend of mine", as Russell described him on a postcard received in 1910 (held in RA): H.T.J. Norton (1886–1937), an undergraduate at Trinity (with H.W. Turnbull and G.N. Watson, and also A.V. Hill, who later became a distinguished physiologist). He wrote some manuscripts on logicism after the second edition of *Principia Mathematica* appeared (University College London, London Mathematical Society Archives, Norton manuscripts). They do not seem to be of especial significance. H. Davies, of the Mathematics Department of Cambridge University, has collected some data on Norton.

The continuance of differing streams of thought was clearly exhibited in several writings of the 1920s. For example, when the twelfth edition of the *Encyclopaedia Britannica* put out in 1922 its three volumes to append to the twenty-nine of the previous edition, "dealing with events and developments of the period 1910 to 1921 inclusive", the writer of the article on philosophy, A.E. Taylor (1869–1945), Oxford-trained in a Bradleian mould but more eclectic in his later thought and peripatetic in his various appointments, freely confessed to the variety of forms of logic. He gave a certain measure of space to Russell's line but also noted the growth in influence of Bergson, the continuing idealist lines, Whitehead's own approach to philosophy, and so on.[94] Interestingly, mathematical logic was also given an entry in the new article on mathematics; the author was J. Nicod (1893–1923), a Frenchman who studied with Russell before his early death.[95]

Another project run by Muirhead gives us a source to study. He invited Russell to contribute a piece to his *Contemporary British Philosophy* book series, and in due course an article on logical atomism appeared in the 1924 volume. Since Bosanquet and Schiller also contributed their views on logic to the book, the resultant mixture reminds one of the 1913 logic volume described in subsection 4.5.[96]

6.3 *The continuing tradition.* The Oxbridgians continued to write after their own wont, though with some signs of ageing. In particular, in 1922 the seventy-six-year-old Bradley put out a second edition of his

[93] See my "Notes on the Fate of Logicism from *Principia Mathematica* to Gödel's Incompletability Theorem", *History and Philosophy of Logic*, 5 (1984): 67–78. Russell himself was getting out of touch with the developments in logic, metalogic and metamathematics in the 1920s (on these, see G.H. Moore, "Beyond First-Order Logic ...", *ibid.*, 1 [1980]: 95–137).

[94] A.E. Taylor, "Philosophy", in *Encyclopaedia Britannica*, 12th ed. (1922), 32: 93–100. Compare his review cited in n. 72.

[95] J. Nicod, "Mathematical Logic and the Foundations of Mathematics", *ibid.*, 31: 874–6. Nicod's best-known contribution is his two doctoral *thèses* at the Université de Paris, published in English translation under the title *Geometry and Induction* (London: 1930). It made little impact, but a new translation appeared under the same title in 1970, at the initiative of R.F. Harrod and with financial help from Russell and "another donor, who wishes to remain anonymous" (p. v). This latter was W.H. Auden (letters from Harrod in RA).

[96] B. Russell, "Logical Atomism", in *Contemporary British Philosophy*, ed. J.H. Muirhead (London: 1924), pp. 357–83; also in *Logic and Knowledge*, pp. 321–43. Bosanquet's piece appeared on pp. 49–74 of Muirhead's volume, and Schiller's on pp. 385–410; see also McTaggart's on pp. 249–69. Bradley gracefully declined to contribute (p. 10).

Logic, gracing it with a dozen "terminal essays" and with "additional notes" to each chapter. The essays were connected pieces, though of varying relevance to the preceding text; but some of the notes only bewilder the reader, with confessions of confusion readily admitted but the required corrections largely omitted.[97] Nothing by Russell or his associates was cited, but there were frequent acknowledgments to the writings of Bosanquet, who had recently attempted in a little book to effect *The Meeting of Extremes in Contemporary Philosophy* (1921). After presenting the dichotomy between realism and idealism, Bosanquet disagreed with Russell's distinction between Napoleon and Hamlet in the context of the referents of definite descriptions, on the grounds that, "In Professor Stout's words, 'Whatever is thought, in so far as it is thought, is therefore real'."[98] As a source of compromise he offered, in a chapter entitled "7 + 5 = 12", the suggestion that such a proposition was an "eternal novelty", that which, "parting from itself, remains within itself, and which being always old is yet perennially new."[99] Russell does not seem to have incorporated this notion into his logicism.

Also at this time appeared a long-awaited work, Johnson's *Logic*, in three volumes in 1921, 1922 and 1924 from Cambridge University Press. The range of the work can be gauged by the sub-titles of the second and third volumes: respectively, "Inference: Deductive and Inductive" and "The Logical Foundations of Science". Unlike the idealists, he was not besotted with judgment but devoted early chapters to propositions and their syntactic structure. However, this enterprise was largely directed only to a rehearsal of the usual details of syllogistic reasoning and some extensions to relations.[100] Again, on inference he sought, among other things, for formal syntactic and semantic conditions for their validity; but he made no particular use of Russell's work and indeed did not always seem to understand it (for example, he reversed Russell's definition of descriptive [mathematical] functions in

[97] F.H. Bradley, *The Principles of Logic*; see n. 24 above. On the new material, see Manser, *Bradley's Logic* (n. 24), pp. 194–209.

[98] B. Bosanquet, *The Meeting of Extremes in Contemporary Philosophy* (London: 1921), pp. 38–50; on p. 41 he quoted G.F. Stout, "The Object of Thought and Real Being", *Proceedings of the Aristotelian Society*, n.s. 11 (1910–11): 187–205 (p. 199). Stout was disagreeing with both Bradley and Russell: Bradley took Stout's stand to be his own view also (*Essays on Truth and Reality* [Oxford: 1914], p. 275).

[99] Bosanquet, *ibid.*, p. 104: compare pp. 111–13 on Russell.

[100] W.E. Johnson, *Logic*, 3 vols. (Cambridge: 1921–24); see Vol. 1, esp. Chaps. 9 and 13. He had stressed "propositional synthesis" in his earlier articles (n. 7).

terms of propositional functions).[101] He also invited confusion when stating that "The current phrase *mathematical logic* is ambiguous inasmuch as it may be understood to mean either the logic of mathematics or the mathematics of logic" and ascribing *Principia Mathematica* to the latter category.[102] We recall from subsection 5.1 Shearman's report of 1906 that Johnson saw all the symbolic logicians as working towards a common goal; it seems that by the 1920s he had not grasped the differences of aim and method which the subject had long been experiencing. This seems strange in a man who had taken the Mathematical Tripos in his youth (and published a textbook on trigonometry later), and was supposed to be familiar with Russell's work. The point is not, of course, that he (or anyone) should *like* Russell's logicism; but from those with such a background as Johnson one could expect a clear *understanding* of the position.

7. CONCLUSIONS: TWO LITTLE QUOTATIONS

The dispute between Oxbridge and Russell was waged at three different levels, each of which has been exemplified in the above survey: the epistemological, including idealism or realism and monism or pluralism; the logical, with the traditional syllogistic logic and its extension into algebraic logic facing the new mathematical logic; and the set-theoretic, in that the part–whole collections theory of the tradition made more modest demands on the properties of classes than the set theory associated with mathematical logic. Of course, Russell and his followers won all three battles, by and large: most of his opponents to whom I have referred are now relics. My purpose in recalling their work has been historical: I make no claim to have disclosed forgotten jewels, and indeed would be hard pressed to encapsulsate in short or even long form the content of stretches of Bradley or Bosanquet, say. On my reading, incidentally, it is the monism more than the idealism which causes the muddles.

However, some discussions by these and the other authors of the philosophical aspects of logic are not without merit for a modern audience. Moreover, to continue the historical theme, the traditionalists did not fall from grace with anything like the speed that some modern his-

[101] *Ibid.*, Vol. 2, Chaps. 1 (inference) and 3 (functions). In Vol. 1: 172, he reversed Russell's dependence of the existence of referents to definite descriptions upon quantificational existence.

[102] *Ibid.*, 2: 137: on p. 138 he correctly stated the logicist position.

torians of logic and philosophy lead us to think.[103] Disputes between Russellian and non-Russellian logicians continued well into the 1930s,[104] and some of the books of the traditionalists were reprinted even up to the 1970s.[105] As for philosophy, Russell's logical atomism did not take a dominant place: already during the 1920s at Cambridge inductive logic was a favoured topic, and at the end of the decade Wittgenstein's new conception of ordinary language philosophy began to take sway. Meanwhile, at Oxford, the Hegelian tradition remained active until the Second World War; after it, logical positivism and ordinary language philosophy took a grip. While the former type of philosophy can be traced back to the revolt of Russell and Moore in the 1890s, the latter was certainly not to Russell's taste. In the 1950s he opined that "bad philosophy has always been an Oxford speciality, and bad philosophy is still philosophy!"[106] Perhaps modern Oxbridge could

[103] For a good specimen of such "history", see G.J. Warnock's misnamed *English Philosophy since 1900* (Oxford: 1958), where all "irrelevant" developments are passed over in silence. The author admitted as much in his second edition of 1969 (p. vii) and added a few pages on Cook Wilson (pp. 8–12), a realist and therefore somewhat approaching a Good Thing. Russell's own *History of Western Philosophy* (London: 1946; 1st ed., 1945) largely passes over the Hegelians (see Book 3, Chaps. 21–2), as do W. and M. Kneale in *The Development of Logic* (Oxford: 1962).

One has to recognize that positivist philosophers are also positivist historiographers, where only the "acceptable" strands are traced (up to today) and the mistakes are passed over in silence. For valuable outlines towards a proper history, see Metz and Passmore (n. 6), and also Manser (n. 24), pp. 1–29.

[104] For example, collectors of the talking past will rate highly this sequence: H.W.B. Joseph, "A Defence of Freethinking in Logistics", *Mind*, n.s. 41 (1932): 424–40; L.S. Stebbing (broadly a Russellian, and carrier of the flag at London University after Shearman), "Mr. Joseph's Defence ...", 42 (1933): 338–51; Joseph, "A Defence ... Resumed", *ibid.*, 417–43; Stebbing, "A Second Reply to Mr. Joseph", 43 (1934): 156–69; Joseph, "A Last Plea for Free-Thinking [*sic*] in Logistics", *ibid.*, 315–20. Well might Schiller express his perplexity soon afterwards in his "Multi-Valued Logics—and Others", 44 (1935): 466–83. Moore was the tolerant editor of *Mind* at the time.

[105] For example, Bradley's *Logic* was reprinted in 1950, Johnson's in 1964, and Joseph's in 1970: in addition, Joachim's *Truth* was reissued in 1969. A library copy of (the original printing of) Johnson which I consulted has been heavily used in recent years (or so I infer from the issue registrations).

Principia Mathematica itself was reprinted until the 1970s. It even received a pirated reprinting in Taiwan in the 1950s.

[106] Russell to R.C. Marsh, as quoted in *Logic and Knowledge*, p. 322 (with the correction of a misprint). Marsh's surrounding commentary, ending "Oxford has finally become a lost cause itself", is well worth reading. I do not treat the history of philosophy education at Oxford here, but I gather from W. Mays that, despite all this idealism, Mill's *System of Logic* was still a set book until the Second World War. Maybe we have here a difference between the teaching and the research levels.

The vastly increased degree of professionalization of philosophy has been an important factor in its development. Philosophy is now a profession, like mathematics and chemistry and ... ; therefore philosophers should work only on their own problems, like mathematicians and chemists and ... .

[107] Bradley, *The Principles of Logic*, p. 388. Note the self-reference of this passage: Bradley mis-stated logicism in the same way as Johnson (n. 102) was to do. I gather from N. Griffin that Bradley's manuscripts (n. 24) contain collections of notes on several of Russell's works.

[108] Russell, "Logical Atomism" (n. 96), p. 359; also in *Logic and Knowledge*, p. 323.

do with another heretic of Russell's calibre. However, the modern predicament falls outside my brief, which has been to show that Russell's revolt from idealism at the end of the nineteenth century made him an intellectual outsider of the Oxbridge of that time, but that gradually over the decades his view came to prominence, initially in logic and then in philosophy more generally.

I began this paper with two little books of the late nineteenth century on logic; let me end with two quotations from the 1920s, respectively due to the chief antagonists, Bradley and Russell. In the second edition of his *Logic* of 1922, Bradley ended the new material for his second Book with this statement, of striking integrity:

> Whether a student of logic, who is incapable of learning mathematics and has therefore to leave out of his theory a recognised part of the facts, should never have written on logic at all, or should later at least suppress all that he once wrote—I will not offer to discuss. And what should be his attitude towards a claim to base the principles of logic on mathematics, I once more hardly know. If a person like myself ventures to point out that something of what is thus offered seems to himself to be untenable and irrational—he can be met with the reply that, if he understood mathematics, he would forthwith think otherwise. And what his answer to this should be, I confess, I can not say.[107]

Russell, who had found Bradley twenty years earlier unable to accept ultimate disagreements, expressed his view two years after this statement by Bradley, in his article on logical atomism for Muirhead: "I hold that logic is what is fundamental to philosophy, and that schools should be characterized rather by their logic than by their metaphysic."[108]

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