The antinomy of dynamical causation in *Leibniz* and the *Principles* and Russell's early picture of physics

by Ian Winchester

IT COULD PROBABLY be said of Russell, as Whitehead said of Kant, that he might have made a good physicist. In his autobiography Russell himself tells us that he doubted that he could ever have mastered all the details of a physical science since his own interests were much too general and his mind tended always to the most abstract. Be that as it may, an interest in physics and in accounting for its foundations (in some sense of "foundations") is a recurrent, if not constant, theme in Russell's thought. As late as 1959, when *My Philosophical Development* was published, Russell offers his present view of the world in Chapter II of that work. Most of that chapter is devoted to questions that relate to physics including the dimensionality of space, the relationship of physics to mathematical logic and the relationship of physics and psychology, and finally the directionality of physical causality.

At a variety of times in Russell's life his thoughts turn explicitly to topics in physical science, and he writes about these thoughts in such a way that we have no doubt about what his picture of physical science at the time actually was. Prior to *A Critical Exposition of the Philosophy of Leibniz* (1900) and after *Principia Mathematica* (1910–13), in *Our Knowledge of the External World*, *The ABC of Relativity*, *The Analysis of Matter* and *Human Knowledge*, his picture of physical science is explicit. So we can follow changes and shifts in his thought easily enough. But this is not so true for the period after 1900 and prior to *Principia Mathematica* even though in both *Leibniz* and *The Principles of Mathematics*, the fruit of his thought in that period, the interest in physical science—particularly rational dynamics conceived as a branch of mathematics—shines through as a central theme.

In *My Philosophical Development* Russell tells us about the period between 1896 and 1898 just prior to his work on *Leibniz*:

> I finished my book on the foundations of geometry in 1896 and proceeded at once to what I intended as a similar treatment of the foundations of physics, being under the impression that the problems concerning geometry had been disposed of. I worked on the foundations

> of physics for two years, but the only thing that I published expressing my views at that time was [an] article on number and quantity [already mentioned]. I was at this time a full-fledged Hegelian, and I aimed at constructing a complete dialectic of the sciences, which should end up with the proof that all reality is mental. I accepted the Hegelian view that none of the sciences is quite true, since all depend upon some abstraction, and every abstraction leads, sooner or later, to contradictions.[1]

A little later in *My Philosophical Development* he says of this early work on physics:

> On re-reading what I wrote about the philosophy of physics in the years 1896 to 1898, it seems to me complete nonsense, and I find it hard to imagine how I can ever have thought otherwise. Fortunately, before any of this work had reached a stage where I thought it fit for publication, I changed my whole philosophy and proceeded to forget all that I had done during those two years. (P. 43)

Russell's own account of what happened in 1898 is that he and Moore rebelled against Hegel, led by Moore's article in *Mind* on "The Nature of Judgment". For Russell this revolt took the form of two new convictions, the first being that he came to believe (with Moore) that fact is in general independent of experience and that the world is made up of many independent things. This latter amounted to the view, in opposition to Bradley in particular, that not every relation between two terms expresses intrinsic properties of the two terms, or, in the end, a property of the whole which the two compose (p. 54). This meant that Russell was free to believe in a plural world made up of many independent things. One of the most striking features of Russell's *Leibniz* is the fluency and ease of his discussions of questions in physics. In this sense, his two earlier years spent on physics were not as wasted as he appears to suppose in his later reflections. On the other hand, since the discussion is essentially about Leibniz's physics, it is not surprising that it is hard to discover from the text, except occasionally, what Russell himself thought about physics as a contemporary problem. This becomes much easier in *The Principles of Mathematics*. However, even in *Leibniz*, Russell himself appears in places. One of the most striking of such appearances is in §41 in Chapter VII, which runs:

> **41.** The most important dynamical argument in favour of force is connected with the relativity of motion. On this point, Leibniz's views present some suggestion of a vicious circle. He seems sometimes to argue that, because force is something real, it must have a subject, and be an attribute, not a mere relation; whence it follows that, in a change of relative situation, the *cause* of change can be apportioned between the bodies, thus giving a sense of absolute motion (e.g. G.M. II. 184). But at other times, he argues that some real change, not merely relative, must underline motion, and can only be obtained by means of force (e.g. D. 60, 61; G. iv. 369). *This argument is interesting, both on account of its difference from the analogous arguments by which Newton proved the need of absolute space, and by the fact that Dynamics, at the present day, is still unable to reconcile the relativity of motion with the absoluteness of force.*[2]

[1] *My Philosophical Development* (London: Allen and Unwin, 1959), pp. 41–2.

[2] *A Critical Exposition of the Philosophy of Leibniz* (London: Allen and Unwin, 1937; 1st ed., 1900), p. 84.

In a footnote to this passage Russell says:

> I cannot here undertake to give the proof of this assertion. It depends upon the fact that, if the laws of motion are to apply, the motion must be referred, not to *any* axes, but to what have been called kinetic axes, i.e., axes which have no absolute acceleration. See Newton, *Principia*, Scholium to the eighth definition. Contrast, in Clark Maxwell's *Matter and Motion*, Arts. XVIII, CV.

In these two spots Russell is putting his logical finger on precisely the problem which animated Einstein to produce his General Theory of Relativity. He even seems to present the situation as one to which physicists contemporary to himself should address themselves. However, a little later in the same section he appears to argue (with Newton) that the only way out of this dilemma is to embrace absolute space, that is to say, the absoluteness of position in space (pp. 86, 87).

There are a number of other passages in the seventh chapter of *Leibniz* which are still more revealing about what Russell himself thought about physics at the time. In §44, Russell expounds the "three great types of dynamical theory", namely those of atomism, of the plenum and of unextended centres of force. On the basis of these three possibilities he says of Leibniz that he "failed to grasp these alternatives, and thus, from his love of a middle position, fell between, not two, but three stools." He goes on:

> His [i.e. Leibniz's] view of impact as the fundamental phenomenon of Dynamics should have led him to the theory of extended atoms, supported by Gassendi, and, in his own day, by Huygens. His belief in the plenum and the fluid ether should have led him to the second theory, and to the investigation of fluid motion. His relational theory of space, and his whole doctrine of monads, should have led him, as it led Boscovitch, Kant and Lötze, to the theory of unextended centres of force. The failure to choose between these alternatives made his Dynamics a mass of confusions. The true Leibnizian Dynamics is not his own, but that of Boscovitch. (P. 91)

In the sections which immediately follow (§§45–7), in which Russell expounds Leibniz's grounds for rejecting atoms, the vacuum and action at a distance, we again catch a glimpse of Russell's own views. For example, Russell, too, did not think atomism of the Gassendi–Huygens type, namely perfectly hard and perfectly elastic, finitely sized objects, to be true, as the last sentence of §45 reveals. Although Russell offers no positive argument for the existence of a vacuum, in §46 he roundly condemns Leibniz's arguments to the effect that a vacuum is inconceivable, finishing with the flourish:

> He has, in fact, no valid arguments whatever against a vacuum. He seems to regard a belief in it as necessarily associated with a belief in extended atoms—"atoms and the void" are always spoken of together. In fact, when action at a distance is rejected, the two are necessarily connected; since unextended atoms must act at a distance, if there is to be any dynamical action at all.

This is followed by a revealing and pithy footnote in which Russell tells us that on the question of whether or not there can be motion in a plenum, Leibniz is unquestionably right as against Locke, who had maintained that there must be empty space or else there would be no room for motion. Leibniz had replied that if matter were fluid there would be no difficulty. In a characteristic condemnation of philosophers who know no physics Russell then says that:

> It should be obvious, even to the non-mathematical, that motion in a closed circuit is possible for a fluid. It is a pity philosophers have allowed themselves to repeat this argument, which a week's study of Hydrodynamics would suffice to dispel. (P. 93)

And in the passage expounding Leibniz's grounds against action at a distance he says he "cannot discover ... anything beyond vulgar prejudice" in Leibniz's grounds against action at a distance (*ibid.*).

The net effect of all this is that according to Russell Leibniz should have embraced the dynamical account of Boscovitch. We are also, it seems to me, led to think that it had found or at least retained considerable favour with Russell himself at the time of his Leibniz lectures and of the book which resulted from them. That is to say, matter is to be conceived of as unextended centres of force which are essentially impenetrable, which inhabit a vacuum that is to be identified with absolute space, and which interact by action at a distance. Russell tells us in *My Philosophical Development* that he was convinced by Whitehead to prefer a plenum to Boscovitch's account. But there is no evidence of this preference in his *Leibniz*.

A puzzle in dynamics which exercised Russell both in Leibniz and later in the *Principles* appears first on page 96 in *Leibniz* and then immediately again in §50, p. 97. What is interesting is that it appears in Hegelian guise in the form of an antinomy, in a passage in which Russell is condemning Leibniz for introducing on metaphysical grounds a primitive force of which no dynamical use is made. Thus Russell says:

> Here again, I think, as in the case of continuity, there is an antinomy which Leibniz refused to face. The total effect on any particle is, dynamically, made up of effects caused by all other particles; thus the separate causation of separate elements seems conceded. But none of these separate effects ever happen: they are all mathematical fictions. What really happens is the sum of effects, i.e., the effect of the sum or of the whole. Thus ... we can hardly escape the admission, which however is directly self-contradictory, that things do, after all, interact. (P. 97)

This passage is remarkable, from the vantage point of Russell's later writing, in that he is still using or appearing to use Hegelian argument against Leibniz. Secondly, it is the first instance in his published writings with which I (at least) am acquainted of his using the term "mathematical fiction", which, once mathematics and logic are identified, is a suggestive model for his later frequent use of "logical fiction".

There is another reason why this passage and the one which follows it is remarkable. Perhaps the most distinctive of all of Russell's doctrines in the period 1898–1900 is the doctrine of "external relations". It seems to me that it is in this "antinomy of dynamical causation" that the problem of external versus internal relations is first driven home to Russell. We see the argument for external relations first developed in *The Principles of Mathematics* in the chapter on Asymmetrical Relations, §§12–16. But its origin appears to be in Russell's struggling with the dynamical puzzle of the additions of vectors. One of the peculiarities of both the passage quoted above and the immediately following §50 in *Leibniz* is that Russell seems to enjoy the antinomy for its own sake and, strikingly enough, doesn't see it as a great challenge either for himself or for a reconstructed physics. This seems to be a remaining Hegelian tendency. In the *Principles*, although he admits the problems for physics which his worries raise, he is apparently so content with having separated mathematical or logical questions from empirical ones, that he can leave the empirical ones to others. (Considering that he had read much the same great books as Einstein subsequently would read (or had read), had an unrivalled mathematical equipment for a young man of twenty-six, and saw clearly what the central problems of physics were with respect to the compatibility of Maxwell's equations with Newtonian assumptions, one can wonder about the value of philosophy as an aid to physical discovery! It is not surprising that, when he later came to read Einstein's work, he wondered by he had wasted his time on "all that rot".)

Writing in December 1902 in the preface to the first edition of the *Principles* Russell tells us that indeed the origin of his logical work is to be found in his investigations into Dynamics:

> About six years ago [i.e., in 1896] I began an investigation into the philosophy of Dynamics. I was met by the difficulty that, when a particle is subject to several forces, no one of the component accelerations actually occurs, but only the resultant acceleration, of which they are not parts; this fact rendered illusory such causation of particulars by particulars as is affirmed, at the first sight, by the law of gravitation. It appeared also that the difficulty in regards to absolute motion is insoluble on a relational theory of space. From these two questions I was led to a re-examination of the principles of Geometry, thence to the philosophy of continuity and infinity, and thence, with a view to discovering the meaning of the word *any*, to Symbolic Logic.

The passages in *Leibniz* and in the *Principles* where Russell discusses the "antinomy of dynamical causation" show him moving from a fairly crude expression of the problem to a much more clear and succinct one. In *Leibniz* the passage in §50 which expands on the passage previously quoted runs as follows:

> There remains one last and principal difficulty, a difficulty which, so far as I know, no existing theory of Dynamics can avoid. When a particle is subject to several forces, they are compounded by the parallelogram law, and the resultant is regarded as their sum. It is held that each independently produces its effect, and that the resultant effect is the sum of the partial effects.... If we are to admit particular causes, each of which, independently of all others, produces its effects, we must regard the resultant motion as compounded of its components. If we do not admit such particular causes, every part of matter, and therefore all matter, is incapable of causal action, and Dynamics (unless the descriptive school is in the right) becomes impossible. But it has not been generally perceived that a sum of motions, or forces, or vectors generally, is a sum in a quite peculiar sense—its constituents are not parts of it. This is a peculiarity of all addition of vectors, or even of quantities having sign. Thus no one of the constituent causes ever really produces its effect: the only effect

is one compounded, in this sense of the effects which *would* have resulted if the causes had acted independently. (P. 98)

A parallel argument appears in at least two places in *The Principles of Mathematics* and is referred to in a number of others. The first of these arises in Russell's defence of the assertion, already canvassed in his treatment of Leibniz, that "the notion of Force is one which ought not to be introduced into the principles of Dynamics."

> Force is the supposed cause of acceleration: many forces are supposed to concur in producing a resultant acceleration. Now an acceleration ... is a mere mathematical fiction, a number, not a physical fact; and a component acceleration is doubly a fiction, for, like the component of any other vector sum, it is not part of the resultant, which alone is supposed to exist. Hence a force, if it be a cause, is the cause of an effect which never takes place.[3]

The second version occurs in §451 of the *Principles*, where Russell is concerned to tackle "a very difficult question ... the question which, when the problem is precisely stated, discriminates most clearly between monism and monadism" (p. 477). His version of a precise question on the point at issue is this: "Can the causal relation hold between particular events, or does it hold only between the whole present state of the universe and the whole subsequent state? Or can we take a middle position, and regard one group of events now as causally connected with one group at another time, but not with any other events at that other time?" Russell's strategy is to take the example of a three-particle gravitating system as follows:

> Let there be three particles *A*, *B*, *C*. We say that *B* and *C* both cause accelerations in *A*, and we compound these two accelerations by the parallelogram law. But this composition is not truly addition, for the components are not parts of the resultant. The resultant is a new term, as simple as its components, and not by any means their sum. Thus the effects attributed to *B* and *C* are never produced, a third term different from either is produced. This, we may say, is produced by *B* and *C* together, taken as one whole. But the effect which they produce as a whole can only be discovered by supposing each to produce a separate effect: if this were not supposed, it would be impossible to obtain the two accelerations whose resultant is the actual acceleration. Thus we seem to reach an antinomy: the whole has no effect except what results from the effects of the parts, but the effects of the parts are non-existent. (§451, p. 477)

There are two things in these quotations to which I want to draw attention. The first is that in dwelling upon dynamical vector components Russell was naturally drawn to the fact that they were examples of asymmetrical relations between particles. And it will be upon the rock of asymmetrical relations that Russell builds his arguments against the Bradleian doctrine of "internal relations", which he finds prominent in all earlier versions of monism. The second is that the terminology and indeed the handling of "fictions", in this case mathematical ones as exemplified by "force" in dynamics, are already in place in much the same way that he will later use them with respect to referential fictions in "On Denoting". Let me just juxtapose the salient bits from *Leibniz* and the *Principles* so that one can see what is going on here.

[3] *The Principles of Mathematics* (London: Allen and Unwin, 1937; 1st ed., 1903), §448, p. 475.

From *Leibniz*:

> But it has not been generally perceived that a sum of motions, or forces, or vectors generally, is a sum in quite peculiar sense—its constituents are not parts of it.

From the *Principles*, §448:

> ... many forces are supposed to concur in producing a resultant acceleration. Now an acceleration ... is a mere mathematical fiction, a number, not a physical fact; and a component acceleration is doubly a fiction, for, like the component of any other vector sum, it is not part of the resultant, which alone is supposed to exist.

And from the *Principles*, §451:

> Thus we seem to reach an antinomy: the whole has no effect except what results from the effect of the parts, but the effects of the parts are non-existent.

As Russell's thought moved from physics ultimately to questions of language in the process of producing the draft of the *Principles*, he discovered that sentences, too, appear to have a unity, in this case, of meaning. (Consider this later line from "On Denoting" [1905]: "According to the view which I advocate, a denoting phrase is essentially a *part* of a sentence, and does not, like most single words, have any significance on its own account.") But it does not follow that each of the apparent component parts has a unity of meaning in isolation. Just as a component of a resultant vector acceleration or force certainly looks as if it must have an effect, the phrase, "The author of *Waverley*", certainly looks as though it had isolated meaning. For the sentences: "The present King of France is bald", or "The Golden Mountain doesn't exist", we can say, as Russell says with respect to the dynamical antinomy: "The whole has no effect except what results from the effects of the parts, but the effects of the parts are non-existent."

I don't wish to press the analogy between Russell's way of proceeding with respect to the antinomy of dynamical causality and the antinomy of sentence meaning too far. But let me just quote a line in §55 of the *Principles* which was almost certainly written after his writings about the dynamical antinomy and considerably before he wrote "On Denoting": "Owing to the way in which the verb actually relates the terms of a proposition, every proposition has a unity which renders it distinct from the sum of its constituents" (p. 52). Is there not an echo of the dynamical antinomy here, as well as a rather Hegelian way of putting matters? But I digress.

Russell's picture of physics in the Principles

If the "antinomy of dynamical causation" and the question as to whether space and motion are absolute or relative are raised centrally in *Leibniz*, they figure as the main problems facing a "rational Dynamics" in the *Principles*. The last part of the *Principles*, Part VII, is entirely devoted to the topic of a rational dynamics and is entitled, not surprisingly, "Matter and Motion". What is surprising is to find such a part, with chapters on Matter, Motion, Causality, Newton's Laws of Motion, Absolute and Relative Motion, and Hertz's Dynamics, included in a book entitled *The Principles of Mathematics* at all! Or at least it would be if Russell hadn't supplied an explanation in the preface to the *Principles* telling us that his path to logicism began with dynamical puzzles, not mathematical ones. But that would only explain why it was included in the book, not why it appeared as the grand finale to the book. I am inclined to think that the obvious answer here is the right one. The chief part of mathematics, and the greatest efforts of the great mathematicians, since the seventeenth century, had been devoted to questions suggested by problems in physics. The general theories of partial differential equations of the second order, the analyses of Fourier series and of mathematical series in general, the theories and practices of numerical approximations, the calculus of variations, the study of continuity, of algebraic geometry, and so on and on, all arise out of the study of topics suggested by problems in physics. So for a book written at the turn of the nineteenth century into the twentieth, the grand mathematical finale would have to include as the final step in the sketched-out chain of future deductions (to come in the second volume which he and Whitehead were to have written) all of the great mathematical stuff inspired by mathematical physics.

Russell announces the topic of "rational Dynamics" in these terms:

> The nature of matter, even more than that of space, has always been regarded as a cardinal problem of philosophy. In the present work, however, we are not concerned with the question: What is the nature of the matter that actually exists? We are concerned merely with the analysis of rational Dynamics considered as a branch of pure mathematics, which introduces its subject-matter by definition, not by observation of the actual world. Thus we are not confined to laws of motion which are empirically verified: non-Newtonian Dynamics, like non-Euclidian Geometry, must be as interesting to us as the orthodox system. (*Principles*, p. 465)

At least with respect to physics, then, Russell is about to describe possible dynamical worlds. But although that is what is promised, and in certain measure delivered, the greater part of these chapters does not discuss non-Newtonian dynamics so much as it discusses contemporary dynamics as Russell understood it. And this discussion proceeds in such a way as to arrive at the kind of mathematical notions necessary to characterize the kind of entities and their relations with turn-of-the-century physics (as it had come to Russell) presupposed.

The easiest way, I think, to understand Russell's notion of rational dynamics as well as the actual picture of what he took received dynamics at his time to be is to start with Boscovitch's account of Newtonian dynamics. This is because in *The Principles of Mathematics* it is from a simplified version of Boscovitch's system that most of the notions crucial to Russell's "rational Dynamics" appear to be derived. Since Russell himself, twenty-three years later in *The Analysis of Matter*, gives a simplified schematic version of the Newtonian system in Boscovitch's terms, I shall start from there by quoting him. In Russell's own words:

> The Newtonian system, stated with schematic simplicity, as, e.g., by Boscovitch, is as follows. There is an absolute space, composed of points, and an absolute time composed of instants; there are particles of matter, each of which persists through all time and occupies a point at each instant. Each particle exerts forces on other particles, the effect of which is to produce accelerations. Each particle is associated with a certain quantity, its "mass", which is inversely proportional to the acceleration produced in the particle by a given force. The laws of physics are conceived, on the analogy of the law of gravitation, as formulae giving the force exerted by one particle on another in a given relative situation. This system is logically faultless. It was criticized on the ground that absolute space and time were meaningless, and on the ground that action at a distance was inconceivable. This latter objection was sanctioned by Newton, who was not a strict Newtonian. But in fact neither objection had any force from a logical point of view. (*The Analysis of Matter*, p. 14)

How much of this system did Russell retain in *The Principles of Mathematics*? The remarkable thing is that he retained nearly the whole of it. He defended absolute space and time. He considered impenetrability to be the central notion for matter. And impenetrability in turn he assimilated to the notion that only one particle can occupy one point in space at one time. The notion of force he abandoned as a real entity. But he retained it as a convenient mathematical fiction. As regards the causal connections between particles of matter, because of his earlier studies of the antinomy of dynamical causation, he argued that there are no causal relations between any two particles, but rather that "the only causality occurring in Dynamics requires the whole configuration of the material world as a datum" (*Principles*, p. 474). I shall shortly turn to each of these in turn. But there are three doctrines which he defends prominently in the Matter and Motion part of the *Principles* which do not, on the face of it, have anything to do with dynamics as such but which offer another opportunity for some of the main doctrines of the book.

The first of these extra-dynamical principles is that analysis is not falsification, although there is a curious qualification of this with respect to propositions. The second is the denial of the importance of the notion of substance for dynamics. Instead, the relations between individual material units and their relations to space and time is emphasized. As Russell puts it: "Matter itself seems to be a collective name for all pieces of matter, as space for all points and time for all instants. It is thus the peculiar relation to space and time which distinguishes matter from other qualities, and not any logical difference such as that of subject and predicate, or substance and attribute" (*Principles*, p. 468). The third of the extra-dynamical principles which is important for Russell in his discussion of "rational Dynamics" is the principle that the empirical must be radically distinguished from the logical or mathematical. This principle, of course, is crucial to this enterprise of emptying dynamics of all empirical content and only retaining the mathematical form.

As regards the doctrine that analysis is not falsification, I think that it appears here because Russell thinks that if analysis were falsification physical science would be impossible. He gives the following curious argument:

> It is also said that analysis is falsification, that the complex is not equivalent to the sum of its constituents and is changed when analyzed into these. In this doctrine, as we say in Parts I and II, there is a measure of truth, when what is to be analyzed is a unity. A proposition has a certain indefinable unity, in virtue of which it is an assertion; and this is so completely lost by analysis that no enumeration of constituents will restore it, even though itself be

mentioned as a constituent. There is, it must be confessed, a grave logical difficulty in this fact, for it is difficult not to believe that a whole must be constituted by its constituents. For us, however, it is sufficient to observe that all unities are propositions or propositional concepts, and that consequently nothing that exists is a unity. If, therefore, it is maintained that things are unities, we must reply that no things exist. (*Principles*, pp. 466–7).

This argument is the last paragraph of §439 in the *Principles*, and it is preceded by a much clearer argument to the effect that all unities are either wholes composed of parts or the unity of the absolutely simple, an argument to be found earlier in Leibniz and later in Wittgenstein's *Tractatus*. It is also followed, in §440, by a discussion of the difference between matter and secondary qualities, a discussion which will set the stage for Russell's neutral monism nineteen years later. Consider these sentences, for example:

The question remains: How and why is matter distinguished from the so-called secondary qualities? It cannot, I think, be distinguished as belonging to a different logical class of concepts; the only classes appear to be things, predicates, and relations, and both matter and secondary qualities belong to the first class. Nevertheless the world of dynamics is sharply distinguished from that of the secondary qualities, and the elementary properties of matter are quite different from those of colours. (*Principles*, §440, p. 467)

The difference, Russell will here argue, is that while two pieces of matter can occupy the same place at the same time, one piece of matter cannot occupy two places at the same time. But for colours, while the impenetrability condition holds, one red, for example, *can* be in two different places at the same time. The assimilation of matter and secondary qualities to the same logical class, although distinguished by the fact that they differ as regards impenetrability and its converse, is Russell's chief ground in this part of the *Principles* for rejecting the notion of substance as having any validity in dynamics. This is because in the mistaken, traditional view, "matter, we are told, is a substance, a thing, a subject, of which secondary qualities are the predicates" (*Principles*, p. 466). But matter, according to Russell, is in the same logical class as secondary qualities, and both are to be distinguished from predicates.

In passing, at least, I have remarked on Russell's treatment of matter and of motion. But why did he defend absolute space, motion and time? And why did he consider that the notion of causality involved the notion of the whole universe at three distinct times?

The short answer to the first question is that he doesn't seem to have any argument for admitting absolute time, and that he believes that since he has given arguments in the earlier part of his book in favour of absolute space (notably in his refutation of Kant's antinomies), there is no good reason not to embrace it. But, if absolute space and time are admitted, then there is no reason to reject absolute motion either. The long answer would take many pages, and I shall not go into it here. Russell does, however, defend absolute motion directly by defending Newton's bucket experiment and by refuting Mach's arguments against absolute motion. I will just give the argument against Mach, because it also clarifies how Russell's logical empiricism (in which mathematics is strictly separate from the empirical world) differs from that of Mach:

469. Mach has a very curious argument by which he attempts to refute the ground in favour of absolute rotation. He remarks that, in the actual world, the earth rotates relating to the fixed stars, and that the universe is not given twice over in different shapes, but only once, and as we find it. Hence any argument that the rotation of the earth could be inferred *if* there were no heavenly bodies is futile. This argument contains the very essence of empiricism, in a sense in which empiricism is radically opposed to the philosophy advocated in the present work. The logical basis of the argument is that all propositions are essentially concerned with actual existents, not with entities which may or may not exist. For if, as has been held throughout our previous discussions, the whole dynamical world with its laws can be considered without regard to existence, then it can be no part of the *meaning* of these laws to assert that the matter to which they apply exists, and therefore they can be applied to universes which do not exist. Apart from general arguments, it is evident that the laws are so applied throughout rational Dynamics, and that, in all exact calculations, the distribution of matter which is assumed is not that of the actual world. It seems impossible to deny significance to such calculations; and yet, if they have significance, if they contain propositions at all, whether true or false, then it can be no necessary part of their *meaning* to assert the existence of the matter to which they are applied. This being so, the universe is given, as an entity, not only twice, but as many times as there are possible distributions of matter, and Mach's argument falls to the ground. The point is important, as illustrating a respect in which the philosophy here advocated is to be reckoned with idealism and not with empiricism, in spite of the contention that what exists can only be known empirically.

Thus to conclude: Absolute motion is essential to Dynamics, and involves absolute space. This fact, which is a difficulty in current philosophies, is for us a powerful confirmation of the logic upon which our discussions are based. (*Principles*, §469, pp. 492–3)

As regards our final question—namely, why did Russell think that causality is a relation holding between events at three times, not two, and that the whole state of the material universe at two of the three times is necessary for the statements of a causal relation?—there is another long story to be told. Essentially, however, it is offered as a way around the antinomy of dynamical causation discussed earlier. That story, however, will have to form part of another paper.

Let me summarize: In both *Leibniz* and the *Principles* Russell shows a thorough knowledge of both the history of physics and of contemporary physics. The physical system he appears to prefer is rather that of Boscovitch, because of its simplicity and purity, although he denies the necessity for believing in actual forces. In his discussion of Leibniz's dynamics he nearly formulates the problems which led Einstein to the special theory of relativity, but when he discovers antinomies in physics he seems to enjoy them too much to consider revising physics as such. This may be a remnant of his Hegelian period still present in *Leibniz* and occasionally rearing its head in the *Principles*. Because of his embracing absolute space and time and motion, it is not clear that his "rational dynamics", although it was offered as a logical system to which any possible dynamics might conform, would have been compatible even with such a tiny tinkering with the Newtonian system as Einstein's special theory represents.

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