Introducing Kurt Gödel

by Billy Joe Lucas

Kurt Gödel. *Collected Works*. Volume I: *Publications 1929–1936*. Ed. Solomon Feferman (Ed.-in-chief), John W. Dawson, Jr., Stephen C. Kleene, Gregory H. Moore, Robert M. Solovay, and Jean van Heijenoort. Prepared under the auspices of the Association for Symbolic Logic. New York: Oxford University Press; Oxford: Clarendon Press, 1986. Pp. xvi, 474. US$35.00.

I

EVALUATING RUSSELL'S CONTRIBUTIONS to logic, his applications of logic, and his views on the nature of logic and on the relationship of logic to philosophy involves answering some serious questions concerning twentieth-century logic, logic both as Russell knew it and as it has evolved since Russell made his contributions. As Gödel is second only to Russell among contributors to logic in the first half of this century, our understanding of Russell will thus be incomplete until we come to appreciate fully the work of Kurt Gödel.

Understanding Gödel is a task that should become easier for most Russell students now that Gödel's collected works are being published in English translation. Volume I contains, or rather was intended to contain (see Section VI below), all of Gödel's writings published in the period 1929–1936. Volume II is to contain the remainder of Gödel's published works. Subsequent volumes are to contain his "unpublished manuscripts, lectures, lecture notes, and correspondence, as well as extracts from his scientific notebooks."

Except for his 1934 lectures on undecidability (which were published originally in English), all of Gödel's publications as of 1936 were in German. Gödel's German texts are printed on left-hand pages, with English translations on the facing pages. Except for his dissertation, original pagination is retained in margin notes, and occasional footnotes (mostly Gödel's) are located at the bottom of the page.

There are 490 numbered pages in this book. In English, the works of Gödel collected in this volume total approximately 145 pages, or thirty percent of the total. Gödel's works, in German, come to another 119 pages; Gödel's writings thus comprise about fifty-four percent of the printed material. The section titled "Textual Notes" occupies only five pages, and fails to mention that Gödel's citations of publications have been altered, sometimes both in German originals and in the translations thereof, to conform to the style used by other contributors to this volume. This *is* mentioned in a four-page section titled, "Information for the Reader." The only index is a fourteen-page index of names of persons, places, and publications that fails to differentiate between occurrences of names in Gödel's writings and occurrences in the writings of the other authors included in this volume.

Dawson's seven-page Gödel chronology is superior to any of the Russell chronologies I have seen, and should prove useful for Russell scholars who wish to integrate the sequence of events in Gödel's life into their image of the sequential flow of Russell's life and work.

Readers with skill at reading photographs will take pleasure in the six well-reproduced photos contained in this volume. These are not listed in the table of contents; one occurs opposite the title-page, and the others between pages 15 and 16.

The third largest portion of this book, after the 145 pages of Gödel in English and the 119 pages of Gödel in German, is the approximately seventy-eight pages comprised of nineteen introductory notes to some thirty-five of the fifty-six works of Gödel collected in this volume. These nineteen introductory notes, easily distinguished from Gödel's text by the occurrence of a vertical line down the outside margin, were written by the following eleven authors: two are by John W. Dawson, Jr.; one is co-authored by Burton Dreben and Jean van Heijenoort; one is by Solomon Feferman; one is by Warren D. Goldfarb; three are by Stephen C. Kleene; one is by Rohit Parikh; five are by W.V. Quine; three are by A.S. Troelstra; one is by Robert L. Vaught; and one is by Judson Webb.

Solomon Feferman has contributed a thirty-six page essay, "Gödel's Life and Work," some fourteen pages of which are an account of Gödel's personal life and career, and the remainder of which contains both a brief description of the contents of this and subsequent volumes of Gödel's works, as well as an accessible overview not only of Gödel's contributions to logic but also of his work and thought in general. This reviewer, as might be expected, cannot agree with Feferman's reference (on p. 1) to Gödel as "the most important logician of the twentieth century", but making the case for Russell's right to be the referent of this description is not, of course, within the proper scope of this review essay. Russell scholars who read Feferman's essay will no doubt be struck by the following parallel between the lives of Gödel and Russell: nearly all of the work in logic for which either is famous was done in an eleven-year period—1900–1910 for Russell, and 1929–1939 for Gödel. (The parallel continues: both did somewhat less well-known work later.)

Gödel also wrote, as did Russell, many reviews of technical books early in his career. Of the fifty-six works of Gödel collected in Volume I, thirty-three are reviews published from 1931 to 1936. Of these, nineteen are of less than half a page in length (five occupy less than ten lines of print each), and only one requires more than one page of print: Gödel's two-page-plus 1933 review of the revised edition of his former teacher Hans Hahn's book *Reelle Funktionen*.[1] Six of these reviews appeared in the *Monatshefte für Mathematik und Physik*, four of them in 1931. The remaining twenty-seven were all published in the *Zentralblatt für Mathematik und ihre Grenzgebiete*, twenty-four of them in the years 1932–35. Another parallel between the careers of Gödel and Russell:

---

[1] Leipzig: 1932.

although both were experienced reviewers, neither ever wrote reviews for *Mathematical Reviews* or *The Journal of Symbolic Logic*. Dawson[2] notes that Gödel was not even asked to review for *Mathematical Reviews* until 1962. It would be interesting to know whether Russell was ever asked to review for either journal.

II

The first three works of Gödel in this volume are his dissertation of 1929 (twenty-one pages in English), a revised and substantially abbreviated version (eleven pages in English) published in 1930, and a brief abstract based on a presentation of Gödel's results in Königsberg on 6 September 1930. Of all of Gödel's longer, published writings, his dissertation has been, until now, the most difficult to obtain, and is here translated for the first time into English, by Stefan Bauer-Mengelberg and van Heijenoort. The 1930 essay is translated by Bauer-Mengelberg, with suggestions and revisions by Gödel and van Heijenoort. The 1930 abstract is translated by Dawson. Dreben and van Heijenoort have contributed a sixteen-page introduction to these three works in which they interpret the historical context in which Gödel saw completeness as a problem and suggest just what his philosophical goals were, sort out some of the tangled history of the origins of the completeness, compactness, and (downward) Löwenheim–Skolem theorems (all of which are proven either in Gödel's dissertation or in the 1930 article), and describe some connections between Gödel's work on completeness and more recent developments in logic. Gödel's way of proving completeness is easily misunderstood: logicians of the calibre of Hilbert, Ackermann, and Church have erred in expounding it. In addition, then, to Gödel's two versions of the proof and Dreben and van Heijenoort's introduction in this volume, the beginning reader may wish to consult section 44 of Church.[3] Of special interest to Russell scholars is the fact that Gödel's results are based on the first-order fragment of *Principia Mathematica*, with axiom $*1.5$ deleted and axiom $*1.3$ replaced by theorem $*2.2$, and are, therefore, among the earliest results describing aspects of Russell and Whitehead's *magnum opus*. Also worth noting is the fact that Gödel here proves the independence of each of the first-order axioms and rules of *Principia*, except, of course, for $*1.5$, which Bernays had already shown (in 1926) to be derivable from the others.

III

Due in part to widespread false claims about how Gödel's incompleteness theorem refutes Russell's logistic thesis, perhaps the writings of Gödel best known to those who study Russell are his three papers on incompleteness published in 1930, 1931, and 1932. These, as translated by Bauer-Mengelberg (1930) and

[2] John W. Dawson, Jr., "The Published Work of Kurt Gödel: an Annotated Bibliography," *Notre Dame Journal of Formal Logic*, 24 (1983): 255–84 (at 272).

[3] Alonzo Church, *Introduction to Mathematical Logic*, Vol. I (Princeton: Princeton University Press, 1956).

van Heijenoort (1931 and 1932), are reproduced here, along with Gödel's "On Undecidable Propositions of Formal Mathematical Systems". The latter consists of notes taken in 1934, by Kleene and J. Barkley Rosser, of Gödel's lectures at the Institute for Advanced Study, together with Gödel's later postscript and corrections. All four of these works have been readily available in English for some time, although not all together in one volume. Kleene's fifteen-page introduction to the 1930, 1931 and 1932 papers on incompleteness is a model introduction: it sets Gödel's work in proper historical context, prepares the reader to understand Gödel's text, and relates Gödel's papers to important developments that have followed. Kleene's eight-page introduction to the 1934 lecture notes is by itself a sufficient reason to purchase this volume.

IV

In addition to these two groups of essays (three on completeness, and four on incompleteness), Volume I contains forty-nine additional items written by Gödel, twenty-seven of them with accompanying introductions. These other writings of Gödel vary both in topic and in length.

The topics covered are diverse and include: Skolem's work on non-standard models for arithmetic (Vaught has contributed a three-page, compact analysis of both Skolem and Gödel's reviews of Skolem); Hilbert's introduction of a restricted, informal "rule" of infinite induction for use in proving completeness (Feferman's five-page introduction to Gödel's review of Hilbert discusses both the meaning of Hilbert's new rule and the "personal" side of Gödel's relationship to Hilbert); Church's early work on foundational lambda calculus that was intended as an improvement over Russell's type theory (Gödel's three reviews of Church are introduced by Kleene); and Gödel's statement, without proof, of his 1936 "theorem" concerning the practical merits of type theory (Parikh's two-page introduction to the topic of Gödel's speed-up theorem is a masterpiece). In addition, Gödel published five short, highly technical papers on geometry (average length: one page) in *Ergebnisse eines mathematischen Kolloquiums* in 1933. Webb has written an interesting four-page technical introduction that puts these notes into an historical context, while relating them both to later developments in geometry and to Gödel's views on completeness. Judging only from his published work, the closest Gödel ever came to returning to work in geometry was in his papers of 1949 and 1952 on physics.

Excluding publications mentioned in earlier sections, Gödel's writings vary in length from that of his eleven-page 1933 essay on the decision problem for first-order logic (which is accompanied by Goldfarb's elegant and helpful five-page introduction to this and two other publications on the decision problem), to that of the following untitled remark published in *Ergebnisse eines mathematischen Kolloquiums* in 1936:

Actually, for each individual entrepreneur the demand also depends on his income, and that in turn depends on the price of the factors of production. One can formulate

an appropriate system of equations and investigate whether it is solvable.

Dawson, in his one-page introduction to this remark, not only manages to set it in a context in which it makes sense, but also provides the reader with references that substantiate the claim that Gödel had a not insignificant influence on Oskar Morgenstern's work in economics. (Serious Russell students will recall, of course, Russell's relationship with Keynes.)

Although Gödel wrote nowhere near as much as Russell on geometry, physics, or economics, this collection of Gödel's papers makes it possible for us to see that, like Russell, Gödel's interest in mathematics was not restricted to work that verified or refuted various philosophical claims, but also included an interest both in math for math's sake and in substantive uses of mathematics to increase our understanding of the world in which we live.

V

Neither Russell nor Gödel accepted the intuitionist philosophy of mathematics associated with Brouwer and Heyting, yet both showed a long-term preoccupation with it. The opening section of Gödel's first work (his dissertation of 1929) reveals his concern with intuitionism,[4] and this concern is paramount in four of the works published in this volume, two of which were first published in 1932, and two in 1933.

Of Gödel's two 1932 publications on intuitionism, his "On the Intuitionistic Propositional Calculus" is far and away the most impressive. Troelstra's introduction to this stunning one-page paper realizes the following goals. In no more than one page the reader is, among other things, given the necessary background to understand this rich little paper by Gödel, told how Gödel's results relate to later work on intermediate logics by Ivo Thomas and Michael Dummett, informed as to which result in Gödel's essay constitutes the first result on intermediate propositional logics, given some insight into the ways in which these latter logics are, and are not, significant, and referred to excellent recent technical surveys and annotated bibliographies on intermediate (between intuitionistic and classical) logics and on many-valued logics.

Gödel's five-page 1933 paper, "On Intuitionistic Arithmetic and Number Theory", shows how to translate every formula of a system of classical arithmetic into a formula of intuitionistic mathematics in such a way as to be able to prove that all the theorems provable in the classical system are translated

---

[4] Concern with intuitionism is also prominent in the last of Gödel's major works to be published in his lifetime, his 1958 essay, "On a Hitherto Unexploited Extension of the Finitary Standpoint". This essay is translated into English by Wilfred Hodges and Bruce Watson in the *Journal of Philosophical Logic*, 9 (1980): 133–42. Hodges and Watson's translation is accompanied by a bibliography (compiled by J.R. Hindley) of work resulting from Gödel's paper, and is not included in the bibliography for this volume. The duration of Gödel's concern with intuitionism is particularly evident in connection with the history of this essay. Dawson (in "Kurt Gödel in Sharper Focus", *The Mathematical Intelligencer*, 6 [1984]: 16) notes that this paper was based on results Gödel had by 1941, and that he worked on an English translation of it as late as the early 1970's.

---

into theorems provable by intuitionistic methods. Troelstra's four-page introduction to this essay explains Gödel's method of translation (there is what can only be a typographical error in Troelstra's presentation—see the list of errata at the end of this review), and sets Gödel's work in the context both of earlier anticipations and of subsequent extensions of his results. Students of *Principia* may find Troelstra's remarks concerning the proof of the consistency of Gödel's system of classical arithmetic relative to intuitionistic mathematics of special interest. Gödel's other 1933 paper on intuitionism is discussed in the following section on modal logic.

VI

At least four works of Gödel included in this volume are concerned with modal logic: his brief 1931 review of Oskar Becker's "On the Logic of Modalities" (this is a suggested correction to the translation of Becker's title in this volume); his article, "An Interpretation of the Intuitionistic Propositional Calculus" (1933); his 1933 review of Lewis;[5] and his 1935 review of Huntington.[6] In terms of influence on subsequent developments within a subfield of logic, Gödel's work on modal logic is not as significant as his work on recursive functions and on set theory, yet it deserves more extensive study than it receives here. Two of these works (the review of Becker and the review of Lewis) are not discussed in any of the twenty introductions to various aspects of Gödel's thought. Two are.

Troelstra has contributed a fine three-page introduction to Gödel's one-page note of 1933 in which Gödel shows how to translate effectively any formula A of Heyting's intuitionistic logic into a formula A' of Gödel's modal logic S4 in such a way that A is a theorem of Heyting's system if and only if A' is a theorem of S4. Troelstra's essay, while describing both Gödel's achievements and subsequent related developments in the field of provability modal logics, manages to unobtrusively integrate references to some seventeen important publications that either influenced or anticipated Gödel or continued the line of work begun in his 1933 essay.

Except for Troelstra's elegant introduction to Gödel's note on the interpretation of intuitionistic propositional calculus, and Feferman's discussion of it in his general introduction, Gödel's contributions to modal logic are not placed in historical context, nor is the reader given any reference to subsequent work in the field. The only other introduction to Gödel's publications in the field of modal logic included in this volume is Quine's introduction to Gödel's review of Huntington's "Independent Postulates Related to C.I. Lewis's Theory of Strict Implication". Quine's introduction consists of these three sentences.

Huntington's system is not a modal logic, for he uses predicates rather than iterable

---

[5] C.I. Lewis, "Alternative Systems of Logic", *The Monist*, 42 (1932): 481–507.
[6] Edward V. Huntington, "Independent Postulates Related to C.I. Lewis's Theory of Strict Implication", *Mind*, n.s. 43 (1934): 181–98.

functors to express necessity and impossibility. Like Lewis, he stops short of quantification. But, if he were to introduce it, he would still be unable, on this approach, to quantify into modal contexts and thus precipitate the referential opacity and related perplexities that beset modal logic.

Quine's claim that both Huntington, in his study of Lewis, and Lewis, in the system studied by Huntington, stop short of quantification is rather perplexing. Huntington's postulate 0 (p. 183) and postulate 12 (p. 184) are existential, and hence appear not to stop short of quantification. Furthermore, on p. 182 of his essay, Huntington says that the axioms of Lewis he is studying include proposition "(20.01) on page 179" of Lewis and Langford.[7] Yet, contra Quine, Lewis's proposition 20.01 is an existential quantification from outside the scope of a modal operator of variables that occur within the scope of the modal operator, i.e., Lewis has quantification "into modal contexts".

In addition to the four papers on modal logic included in this volume, Gödel also wrote, in collaboration with William Tuthill Parry, another paper on modal logic that was not included in this volume. On 18 February 1932 Parry gave a lecture at Karl Menger's colloquium in Vienna. Based on Parry's lecture, and without the use of any notes written by Parry, Gödel actually wrote the text of the published report of Parry's lecture. What's more, Gödel also contributed original, substantive additions to the material contained in Parry's lecture. The result of this joint effort, "Zum Lewisschen Aussagenkalkül", was published in *Ergebnisse eines mathematischen Kolloquiums* in 1933. Gödel added a footnote to the report, which said, "the proof given below differs in some respects from that given by Parry."[8] This footnote and my correspondence with Professor Parry, from which the account just given is derived and in which Parry says, "Gödel had, of course, greatly improved my proof; so the paper as published was a joint paper, and in my (not unbiased) opinion, belongs in the collected works of Gödel",[9] seem sufficient to establish "Zum Lewisschen Aussagenkalkül" as a paper co-authored by Gödel and Parry.

This collection is, then, incomplete. Gödel's hitherto generally unknown collaboration with Parry raises an interesting question: are there other papers in the proceedings of Menger's colloquium during the years Gödel served on the editorial staff that are also the result of the creative additions of Gödel?[10] If there are, and if there is no decisive textual or archival evidence of this in cases where the lecturer in question is now dead, then there is no effective and practical way to compile a complete edition of Gödel's papers.

VII

Naturally following upon the question of the completeness of this volume of Gödel's publications is the question of its soundness. An examination both of the content and of the evidence concerning the authorship of a brief, two-paragraph note that the editors have included and given the title, "On Parry's Axioms", reveals that the second paragraph of this publication is not, in an acceptable sense, the work of Kurt Gödel.[11]

VIII

The bibliography, or list of references, for this volume is decidedly odd. The following features deserve attention.

The list of references takes up 53 pages, or just over a third as much space as is occupied by Gödel's writings in English. Yet, it fails to include such basic items as the following: a complete list of translations of Gödel's works into English (e.g., neither Meltzer's 1962 translation nor Mendelson's 1965 translation of Gödel's major undecidability paper of 1931 is listed); reviews of earlier translations of Gödel (e.g., Bauer-Mengelberg's 1965 review of Meltzer's translation is not listed); translations of Gödel's publications into languages other than English (e.g., not even Mosterín,[12] the most comprehensive edition of Gödel to appear in any language as of the date of this volume, is cited); previously published correspondence of Gödel (none of the correspondence in Dawson[13] is counted among Gödel's publications); some major philosophical and/or expository works on Gödel are omitted (e.g., neither Myhill 1952[14] nor Nagel and Newman 1958[15] is here); some landmark papers in the history of logic that are of major significance in interpreting Gödel's work (e.g., Henkin 1950[16] is not included); certain basic texts that would be important for, but possibly unknown to, readers outside the field of logic, at whom this collection is also aimed (e.g., although Corcoran's revised edition of Tarski's papers is cited, the inexpensive, paperback edition published by Hackett is not mentioned). All of this is compounded by the fact that there are some 316 items entered in the list of references that are not by Gödel, and yet are not cited anywhere in this volume, either by Gödel or by the other contributors.

There is a questionable choice in the citation of works by Russell: the 1920

---

[7] *Symbolic Logic* (New York: Century, 1932).

[8] William Tuthill Parry and Kurt Gödel, "Zum Lewisschen Aussagenkalkül", *Ergebnisse eines mathematischen Kolloquiums*, 4 (1933): 15–16 (at 15).

[9] Letter to Billy Joe Lucas, 31 July 1986.

[10] On p. 38 of this volume, Gödel is said to have assisted in the editing of volumes 1–5, 7 and 8. In Dawson's "Kurt Gödel in Sharper Focus", the list includes only volumes 2–5, 7 and 8 (p. 12). Gödel's work as reporter and editor deserves further investigation, for the reason just given, as well as in connection with the issue raised by section VII below.

[11] For additional details, see my "Kurt Gödel's Contributions to Relevance Logic", abstract in *The Journal of Symbolic Logic*, 54 (1989): 244–5.

[12] Ed. Jesús Mosterín, *Obras completas* (Madrid: 1981).

[13] "The Published Work of Kurt Gödel: an Annotated Bibliography".

[14] John Myhill, "Some Philosophical Implications of Mathematical Logic", *The Review of Metaphysics*, 6 (1952): 165–98.

[15] Ernest Nagel and James R. Newman, *Gödel's Proof* (New York: New York University Press, 1958).

[16] Leon Henkin, "Completeness in the Theory of Types", *The Journal of Symbolic Logic*, 15 (1950): 81–91.

printing of *Introduction to Mathematical Philosophy* is incorrectly listed as a "second edition".[17]

This volume is remarkably free of typographical errors. I have noticed only the following: p. 183, line 13 for "χ_1" read "χ_i"; p. 282, line 15, insert "$(\neg F)' := \neg F'$"; p. 354, line 12, delete "$<$"; p. 386, line 3 of title, p. 387, line 3 of title, and p. 430 in *Huntington 1934* for "Lewis'" read "Lewis's"; p. 442 for *Parry 1933a* change "17" to "16"; p. 470 to entries under "Parry, William T." add "266".

The production of this volume involved not only the work of the six editors and seven additional contributors of introductory notes, but also the efforts of, and support by, a variety of other individuals and organizations. The result is a well-made, well-organized, landmark book in the history of western intellectual thought. No one's personal library, however small, should be without this book.

Department of Philosophy
Manhattanville College

[17] See Kenneth Blackwell, "A Non-Existent Revision of 'Introduction to Mathematical Philosophy'," *Russell*, no. 20 (1975): 16–18.