PHILOSOPHY WITHOUT MATHEMATICS

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Peter Hylton. *Russell, Idealism and the Emergence of Analytic Philosophy.* Oxford: Clarendon Press, 1990. Pp. xvii, 420. £48.00.

I

Interest in Russell's philosophical career naturally takes as a main focus his transition from the prevailing neo-Hegelianism of his early manhood to the realism which he pioneered with G. E. Moore. These two standpoints seem to be diametrically opposed; yet it has become clear to researchers that the change was not a simple pendular swing, but that various elements and concerns of the former standpoint were retained in the latter, albeit in substantially changed contexts. Further, it was already clear from his publications of that time that questions on the foundations of mathematics, and the bearing upon them of the new mathematical logic which he learnt largely from G. Peano and enriched with his own contributions, played a major role in the change.

The purpose of the book under review is to examine this story in detail with especial concern for the philosophical differences. The account is largely based upon Russell's publications, but good though limited use is made of some manuscripts, especially from the Russell Archives. The book falls into three parts, following a chronological order.

"The Idealist Background" (pp. 19–101) begins with two welcome chapters on T. H. Green and F. H. Bradley, followed by a summary of Russell's own standpoint of this genre. J. M. E. McTaggart is given much less treatment (p. 80), although the fact that "Moore said that he was more influenced by McTaggart than by any of his other teachers at Cambridge" (p. 116) is one reason why he should have been treated in greater depth, the other reason being of course Russell's regular interaction with his slightly older adversary.

"Platonic Atomism" (pp. 103–275) is the somewhat anachronistic title given to a part which covers Russell from his reform with Moore through *The Principles of Mathematics* (1903) up to "On Denoting" (1905). The word

"atomism" in Russell connotes for us the logical atomism and related doctrines of the 1910s; while brick-building was going on also in this earlier period, Russell's ontology was then still too luxuriant for the strategy to be as dominating as this title suggests.

"Logic, Fact and Knowledge" (pp. 277–391) has two chapters devoted respectively to *Principia Mathematica* and to the doctrines concerning epistemology and judgement that characterized his immediately succeeding studies and the first interactions with Wittgenstein. It ends appropriately with Russell's hopes for the new scientific philosophy that he had forged.

With regard to the two aspects of Russell's development mentioned at the head of this review, the first is fairly well conveyed, although a shorter account could have left a clearer picture. The second, however, is most disappointingly treated. The difficulty seems to arise from the author's misconception that "The philosophy of mathematics was, for Russell, a sort of crucial experiment—a testing ground on which the relative merits of idealism and Platonic Atomism could be definitively judged" (p. 115). This statement is of course correct as far as it goes, but it does not go far enough; students of Russell's mathematics and logic know that *these provided much of the inspiration* for the change of standpoint, and for many tactics followed in the new position. As a result, unlike Russell, the author does not take mathematics seriously; in consequence many features of the logic fade from sight also. Let me mention some of the chief omissions, dwelling in turn with Russell's logic, his logicism, and his mathematics.

II

For all that Chapter 7 is entitled "The Logic of *Principia Mathematica*", several aspects of Russell's logic are discussed but little or not at all. For example, a decision is taken on page 286 not to analyze the substitutional theory, as the author has treated it in a separate paper; but then we lose not only a crucial use of the theory of definite descriptions to create (between 1905 and 1907) a mathematico-logical system, but also several traces of the theory of types adopted in *Principia Mathematica.* Further, while good points are made about that theory—for example, that it contains types within orders rather than the normal converse interpretation (pp. 306–11)—nothing is said of the different version presented in the 1908 paper "Mathematical Logic as Based on the Theory of Types", which is not discussed at all and only mentioned on page 282 as if it contained the same details as in *Principia Mathematica.* Finally, the long account of the multiple-relation theory of judgment (Chap. 8) does not fully grasp the relationship between mathematics, logic and type theory itself; for example, the brief but important manuscript of

1912 on logic as a science of forms is not mentioned, especially not on page 268.[6]

III

The maltreatment of logicism is most evident in the consistent basic misstatement: that Russell wished to reduce mathematics to logic. This mistake occurs most noticeably on page 190, where the opening clause of the *Principles* that "Pure mathematics is the class of all propositions of the form *p* implies *q* ... " and yet talk of "mathematics" are adjoined once again. The differences between mathematics and pure mathematics are quite crucial for Russell, for (to the extent that his definition of the latter is understandable at all) it leads to a "categorial" formulation of logicism: that the category of logic is reducible to that of mathematics. Another missed opportunity occurs on page 384, where a useful account of a manuscript of 1912 (a brother to the unnoticed one mentioned in sec. 11) includes a statement about "pure mathematics" as involving "variables, concerning which certain hypotheses are made" and waiting for application to give cases (in this context, of physics).

Some of the silence may be due to the very light treatment of the paradoxes. Only Russell's own is discussed, with no original points made; thus his aim of constructing a logic in which all of them will be Solved, and moreover uniformly, does not receive treatment.

IV

The treatment of mathematics in general is well characterized by the lamentable pages 184 to 187. After the usual passing mentions of G. Cantor and Peano—whose work requires of the Russell scholar the same detailed and serious attention which Russell himself gave it—some forays into mathematical concepts are essayed. "The dynamical or kinematic approach to the calculus has the advantage of avoiding the Leibnizian appeal to infinitesimals", as if Newton's version, mentioned six lines earlier, did not use its own version (p. 185). The continuity of a mathematical function is actually uniform continuity over an unspecified interval (p. 186), an important distinction in the mathematical analysis of Russell's time. Again, the definition of convergence of an infinite series lacks specification of positive values of variables.

Elsewhere the finitude of Russell's theory of types is regarded as "of the first importance" for "the subsequent history of logic" (p. 304) without any note being taken of its consequences for Russell's logicism: namely, that the

[6] "What Is Logic?"; due to appear in *Papers* 6 (forthcoming 1992).

upper reaches of Cantor's transfinite numbers cannot be defined. A few pages later "the axiom of infinity has the same status in *Principia Mathematica* as does the axiom of choice" (p. 319); Russell, of course, was more perceptive, and discussed the matter in detail in 1911 in a little-known paper which I made easily available some years ago.[7]

At least these areas of pure mathematics (to use that phrase in its traditional form) are mentioned; applied mathematics is passed over in almost complete silence. Yet the change from idealism to realism is particularly marked in these contexts. Take for example, mechanics. In his Hegelian phase Russell was concerned with, for example, the "dialectical transition" from space to matter, of space as "an adjective of one substance", and such funny stuff;[8] by the time of the *Principles*, E. Mach and H. Hertz are centre stage, and quite a different epistemological framework is evident. Here is a perfect example of a "testing ground" of philosophical change; yet "mechanics" does not occur in the index (an excellent one, prepared by B. Johnsen).

V

Lest it be thought that these omissions are inevitable consequences of pioneering research in a little-studied area, all the points discussed above, and many related ones, have been examined quite intensively in the last decade, and usually with greater penetration than is shown here. The disappointments are clarified by the bibliography (pp. 393–400), which contains no items from this journal and also no writings by N. Cocchiarella, N. Griffin, G. Landini or F. A. Rodríguez-Consuegra.[9]

[7] B. Russell, "Sur les axiomes de l'infini et du transfini', *Comptes rendus des séances de la Société Mathématique de France*, no. 2 (1911): 22–35; translated into English in my *Dear Russell—Dear Jourdain. A Commentary on Russell's Logic, Based on His Correspondence with Philip Jourdain* (London: Duckworth; New York: Columbia U. P., 1977), pp. 161–74.

[8] These texts are now conveniently gathered together in *Papers* 2; see especially papers 2 (*passim*) and 9–11.

[9] For a survey of recent Russell scholarship, see my "Bertrand Russell (1872–1970) after Twenty Years", *Notes and Records of the Royal Society*, 44 (1990): 280–306.