Russell's Argument against Frege's Distinction

Paweł Tarnau
Philosophy / Jagiellonian University
Cracow, Poland

In "On Denoting," Russell argued that Frege's theory of sense and reference was an "inextricable tangle," but, ironically, many readers found the argument even more knotty. In an effort to make sense of it, commentators were often driven to attribute to Russell quite obvious and simple fallacies. A different approach was taken by Peter Geach, who suggested that Russell's argument could be given a consistent reading if it were construed as arguing, not against Frege's theory, but against Russell's own earlier theory, which he put forward in The Principles of Mathematics. It is not an implausible hypothesis, considering how prone Russell was to misrepresent other philosophers' views and read his own powerful ideas into their writings. In order to justify Geach's hypothesis properly, one would have to show the crucial difference between Russell's and Frege's theories that made only one of them susceptible to the attack in "On Denoting." Russell explicitly considered the two theories to be "very nearly the same"; this self-interpretation would have to be questioned. Geach's hypothesis would be further strengthened by providing an actual exegesis of the "On Denoting" argument as directed against The Principles of Mathematics theory. In this paper I will try to accomplish both of these tasks.

Russell is often accused of confusing use and mention. The accusation is not fair, however. It is true that he does not insist on this distinction; he does not employ it extensively. But he is certainly aware of it; he simply does not think that it is an important logical distinction. Today this view may seem almost incomprehensible, but that only shows the strength of Frege's influence on our ideas.

There are two striking symptoms of Russell's lack of concern for the use-mention distinction: firstly, he does not observe the convention which says that putting an expression in quotation marks produces a name of that expression; secondly, he does not feel any need to employ the pair of terms "sentence" and "proposition," but contents himself with the latter only. The claim about Russell's use of inverted commas needs to be proved; I will come to that later.

Russell's assertions are often ambiguous between use and mention of expressions. For example, he asserts that "propositions do not contain words" (PaM, p. 47). However, a few pages later, he speaks of propositions that contain the phrase "any number" (p. 53). If this is not to be self-contradictory, then either the term "proposition" must sometimes signify a linguistic expression, or the term "phrase" must sometimes be understood without implying that a phrase consists of words. This ambiguity is openly admitted in a later article, where he writes:

"[we shall be concerned with] the distinction between verbs and substantives, or, more correctly, between the objects denoted by verbs and the objects denoted by substantives. Since this more correct expression is long and cumbersome, I shall generally use the shorter expression to mean the same thing. Thus, when I speak of verbs, I mean the object denoted by verbs, and similarly for substantives."  

Russell's attitude towards the distinction between use and mention is not unjustified. It has its reason in his view of language and the subject-matter of logic. In Russell's view, the subject-matter of logic has nothing to do with language. Words, both spoken and written, are entirely outside its realm. Logic, just as other sciences (except linguistics), is concerned with the entities indicated by words, and not with words themselves. Even the relation of indicating, which holds between words and things in the world, is of no interest to it: "mean-

1 In LK, pp. 39–56.

Russell's Argument against Frege's Distinction

Mont Blanc or Etna, could be a constituent of a thought which one can assert, seemed to him absurd: otherwise, "each individual piece of frozen, solidified lava which is part of Mount Etna would also be part of the thought that Etna is higher than Vesuvius. But it seems to me absurd that pieces of lava, even pieces of which I had no knowledge, should be parts of my thought."4 Needless to say, Russell was unmoved by such arguments.

The position taken by Russell in the letter to Frege quoted above does not seem, however, to be quite the same as that of The Principles of Mathematics, page 47, which he published shortly before. He must have been already well advanced on his way towards the view expressed in "On Denoting". In the letter, Russell argued against Frege that if a real object like Mont Blanc could never occur in a proposition, then we would "know nothing at all about Mont Blanc". But this argument is actually incorrect from the standpoint of the Principles. And indeed, whereas in the book, in the Appendix devoted to Frege, Russell wrote that he did recognize a distinction roughly similar to Frege's sense-reference distinction, in the letter he already denied it: "I do not admit the sense at all."

What was the theory put forward in the Principles, page 47, then? In addition to things and ordinary concepts, Russell also recognized the existence of a third type of objects, the so-called denoting concepts. Of a thing or of an ordinary concept it is always true that if it occurs in a proposition, then the proposition is about that object. Thus, the proposition John is taller than Jane consists of John, Jane and the relation is taller than. But if we take a proposition like I met a man, then it is not about the concept a man, even though that concept is a constituent of the proposition. The proposition is about something quite different, some actual biped denoted by the concept (PoM, p. 47). Denoting concepts, when inserted into propositions, cause what might be called a "shift of subject-matter". This phenomenon is indeed their defining characteristic. Thus, they are an exceptional case in Russell's semantics, and do not cohere with it very well.


4 Letter to Jourdain, n.d., in Frege, Philosophical and Mathematical Correspondence, p. 79.
Not surprisingly, Russell was only too glad to dispose of them as soon as he discovered a way of doing that.

Before we consider his rejection of denoting concepts, we must consider reasons that led him to recognize them in the first place. He distinguished six kinds of denoting concepts, of which the following are examples: a man, some man, any man, every man, all men, the richest man. Let us confine our attention to the last kind of denoting concepts, i.e. ones indicated by descriptive phrases beginning with “the”.

The question arises: Why, say, should Mount Everest not be a constituent of the proposition The highest mountain on Earth is over 8,000 m. high? It is conceded, after all, that the proposition is indeed about Mount Everest. Russell adduced two arguments to demonstrate the indispensability of denoting concepts, i.e. to demonstrate that descriptions cannot be taken to be proper names which merely indicate their objects. One of the arguments has to do with identity, the other with definitions.

Identity is a relation between an object and itself. But how can an identity proposition fail to be trivially true or trivially false? If it is a true identity proposition made by means of two names it must be trivially true, because someone who does not recognize it as true must clearly fail to know the meaning (= what it indicates) of at least one of the two names. But if such a proposition contains at least one denoting concept, then it carries non-trivial information about what is denoted by a given concept. This argument is essentially the same as that given by Frege in the beginning of “On Sense and Reference”.

The other argument concerns definitions. The puzzle is that definitions, theoretically, are nothing but statements of symbolic abbreviation, irrelevant to the reasoning and inserted only for practical convenience, while yet, in the development of a subject, they always require a very large amount of thought, and often embody some of the greatest achievements of analysis. (PoM, p. 63)

This case is therefore the opposite of the case of identity propositions. A definition also states an identity, but here the thing is given, while the denoting concept is missing: “An object may be present to the mind, without our knowing any concept of which the said object is the instance; and the discovery of such a concept is not a mere improvement of notation…. In the moment of discovery the definition is seen to be true, because the object to be defined was already in our thought.”

Russell thought that Frege’s theory was roughly the same as his. He equated Frege’s distinction between sense and reference with his own distinction between “a concept and what the concept denotes” (PoM, p. 502). The only difference which he mentioned is that Frege’s distinction was more sweeping, since he considered all proper names to have both sense and reference. Russell thought, rather, that “only such proper names as are derived from concepts by means of the can be said to have meaning [= sense], and that such words as John merely indicate without meaning” (ibid.).

Russell’s comparison of the two theories does not appear to be correct, however. The difference he mentions might appear a minor one in itself, but in fact it is symptomatic of a very radical incompatibility of the two views. In Russell’s theory, denoting phrases do not differ essentially from other expressions: they serve to indicate something. The difference lies in what they indicate. Russell recognizes two kinds of objects, which differ with respect to their behaviour as constituents of propositions. Nevertheless, there is only a single, undifferentiated notion of meaning of an expression, namely, the relation of indicating. Denoting phrases may be said to bear an additional relation to things, but that relation is derivative, being based upon the relation of denoting which obtains between denoting concepts and their denotation. Paradigmatically, words are used to speak about what they indicate. Denoting concepts behave abnormally in that they cause a shift of the subject-matter of propositions in which they occur. Frege’s theory is different. Every expression plays a double semantical role: it expresses its sense and refers to its reference. This is true of all expressions,

5 The italics do not mean, of course, that we are mentioning expressions. As in other cases, we should forget about expressions and speak about objects indicated by the expressions, in this case, about denoting concepts.

6 PoM, p. 63. Incidentally, it is interesting to notice that this statement already contains the rudiments of what later came to be known under the name of the Theory of Knowledge by Acquaintance.
including complex ones. It is true even of whole sentences.

It appears, therefore, that the partial translation of Fregean terms into Russell's language proposed by Peter Geach is more correct than Russell's own. According to Geach, the following translation is "more fruitful and less misleading":

\[ \text{[no counterpart]} \leftrightarrow \text{Sinn} \]
\[ \text{indication} \leftrightarrow \text{Bedeutung} \]
\[ \text{denotation} \leftrightarrow [\text{no counterpart}] \]

I have just commented on the lack of a counterpart in Russell's language for the notion of sense. The lack of a counterpart in Frege's language for the notion of denotation is also apparent. Russell admitted that Frege's notion of sense was more widely applied than the notion of denotation; however, he failed to point out that the opposite was also true. Frege did not need, and could have no sympathy for, such denoting concepts as a man, some man, every man, any man or all men. It was precisely his major contribution to logic—viz. the device of quantification—that rendered all of these superfluous. In Frege's theory, the phrase "a man" does not indicate, or refer to, any object; yet, a sentence in which it occurs is not meaningless. "I met a man" is paraphrased by "There is \( x \) such that I met \( x \) and \( x \) is human", a sentence in which the phrase "a man" does not occur at all. It is remarkable that quantification is not even mentioned in the *Principles*. In its stead, Russell seeks to employ the complicated machinery of denoting concepts.

Bearing in mind all that has been said about the differences between Frege's and Russell's approaches to language, we can now attempt to reconstruct the argument Russell puts forward in "On Denoting", pages 48–51. To facilitate discussion, I will adopt Blackburn and Code's method of referring to single paragraphs of "On Denoting". Thus, paragraph \( A \) begins on page 48 with the words "The relation of the meaning ..." and ends on the same page with the words "... must be wrong". The next paragraph is marked \( B \), and so on until we reach \( H \), which begins on page 50 with the words "... must be abandoned".

Even before we go into details, it should be clear that Russell's notion of a denoting concept as some special kind of object with extraordinary properties is dubious and that it does not agree very well with the rest of his views. Certainly the theory of descriptions was a major purification of his view, and he must have felt a great relief after inventing it—of which he indeed often wrote afterwards.

I will first summarize Russell's argument without following the text sentence by sentence. I will then comment on some particular passages.

Two versions of the theory of denoting are considered and rejected. In the first version, a denoting concept, or complex, as he now calls it, has both sense and denotation. Both versions, however, assume that "the relation of meaning and denotation is not merely linguistic through the phrase", but it is a logical relation, "which we express by saying that the meaning denotes the denotation" (C). That is to say, a proposition containing a denoting complex is not about the complex, but about the object it denotes. The second version, therefore, corresponds to the *Principles* theory, and it is this version that is relatively closer to Frege's theory, since the latter does not postulate four entities, viz. expression, denoting complex (a non-linguistic entity), sense and reference, as the first version would have it. It appears that Russell devised the first version merely to make the argument more exhaustive and to remove any suspicion that perhaps some possibility has been overlooked. However, the argument against both versions is basically the same, and it even turns out in its course (F) that the first version is just a special case of the second.

The difficulty that both these theories encounter will appear as soon as we ask how anything could be said about the sense of a denoting phrase. If we put the sense into our proposition, we will find out-

---

8 This has been noticed by P. Dau, "The Complex Matter of Denoting", *Analysis*, 45 (1985): 190–8.
10 Russell's translation of *Sinn*.
11 According to the second version of the theory this sense is just the denoting complex indicated by the denoting phrase. Perhaps it would be better to speak about a *denoting complex*, rather than about the sense, in order to stress that the argument is meant to apply to the *Principles* theory.
selves speaking about something else. The sense evades us, hiding, as it were, behind another object. What other proposition could we use, then, to speak about the sense? Its subject cannot be that sense which we want to speak about, let us call it sense 1, but another, sense 2, which denotes sense 1. If we want to speak about some sense we must not focus our attention directly on that sense, but on a different object of the same kind which would in turn have the extraordinary property of “hiding” behind the sense we want and “pushing” it forward for us to see and speak about. Everything would indeed be in order if only we could find such an object. How should we look for it? We know that it cannot contain sense 1 as its constituent, since then sense 1 would also be a constituent of any proposition we might form from such an object. Such a proposition would again say something about the denotation of sense 1, and not about the sense itself. We might yet hope that the object we sought was the value of some function applied to sense 1. But no such function can exist, argues Russell, for many senses correspond to one denotation. In our case there is no function leading from sense 1 to a new sense which denotes it. “There is no backward road from denotation to meanings [= senses]” (F). Therefore, there is no systematic correlation of senses, a correlation which would guarantee that, given sense 1, we can always find an appropriate sense 2. Perhaps, in a given case, we might, by sheer luck, hit on the appropriate sense (especially since there are many of them!). But in a situation in which we lack a reliable method of forming propositions about senses, we cannot regard the very notion of sense as theoretically satisfactory. We would do better if we managed to dispense with it altogether. The theory of descriptions which Russell put forward in “On Denoting” enables us to do exactly that.

There are at least two objections to this argument. First of all, it might be argued, it cannot be correct, because otherwise we could prove in the same way that we cannot speak about ordinary objects—a conclusion which is surely false. The proof would run: if there is no backward road from sense 1 to sense 2, then, likewise, there is none from, say, Mont Blanc, to a sense which would denote it. We are given an object; we want to speak about it: how do we find a suitable sense?

Russell would apparently think that this is indeed a highly non-trivial task. We may be acquainted with an object (it is “given” to us), but we may be at a loss how to find a sense which would denote it uniquely (this is exactly the situation that Russell spoke about in connection with the importance of definitions—see page 56 above). But the lack of an appropriate sense would not prevent us from speaking about the object. If, as we have assumed, we were acquainted with the object, we could always give it a name and use that in order to speak about the object. This possibility is not open to us in the case of a sense. Even if we could give it a name, we could not use the name to speak about the sense, because using a name means that the bearer of the name becomes a constituent of the proposition. Unlike in the case of an ordinary object, such a proposition would not be about the sense because of the “shift of subject-matter” characteristic of senses. Therefore, we really need to have a new sense, whereas what we have got is rather a direct acquaintance, which “gives” the sense but does not provide a higher-level sense which would denote it. We can only use the sense, put it into a proposition when we want to speak about its denotation, but we really know nothing which would characterize it uniquely; save, perhaps, that it is the sense of such-and-such an expression.

This reply, however, immediately suggests a second objection to Russell’s argument. One might think that the sense of “the sense of ...” is exactly what we need in order to resolve our problem. That would be a “roundabout way from denotation to sense” instead of a “backward road”. It could be replied that there is still no guarantee that this roundabout road will take us to the right place. Perhaps this is a good objection, but all the same it is not a point which Russell makes. In fact, Russell does not have any argument against the present suggestion, and the apparently astonishing fact that he does not seem concerned about this gap in his argumentation can be explained by appeal to his general view of language.

The study of words is confined to linguistics, and logic should have nothing to do with that. It is inconceivable that a word should be an indispensable constituent of a proposition about something entirely...
extra-linguistic, and belonging to logic. Sense must not be anything linguistic, identifiable only by its correlation with a word or phrase. It is an objective constituent of propositions, just as the concept white or the relation higher than or Mont Blanc are. From Russell's viewpoint, Frege's proposal that a sense is nothing but the sense of an expression is tantamount to considering it as a complex, of which a linguistic phrase is a constituent. It would not be objectionable if a sense could be spoken about only by means of some composite sense; but it is objectionable if that composite sense must contain a linguistic expression as one of its constituents.

I am not claiming that this is a valid argument against Frege's theory, although the question of language independence and objectivity of the Fregean realm of sense might be taken seriously. But the argument is fatal to the theory in the Principles precisely because the only way out has been blocked by a virtual ban on mentioning linguistic expressions.

Evidence for the above interpretation of Russell's argument comes from two sources: one internal, i.e. the text itself, and the other external. To take up the external first: this interpretation, unlike, for instance, Searle's, Butler's or Jager's, avoids attributing to Russell excessively simple mistakes. If it is correct, Russell was guilty only of mistaking his own former theory for Frege's theory. But applied to the right theory, the argument turns out to be perfectly sound and to be free from any simple logical blunders.

A few authors have made valiant attempts to show that indeed Russell's argument knocks down the real theory of sense and reference. I do not think that any single logical argument can prove such a coherent and well developed structure as Frege's theory to be simply wrong. Its acceptance or rejection must depend rather on more fundamental assumptions and differences in the general approach to language and logic. I tried to show what these assumptions were in Russell's case. The views expressed in the Principles, however, are eclectic and incoherent. That is why Russell was able to prove them wrong in "On Denoting": assuming the main tenets, he showed that a certain detail was incompatible with them. There is much more continuity in Russell's philosophical development than is usually allowed. On the other hand, there is much less similarity between his views and those of Frege.

All this does not demonstrate that Blackburn and Code's or Hochberg's papers do not contain a refutation of Frege's views, which, moreover, would be a reconstruction of Russell's old argument. Lack of space, however, does not permit me to attempt any such demonstration.

I will now deal with the internal evidence, by way of commenting on some crucial passages in "On Denoting", B–H. First, a justification of the claim about inverted commas. The claim is that Russell did not use them to mention expressions. Some evidence can be found in B, where the problem to be discussed is formulated and terminological conventions are established. Russell states that inverted commas will be used to speak about the denoting complex (not about a linguistic expression!):

The centre of mass of the solar system is a point, not a denoting complex; "The centre of mass of the solar system" is a denoting complex, not a point. Or again, The first line of Gray's Elegy states a proposition. "The first line of Gray's Elegy" does not state a proposition.

Later in the same paragraph he writes: "We wish to consider the relation between C and 'C'." Of course, what he wishes to consider is the problematic relation between a sense and its denotation, and not between the denotation and a linguistic expression.

The last quotation raises a further point: Russell uses the letter "C" as short for any denoting phrase, and not as a metalinguistic variable running over expressions. As a result, when he uses the letter "C", he does not speak about a phrase, but rather uses that phrase (for which "C" is short). Hence he writes: "if we speak of the 'meaning of C' that gives us the meaning (if any) of the denotation" (D), or again, "suppose C is our complex" (F). Wherever "C" occurs we may put in

---

its place any description without inverted commas.

The technical use of inverted commas in "On Denoting" is well illustrated by the following passage in D:

... let "C" be "the denoting complex occurring in the second of the above instances". Then

\[ C = " \text{the first line of Gray's Elegy} \]

and the denotation of \( C \) is The curfew tolls the knell of parting day. But what we meant to have as the denotation was "the first line of Gray's Elegy". Thus we have failed to get what we wanted.

When "C" occurs in this passage for the first time, it is put in inverted commas, because we want to speak about the denoting complex, not about its denotation. The very next use of inverted commas is to be explained in the same way, because we again have a denoting complex. Thus, our exemplary denoting complex is "the denoting complex occurring in the second of the above instances". We have the following chain of denotations:

"the denoting complex occurring in the second of the above instances" \( \leftrightarrow \) "the first line of Gray's Elegy" \( \leftrightarrow \) The curfew tolls the knell of parting day.

It is significant that Russell does not put the last item above in inverted commas, because we want to speak about the denoting complex, not about its denotation. The very next use of inverted commas is to be explained in the same way, because we again have a denoting complex. Thus, our exemplary denoting complex is "the denoting complex occurring in the second of the above instances". What it denotes is again a denoting complex, namely, "the first line of Gray's Elegy". We have the following chain of denotations:

"the denoting complex occurring in the second of the above instances" \( \leftrightarrow \) "the first line of Gray's Elegy" \( \leftrightarrow \) The curfew tolls the knell of parting day.

It seems that this argument is perfectly sound within the Russellian framework. What is asserted is a proposition. It consists of several objects put together. If the proposition Scott is the author of Waverley, in which the denoting concept the author of Waverley occurs as a part, is merely about the denotation, and not all about the sense (only a completely different proposition is about that), then to know this proposition must be the same as to know that Scott is Scott. The reason for this is that a proposition, i.e. a complex indication of a string of words forming a sentence, cannot change when the sentence is merely embedded in some wider context. Russell's framework rules out that oblique contexts should induce expressions to assume indirect sense and indirect reference. Russell's theory is, to use Davidson's phrase, "semantically innocent". The very same argument from \( H \), on the other hand, must seem to be based on a gross mistake when applied to Frege's theory. "Scott is the author of Waverley" standing
alone is simply about Scott (assuming, as Frege does, that a sentence is about the references of its component words). But in “George IV wished to know whether Scott was the author of Waverley” the last phrase does not refer to Scott.

I conclude that Church correctly observed that in the argument against Frege contained in “On Denoting”, Russell applies quotation marks to distinguish the sense of an expression from its denotation, but leaves himself without any notation for the expression itself; upon introduction of, say, a second kind of quotation mark to signalize names of expressions, Russell’s objections to Frege completely vanish.¹⁹

I have tried to show, however, that the use of any such device to mention expressions would be inconsistent with Russell’s general view of language.