

RUSSELL AND G. H. HARDY: A STUDY OF THEIR RELATIONSHIP

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I. INTRODUCTION

From time to time the name of Hardy turns up in Russell's career: a common interest in set theory and the philosophy of mathematics, similar political and religious sentiments, and certain matters of mutual concern arising at Trinity College Cambridge and in the university in general. However, there is no connected account of their contacts. The purpose of this article is to fill a gap in the literature on both men.

After enduring private instruction at home, Bertrand Russell gained a minor scholarship in mathematics at Trinity College in 1890; he took Part I of the Mathematical Tripos until 1893 and then Part II of the Moral Sciences Tripos in the following year. In 1895 he gained a Prize Fellowship, which he held until 1901; in 1910 he was awarded a college lectureship which he lost six years later under circumstances noted in section IV below. Much of the intervening time was spent in Oxford or London; however, he spent part of each year in Cambridge.¹ The later years are briefly recorded in section IV.

Born into an intellectual family of modest means in 1877 (five years younger than Russell, therefore), Godfrey Harold Hardy won a scholarship to Winchester College and then went to Trinity College in

¹ Standard biographical information on Russell is easily available; see, for example, R. W. Clark, *The Life of Bertrand Russell*; specific phases and events are dealt with in succeeding footnotes. *The Autobiography of Bertrand Russell* is of lower quality than one might imagine; for example, Hardy is not mentioned at all!

1896, just after Russell had gained his fellowship. He was elected to the Apostles (the "Society") in 1898, the year after Russell "took wings" after five years' active membership; he seems to have been less active than Russell as a speaker during his own three years there.² At some stage he also joined the Cambridge University Moral Sciences Club, which Russell had joined in 1891; a surprising contribution will be described in section iv.³

Graduating as fourth wrangler in Part I of the Mathematical Tripos in 1898 and gaining a high place in Part II in 1900, Hardy received at once a Prize Fellowship (like Russell). Upon its termination six years later, he obtained a college lectureship, which he held until 1919. He was elected a Fellow of the Royal Society of London in 1910 (two years after Russell),⁴ and in the following year he began his life-long collaboration with his younger colleague J. E. Littlewood (1885–1977), also a Trinity College graduate and then Fellow, and also a friend of Russell.⁵ The next stages of Hardy's career will be described in section iv: his personality is assessed in a concluding section v.

² On Russell's and Hardy's roles in the Apostles, see *passim* in P. Levy, *Moore: G. E. Moore and the Cambridge Apostles* (London: Weidenfeld and Nicolson, 1979).

³ The chief obituary of Hardy is by E. C. Titchmarsh, published in slightly different forms in *Obituary Notices of the Royal Society of London*, 6 (1949): 447–61, and *Journal of the London Mathematical Society*, 25 (1950): 81–101 (followed by more technical surveys of his mathematics by Titchmarsh and eight other authors on pp. 102–38). The first version is reprinted with a few changes in *Hardy's Collected Papers*, 7 vols. (Oxford: Clarendon P., 1966–79), 1: 1–12. No major study has been written on him since.

⁴ Hardy was nominated for a Royal Society fellowship in 1907 by A. R. Forsyth, seconded by J. W. L. Glaisher, and supported "on personal knowledge" by W. Burnside, E. W. Hobson, P. A. MacMahon, A. N. Whitehead, A. E. H. Love, E. B. Elliott and T. J. d'A. Bromwich; the proposal was "suspended" (put up for voting) in 1908, 1909 and 1910 (Royal Society Certificates, 12: fol. 311). Although Russell was a Fellow by 1910 (see my "Russell's Election to the Royal Society", *Russell*, no. 17 [spring 1975]: 23–6), he did not exercise his right to add his name to the suspended certificate; but this will not reflect any distancing from Hardy. See also the next footnote.

⁵ Littlewood gives some impressions of Hardy and Russell in his *A Mathematician's Miscellany*, 2nd ed., ed. B. Bollobas (Cambridge: U. P., 1986), esp. pp. 118–20, 128–31. Both men supported Littlewood's candidature for a Royal Society fellowship, "on personal knowledge", when it was proposed in 1913 by Hobson and seconded by Forsyth. The other supporters were Glaisher, E. W. Barnes, H. F. Baker and F. W. Richmond, and election was gained in 1916 after suspension in 1914 and 1916 (Royal Society Certificates, 13: fol. 104).

Russell and Hardy belonged to some of the same Trinity societies, such as the First Trinity Boat Club and the Magpie and Stump and the Decemviri debating societies, although not at the same time. However, around 1910 and 1911 they were both members of the Sunday Essay Society (a theological discussion group), and Russell spoke there on "The Nature of Truth" on 6 November 1910; no attendance list was kept, so it is not known if Hardy was present.⁶

Hardy left few manuscripts after his death in 1947;⁷ indeed he was well known for returning letters to their senders along with his replies. But this means that elements of both sides of his correspondence with Russell survive in the Russell Archives. They form a basis of the next two sections, which are devoted to their common interests in mathematics and set theory.

II. TWO CONVERTS TO CONTINENTAL MATHEMATICS

A common feature of their work is that both men learnt little of interest from Cambridge mathematics and gained their inspiration from Continental sources.⁸ This gave them a common desire for the reform of the Tripos system. Russell was not in fact very active in this movement; Hardy managed over many years to get some changes made.⁹

Russell's retraining began in the philosophy of geometry, and then especially in the set theory of G. Cantor and the mathematical logic of G. Peano; he came to Cantor by a strange route, first learning of set theory from a French philosophical work in 1896. Another significant influence was to be the *Universal Algebra* of 1898 by (fellow Apostle)

⁶ A version of this talk probably constitutes Chap. vii of Russell's *Philosophical Essays* (London: Longmans, Green, 1910). For the information given in this paragraph, I thank Miss Diana Chardin of Trinity College Library.

⁷ Some mathematical notebooks and papers of Hardy are kept at Trinity College Library (Add. ms. b. 55–6); a few small collections, not relevant to this paper, are held elsewhere in Cambridge and in Oxford.

⁸ On Russell's mathematical background see N. Griffin and A. C. Lewis, "Bertrand Russell's Mathematical Education", *Notes and Records of the Royal Society*, 44 (1990): 51–71; and Griffin, *Russell's Idealist Apprenticeship* (Oxford: Clarendon P., 1991), esp. Chaps. 4 and 6.

⁹ See especially Hardy's presentation of "The Case against the Mathematical Tripos" in 1926, in *Collected Papers*, 7: 527–37; p. 531 contains an example of Cambridge ignorance provided by Russell.

A. N. Whitehead, after he had taught Russell as a student and before he was to collaborate in Russell's logicist programme.¹⁰

Hardy was somewhat luckier, since after grinding through the Tripos he was advised by A. E. H. Love, a real mathematician, who guided him towards C. Jordan's *Traité d'analyse*:

I shall never forget the astonishment with which I read that remarkable work, the first inspiration for so many mathematicians of my generation, and learnt for the first time as I read it what mathematics really meant.¹¹

Although Hardy is not specific, he probably read the second edition (1893–96) of Jordan; and an important aspect of its impact would have been the elegant and detailed treatment of Cantorian point-set theory in the opening pages of the first volume (in the first edition of 1882–87 this topic was consigned to the end of the third volume).¹² This work, and presumably the other writings to which he was led,¹³ placed his interests firmly in mathematical analysis of real and complex variables, including point-set theory and applications to integration, Fourier series and related topics; later on, applications to number theory were adjoined. In those days the word “class” was usually preferred to “set”, and among English writers set theory took names such as “theory of aggregates”.

Hardy attended the lecture course on “applications of symbolic logic” in the academic year 1902–03 given by Whitehead,¹⁴ and the places of intersection between logic and set theory gave him many points of contact with Russell. Thus, although he never worked with Russell and Whitehead's mathematical logic, he knew its main features and was sensitive to many of the philosophical questions attending

¹⁰ Russell's early research career in logic and mathematics is excellently traced in his *Papers 2*.

¹¹ Hardy, *A Mathematician's Apology* (Cambridge: at the U. P., 1940), p. 147 (citing the 1969 reprint with a new introduction by C. P. Snow).

¹² This conjecture is corroborated by Hardy's comments on the two editions of Jordan's treatise in the opening of his obituary of Jordan (*Collected Papers*, 7: 721–4).

¹³ Later Hardy praised even above Jordan the work of J. Liouville on integration; see the preface to his tract *The Integration of Functions of a Single Variable* (Cambridge: at the U. P., 1905).

¹⁴ Hardy, *Collected Papers*, 7: 434.

Russell's logicist programme. This is evident in his review of Russell's *The Principles of Mathematics* of 1903. He gave rather more attention to the range and survey of mathematics that Russell presented than to its philosophical and logical basis, and wrote rather critically of the treatment of dynamics; but, for example, he was also glad to learn of the work of G. Frege, “of whom we must confess we had never heard”, and he praised Russell's analysis of the Zeno paradoxes.¹⁵ However, he does not seem fully to have grasped the import of the reductionistic position which Russell and G. E. Moore had adopted in 1899 and which was (supposed to) underlie many of the ontological concerns of the book.

Hardy wrote five papers in or around set theory and transfinite arithmetic between 1904 and 1910.¹⁶ He had three main concerns: properties of transfinite ordinals, especially in view of Burali-Forti's contradiction (as he took it over from Russell); the use of the axioms of choice, of which E. Zermelo and E. Schmidt had published the first explicit example in 1904; and the cardinalities of particular kinds of sets of points. The first two of these aspects brought him into closest contact with Russell: in particular, he knew that Russell had found an axiom of choice for himself in 1904, calling it “the multiplicative axiom” in connection with the context of defining infinite products. (In one way or another, the axioms asserted that if one member were chosen from each of an infinitude of classes, each choice made independently of the others, then the ensemble obtained was a class on a par with the originals.) During these years both men were together sometimes in Cambridge, but Russell did much of his logic and philosophical work from his home at Oxford, so that (luckily for us) letters were exchanged.

III. LETTERS BETWEEN HARDY AND RUSSELL

It seems that each man was active in the initiation of letters. This makes a contrast with a frequent mutual correspondent, their slightly

¹⁵ Hardy's review was published anonymously in *The Times Literary Supplement*, 18 Sept. 1903, p. 263; also in *Collected Papers*, 7: 851–4. The Zeno paradoxes also begin his next review in that journal, of the Heath edition of Euclid (*ibid.*, p. 855).

¹⁶ They are conveniently published together in Hardy, *Collected Papers*, 7: 421–51.

junior colleague P. E. B. Jourdain (1879–1919); he had also graduated from Trinity College (in 1902), having taken Russell's lecture course on mathematical logic in the academic year 1901–02. Jourdain wrote to them both frequently throughout the 1900s on these matters, especially on the possibility of proving the axioms of choice, and they replied. As he knew himself, his work was not of the calibre of that of his seniors, but he raised many interesting questions with them;¹⁷ he also alerted them to historical matters, on which he was a considerable expert.¹⁸

On ordinals, one of Hardy's first contributions was to prove in 1904 that the cardinality of the continuum was greater than or equal to that of Cantor's second number-class of transfinite ordinals. His proof was more complicated than Cantor's own diagonal argument of 1891, and so was not significant; but its deployment of the ordinals led him to detailed analyses of the series, and Russell sent him long accounts of his own methods of constructing ordinals.¹⁹

Cantor's argument had provided a means of defining the exponentials of cardinal numbers (a^b , where one or both of these cardinals were transfinite), and this technique and the Burali-Forti contradiction were two of the matters being analyzed anew as possible consequences of the discovery of the axioms of choice. On 30 June 1905 Hardy wrote a string of nine rather puzzled questions to Russell in this territory, and received a magnificent answer, in which among other things Russell showed how he had found his own paradox by applying a variant of Cantor's argument on the class of "all" classes.²⁰ He also

¹⁷ Jourdain's correspondence (with many figures) is kept at the Institut Mittag-Leffler, Stockholm. His exchanges with Russell form the basis of my *Dear Russell—Dear Jourdain: a Commentary on Russell's Logic, Based on His Correspondence with Philip Jourdain* (London: Duckworth; New York: Columbia U. P., 1977). Those with Hardy, which remain unpublished, deal with current researches in these areas and with the possibility of Jourdain publishing his work.

¹⁸ Jourdain's principal writings on the history of mathematical logic and set theory are reprinted in *Selected Essays on the History of Set Theory and Logics (1906–1918)*, ed. I. Grattan-Guinness (Bologna: CLUEB, 1991), including an editorial introduction and indexes.

¹⁹ Hardy in *Collected Papers*, 7: 427–34; Russell, letter of 29 January 1909 and an undated but I think associated supplement beginning "The problem I was".

²⁰ This letter and some others between Russell and Hardy are to appear in A. Garciadiego, *Bertrand Russell and the Origins of the Set-Theoretic "Paradoxes"* (Basel:

distinguished problems concerned with the paradoxes from those of the axioms.

In response to another question, Russell affirmed his preference for definition by intension:

Df. by extension. Logically, there is no such thing. The class whose members are a and b is defined by the *intension* "identical with a or identical with b "; and what one commonly calls definition by extension is really definition by intensions of this type....

In fact, Hardy's question was very acute, for the epistemological balance between intensions and extensions was to be one of the mysteries of existence which Russell left to his successors in logic, especially after the foray into full-blown extensionalism in the second edition of *Principia Mathematica* in the 1920s.

Another matter raised by Hardy in this letter concerned a recent proposal by their Cambridge colleague E. W. Hobson (1856–1933) to solve the paradoxes and handle the axioms of choice. His idea was that classes, and relationships between them, should be admissible only if expressed by means of a "norm", which was a finite collection of "specified conditions". He pointed to various examples where norms had not been obtained, but some of them did not involve paradox-generating mathematics; further, he also failed to block paradoxes of finite definability, which had recently been brought to light by D. König and others. Both men replied in print to Hobson at the time,²¹ and discussed him in letters. Hardy opined to Russell that

Birkhäuser, forthcoming). It formed the basis of my paper on "How Bertrand Russell Discovered his Paradox", *Historia Mathematica*, 5 (1978): 127–37. In another question Hardy asked about some aspects of the work of Cantor's contemporary P. du Bois Reymond, upon which Russell had touched in the *Principles*; later he popularized in Britain du Bois Reymond's theory of the ways in which functions $f(x)$ can increase in value with x , in the tract *Orders of Infinity: the "Infinitärkalkül" of Paul du Bois Reymond* (Cambridge: at the U. P., 1910); see also his two-part paper of 1913 in *Collected Papers*, 4: 280–341.

²¹ E. W. Hobson, "On the General Theory of Transfinite Numbers and Order Types", *Proceedings of the London Mathematical Society*, (2), 3 (1905): 170–88; Hardy, "The Continuum and the Second-Number-Class", *ibid.*, (2), 4 (1906–07): 10–17 (also in *Collected Papers*, 7: 438–45); Russell, "On Some Difficulties in the Theory of Transfinite Numbers and Order Types", *ibid.*, 29–53 (reprinted in *Essays in Analysis*,

Hobson “hasn’t really got hold of the point” when raising a particular use of an axiom of choice, and Russell agreed about him being “confused”.

In a further letter on the point, of 22 June, Russell gave perhaps his first interpretation of the axioms in terms of boot ownership:

I can’t prove that if you have an infinite number of pairs of boots, you must have an even number of boots, unless (as is likely) your right boots can be distinguished from your left boots, in which case the number of right boots = the number of left boots. But if you are faddy, and have no difference, there is no way of selecting one out of each pair (no *norm* in Hobson’s phrase); thus if k is the class of pairs, X^k [the multiplicative class of k] may be null, and you can’t prove that the number of boots is even.²²

The axioms of choice raised great controversy at the time, both in their philosophical (in)admissibility, their epistemological status as axioms, and the places for their mathematical need. Hardy’s position was probably influenced by his own proof of the theorem on the cardinality of the continuum which was written shortly before Zermelo’s paper was published, for infinite selections were used in his constructions. In a fresh discussion of cardinal exponentiation on 5 July he told Russell that

I should not like to say that anything so abstract and general as Zermelo’s axiom strikes me as obvious, exactly; but I will go as far as saying that the more one thinks of it the more paradoxical the contrary seems, so that unless it appeared to lead to contradictions I should (in default of proof) be disposed to assume it and hope for the best.²³

Russell took a similar view, except that for him as a logicist, the situation was exacerbated by the difficulty of expressing an axiom which allowed an infinitude of independent selections of members from sets within the *finitary* language to which his logic was restricted.²⁴

ed. D. Lackey [London: Allen & Unwin, 1973], pp. 135–64).

²² Russell gave this illustration to Jourdain soon afterwards; see my *Dear Russell—Dear Jourdain*, pp. 55, 64.

²³ This letter is not dated as to year, but I think that it belongs to 1905. Hardy expressed this view in similar terms in his reply to Hobson (*Collected Papers*, 7: 445), and ignored the notion of norms in his 1934 obituary of Hobson (*ibid.*, 7: 749–61).

²⁴ On this neglected aspect of Russell’s logicism, see my *Dear Russell—Dear Jour-*

Hardy seems to have kept abreast of the main lines of Russell’s mathematico-logical systems. In particular, on 15 December 1905 Russell outlined in detail the main features of his substitutional theory, which he then thought would be the definitive version; at the end of his letter he offered to give more details and Hardy seems to have responded positively, for a week later Russell wrote out a fifteen-page manuscript called “On Substitution”, which outlined the main philosophical techniques of the theory, including an early instance of the contextual definition of definite descriptions (the central device of the theory).²⁵

When Russell and Whitehead published the first volume of *Principia Mathematica* in 1910, Hardy was one of the reviewers. As in his earlier review, he expressed sympathy with the project, while also isolating the cleft into which it would fall: “It is a strange and discouraging fact that mathematicians as a class are utterly impatient of enquiries into the foundations of the subject” but also the hope (against hope?) that “the time has passed when a philosopher can afford to be ignorant of mathematics.” He praised the theory of definite descriptions but in a surprising way, by asserting that “mathematics, one may say, is the science of propositional functions”; in fact it is the apparatus of descriptions that allows mathematics to be expressed within this logic. On type theory he picked a good point; that, in line with the finitariness mentioned above, only a finite number of types were permitted.²⁶ If he read the succeeding volumes when they

dain, pp. 68–9; and for a little-known paper of 1911 in which Russell squirmed around the matter, see pp. 161–74. On the roles of Russell and Hardy in the debate over the axioms, see J. Cassinet and M. Guillemot, “L’Axiome du choix dans les mathématiques de Cauchy (1821) à Gödel (1940)”, 2 vols. (U. of Toulouse double *docteur d’état des sciences*, 1983), esp. 1: Chap. 2; G. H. Moore, *Zermelo’s Axiom of Choice ...* (New York: Springer, 1982), Chaps. 1 and 2; and F. A. Medvedeff, *Rannyya istoriya aksiomi vybora* (Moscow: Nauka, 1982), Chap. 5.

²⁵ This manuscript was originally found in the Hardy file of letters. On the substitutional theory, see my “The Russell Archives: Some New Light on Russell’s Logicism”, *Annals of Science*, 31 (1974): 387–406; and G. Landini, “New Evidence concerning Russell’s Substitutional Theory of Classes”, *Russell*, n.s. 8 (1989): 26–42.

²⁶ Hardy’s review was published anonymously in the *Times Literary Supplement*, 7 Sept. 1911, pp. 321–2; also in *Collected Papers*, 7: 859–62. The identity of the reviewer, and also of its predecessor cited in n. 15, is established on p. 862 of the reprint. For reviews of other books of a philosophical character, see pp. 832–8, 864–6.

appeared, then he would have found that the series of transfinite ordinals could not be laid out, thus causing a serious limitation to the logicist construction of mathematics.

IV. LATER RELATIONS

Hardy became an important figure in the London Mathematical Society, and held the offices of Secretary or President for many years after 1917. By contrast, after publishing *Principia Mathematica* Russell largely abandoned work in logic and mathematics, so that his contacts with Hardy decreased. However, there was a revival in 1916 when Russell's pacifism led the Council of Trinity College to expel him from his lectureship. Hardy was one of those who defended Russell and helped in his reinstatement in 1919;²⁷ during the Second World War, when a serious illness in 1939 had impeded his research powers but with that issue still alive in his soul, he wrote a short book on the affair.²⁸

In 1917 Hardy published a paper on an unusual topic for him: Russell's religious position. This was the text of a talk that he had given at the Cambridge University Moral Sciences Club in 1915, although (according to a note to his title) it had been written before

²⁷ For an excellent survey of this affair, drawing on Russell-Hardy letters of the time among other sources, see P. Delany, "Russell's Dismissal from Trinity: a Study in High Table Politics", *Russell*, n.s. 6 (1986): 39-61. He makes clear that Hardy's position over the matter contrasted with the vacillations of Whitehead (see his "To the Master and Fellows of Trinity College, Cambridge", *ibid.*, pp. 62-70), who never seems to have had any particular relationship with Hardy. In a letter to Hardy of 25 September 1916, Russell found that the position of Whitehead and some other Fellows "does not seem to me any use"; he also felt "it unlikely that I can ever again endure the stuffiness of a high table, even if it could endure me, with all the cold draughts that I should let in", and that "There is no hope of any freedom of thought in the British empire if it is to be banished from Trinity. We may as well shut up shop and get ourselves expunged from the list of civilised nations". In the end, due to personal circumstances Russell did not take up his restored appointment. A propos of n. 1, this letter is marked by Russell as to be "Rej.[ected]" from his planned autobiography.

²⁸ Hardy, *Bertrand Russell and Trinity: a College Controversy of the Last War* (Cambridge U. P., 1942); reprinted 1970 (with an introduction by C. D. Broad) and also text only (New York: Arno Press, 1977). Russell was in the USA in the early 1940s, and he was not involved in its preparation; he only pointed out an error in the manuscript, which Hardy dealt with in a postscript on p. 61.

the War.²⁹ This was part of his strategy in the affair with Trinity College, for he sided with Russell's atheism (a standpoint which they shared) against the Christianity (and, by implication, the pro-War stance) of (fellow Apostle) J. M. E. McTaggart. As a contribution to the philosophy of religion, the essay is exceedingly modest; but for our concerns it is striking that Hardy wrote and published it at all.

One of the consequences of the Russell affair for Hardy was a disenchantment with Trinity College; and at the end of 1919 an opportunity arose for a move to a chair at Oxford. Another candidate was W. H. Young, Hardy's senior by fourteen years and equally distinguished in mathematical analysis, who had passed most of his career on the Continent. The two men corresponded on the matter, Hardy offering that "if there were a question between us, my candidature should not be pressed." But soon afterwards he changed his mind, being of course "sorry to have to go back on my word but I must do so" and pleading the apparent Cambridge tendency "to pile up work on the staff to a point which may rapidly become intolerable". Outsider Young, knowing that connections score most points in academic life, withdrew with dignity.³⁰

From this time onwards the contacts between Russell and Hardy were pretty occasional and routine. Some of them in 1918 and 1919, for example, arose in connection with third parties, such as Jourdain's continuing (and by now tiresome) concern with proving the axioms of choice, or Dorothy Wrinch's interest in questions surrounding *Principia Mathematica*.³¹ In July 1919 Russell accepted Hardy's request

²⁹ Hardy, "Mr. Russell as a Religious Teacher", *The Cambridge Magazine*, 6 (1917): 624-6, 650-3; also in *Russell*, n.s. 1 (1981): 119-35. See J. Pitt, "Russell and the Cambridge Moral Sciences Club [*sic*]", *ibid.*, pp. 103-18.

³⁰ See my "A Mathematical Union: William Henry and Grace Chisholm Young", *Annals of Science*, 29 (1972): 105-86 (at 161-2). Young's wife, also a mathematician, had known Russell through family connections when they were in their youth. According to the Young's children, they held a low opinion of *Principia Mathematica*, annotating their copy with liberal doses of "rot!" and "tommy rot!" I have not been able to find their copy, but as it was disposed of when they lived in Lausanne, the story has good credibility.

³¹ On Wrinch see P. Abir-Am, "Synergy or Clash: Disciplinary and Marital Strategies in the Career of the Mathematical Biologist Dorothy Wrinch", in Abir-Am and D. Outram, eds., *Intimate Lives* (New Brunswick and London: Rutgers U. P., 1985), pp. 239-80 and 342-54 (n. 16 and text).

to referee a paper submitted to the London Mathematical Society by Norbert Wiener;³² conversely, when in 1924 Russell was helping to launch the British Institute of Philosophy, he sought Hardy's support.

Hardy's continuing sympathy with Russell's philosophy was evident when in 1929 he chose to speak at Cambridge on "Mathematical Proof"; for he included several passages on *Principia Mathematica*, preferring this philosophy of mathematics to its competitors of the time. In 1931 he returned for good to that city when he succeeded Hobson in the Sadleirian Chair of Pure Mathematics; he retired in 1942. By then Russell was in his popular books phase, and when he tried to return to academic life in 1935 it was within the philosophical rather than the mathematical ambit.

However, Hardy's book of 1942 on the affair over Russell's lectureship, mentioned in section II, seems to have had some consequences for Russell's academic life; for it probably played a role in his appointment to a non-residential fellowship at Trinity College in 1944. He spent parts of that and the following three years at the college, completing his *History of Western Philosophy* (1945) and writing much of *Human Knowledge, Its Scope and Limits* (1948). Presumably he saw Hardy during this time until Hardy's death in December 1947.

The relationship between these two men can be assessed from various points of view. Like Russell, Hardy held a left-wing position, which manifested itself to the extent of displaying a photograph of Lenin in his study and serving between 1924 and 1926 as President of the Association of Scientific Workers; however, unlike Russell he never stood for Parliament. As with Russell, he hated the First World War, but he did not participate in the kind of activity which led to Russell's dismissal.³³ On the other hand, he was active in the 1920s in encourag-

³² Russell's report was published in my "Wiener on the Logics of Russell and Schröder. An Account of His Doctoral Thesis, and of His Subsequent Discussion of It with Russell", *Annals of Science*, 32 (1975): 103-32 (at 104), and again in Wiener, *Collected Works*, ed. P. Masani (Cambridge, Mass.: MIT P., 1975), 1: 86, and in *Papers* 9: 470. On Wiener's relations with Russell and with Hardy see *passim* in P. Masani, *Norbert Wiener* (Basel: Birkhäuser, 1990). On 31 October 1921 Russell also recommended for Hardy a paper by the American logician H. M. Sheffer, which was never published. It may well have been Sheffer's essay "The General Theory of Notational Relativity" of that year, of which a copy is held in the Russell Archives.

³³ Hardy unwillingly volunteered for the First War (*Bertrand Russell and Trinity*, p.

ing the reconciliation of the international mathematical community.³⁴

But the most important aspect is their common intellectual interests, since the relationship was basically grounded there. The similarities and differences are well characterized by the general recognition given to Cantor's set theory and transfinite arithmetic by mathematicians and (some) philosophers in the 1890s, when they both became deeply involved in it. Many workers, such as Hardy, took a principal interest in the mathematical aspects, while sympathetic to the philosophical issues; others, such as Russell, concentrated on the general and philosophical sides, while also working on parts of the technical side. The different areas of main concern of these two sides separated Russell and Hardy in the main thrusts of their research; however, the common regions brought them together to a notable extent. It is a pity that the relationship has been ignored (including in Russell's *Autobiography*), for it involves an unusual combination of concerns.

V. RUSSELL ON HARDY'S CHARACTER

Hardy made a great impression on his colleagues and contemporaries; but the nature of the image is not clear, for he was shy and withdrawn from all but his *intimes*. One contrast with Russell would appear to be in sexual orientation; for while Russell pursued the conventional mode with some enthusiasm, Hardy never married. Some assertions have been made of his homosexuality, and even that his support of the Indian mathematician S. Ramanujan in the 1910s carried overtones of passion.³⁵

A strongly qualified version of such views is perhaps tenable, asserting merely suppressed or even latent modes of homosexuality. But a different interpretation can be formed from a letter which Hardy sent

4). In 1940 he expressed his pacifism in a passage of an essay where he also referred to Russell (*Collected Papers*, 7: 634; again in *A Mathematician's Apology*, p. 143).

³⁴ See J. W. Dauben, "Mathematics and World War I: the International Diplomacy of G. H. Hardy and Gösta Mittag-Leffler as Reflected in Their Personal Correspondence", *Historia Mathematica*, 7 (1980): 261-88.

³⁵ I have in mind in particular A. Nandy, "Ramanujan in England: an East-West Collaboration in Science", *Psychoanalytic Review*, 66 (1979): 423-42. We find before-breakfast analyses such as Hardy "manifested his femininity [in] his intense, uncompromising pacifism" (p. 433: therefore Russell was ... ?).

to Russell on 30 November 1919. The issue of the Oxford chair was still unsettled, and, more importantly,

I wish you could find some tactful way of stirring up Littlewood to do a little writing. Heavens knows I am conscious of my huge debt to him. But the situation which is gradually stereotyping itself is very trying for me. It is that, in our collaboration, he will contribute ideas and ideas *only*: and that *all* the tedious part of the work has to be done by me. If I don't, it simply isn't done, and nothing would ever get published. It is of course true that he has not been fit; and he has had a course of lectures demanding much thought and preparation. But I'm sure he's fit enough now: and that the work he does is nothing to what I do. He has 2 lectures, I 3. I have also 2 secretary-ships.³⁶ And yet he says he can do *no* research, and I do it all. Really its [*sic*] got very badly on my nerves. It is the fear that, unless I can get some more leisure somehow, I shall work myself to pieces, that has driven me, more than anything else, into standing for the vacant Oxford professorship (which I probably shant [*sic*] get, as it is a chair of "Geometry").

The real truth, I think, is that L. has acquired an *idée fixe* that he will break down if he does anything which he finds at all a bore. And writing papers for press is, after a certain point, a hell of a bore.

At the moment I am committed to write out two joint papers for publication in Germany, inside about 2 months. And I can get absolutely no help from him at all: not even an inquiry as to how I am getting on! The effect on morale is most disheartening.

It is known that Hardy saw Littlewood as the more creative partner in the collaboration;³⁷ but this letter shows in a new way the extraordinary extent of Hardy's dependence upon him. Russell's reply does not survive, but he is not likely to have been surprised by the manner in which Hardy expressed himself; for he had given his own character analysis of Hardy to Ottoline Morrell eight years earlier:

After luncheon I went a walk with a mathematician named Hardy, whom I find congenial on the purely intellectual side, tho' not as a human being.... He is a vampire—the only male vampire I have ever known. He had a friend

³⁶ Hardy was secretary of the London Mathematical Society and also of Section A (Mathematics and Physics) of the British Association for the Advancement of Science.

³⁷ See, for example, Snow's remarks in Hardy, *A Mathematician's Apology* (1969 reprint), pp. 12, 29, and Hardy's own comparison of himself with Littlewood on p. 148.

called Gay, also a fellow here, whom he vampired until Gay committed suicide.³⁸ He began to vampire Norton, but the Stephens³⁹ or something just rescued Norton. He loves excitement, and the life here provides no legitimate excitements.⁴⁰ But he is likeable in many ways.⁴¹

For Russell, therefore, intellectual depletion rather than sexual gratification lay at the heart of Hardy's impulses towards others. Russell knew his friend rather acutely.⁴²

³⁸ *Sic*; this is R. K. Gaye (1877–1909), who was admitted to Trinity College in the same year (1896) as Hardy and became a classicist. He held a fellowship until his suicide in April 1909 (J. A. Venn, *Alumni Cantabrigiensis*, Pt. 2, 3 [1947]: 27).

³⁹ Presumably this is a reference to the clan Stephen of Bloomsbury fame: Adrian, Virginia, Vanessa.

⁴⁰ H. T. J. Norton (1886–1937) also went up to Trinity College, joined the Apostles in 1906 and became a College Fellow four years later. He became interested in both Russell's and Hardy's work, and wrote some (unremarkable) commentaries on the second edition of *Principia Mathematica* after it was published in the 1920s (Archives of the London Mathematical Society, University College, London). He also supplied a new proof for the second (1916) edition of Hardy's tract cited in n. 13. He was the dedicatee of L. Strachey's *Eminent Victorians*.

⁴¹ Russell to Morrell, no. 39, 23 April 1911, Ransom Humanities Research Center, U. of Texas at Austin, Texas. This letter was written just as Hardy's collaboration with Littlewood was starting, so no attitudes towards him were then evident. Compare Russell's comments to Morrell on Hardy in 1914, quoted in Delany, "Russell's Dismissal from Trinity", p. 43.

⁴² I am very grateful for recollections and observations from Dr. F. Smithies, who was a research student of Hardy. The Royal Society kindly allowed me to consult their volumes of election certificates.