

RUSSELL'S LOGICISM

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Denis Vernant. *La Philosophie mathématique de Bertrand Russell*. Paris: J. Vrin, 1993. Pp. 509. 174F.

This is a massive work which details the development of Russell's mathematical philosophy from *The Principles of Mathematics* to the second edition of *Principia Mathematica*. Its thoroughness is impressive. It will make available to the French academic world the kind of Russell scholarship previously available only in English. In this way it is an important successor to Jules Vuillemin's *Leçons sur la première philosophie de Russell* (1968).

Vernant begins with an extremely detailed exposition of the main points of the *Principles*, from the opening remarks on meaning and denotation to the discussion of geometry. He focuses on the evolution of Russell's logicism from the Platonism of the *Principles* through the ramified theory of types and finally to the amended logic of the second edition of *Principia*. Vernant argues that Russell built his various philosophical systems in response to problems which arose in the development of this logicism, and he further argues that the evolutions and modifications of Russell's philosophy are justified by these problems, and that they do not involve a rejection of the

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initial project. Throughout, Vernant is extremely careful in his exposition of Russell's various problems. Despite the fact that he does not see Russell's logicist project as successful, Vernant is always respectful of Russell, and at pains to point out the value of Russell's work.

Vernant has a clear sense of Russell's general philosophical outlook and of the project which began with the *Principles*. Here is his summation of Russell's position, as opposed to Hilbert's formalism:

L'entreprise philosophique ne prétend pas donner de l'extérieur un sens, mais seulement révéler une signification que le discours logico-mathématique contient déjà *pour et par lui-même*. La construction logico-mathématique se fonde sur ses propres principes et n'a pas à être autrement fondée. Le regard philosophique ne vise qu'à élucider ce procès d'*auto-fondation de la logique* en contribuant à clarifier ses principes. (P. 29)

It is this aspect of Russell's logicist project that most interests Vernant, namely the notion of logic as the most general science not requiring and not being capable of a foundation other than what it itself contains, and the idea that mathematics is a part of this logic, in the sense that the mathematical notions are themselves definable in terms of logical notions, and mathematical principles are derivable from logical principles. He sees Russell's early theory of meaning as being part and parcel of the logicist project, and he sees the modifications which began with "On Denoting" as being crucial to the development of the project.

With respect to his chronicling of the development of Russell's theory of meaning and denotation, Vernant gets the overall picture right. I would quibble, though, with some of the details. For example, his analysis of the main arguments in "On Denoting" relies on the outdated interpretation which "explains" the crucial part of the argument in terms of a confusion between use and mention. With respect to this part, Vernant would have done well to consult some of the manuscripts in the Russell Archives. He is clearly aware of them, and makes reference to "On Fundamentals" (1905),²³ but doesn't appear to have examined this manuscript with an eye to the arguments in "On Denoting".

The section on *Principia Mathematica* begins with an account of the resolution of the paradoxes by the theory of types. This topic is very difficult since Russell was not always clear as to what exactly the theory was. Many commentators have, after Ramsey's work, seen the ramified theory of types as imposing the orders on a simple theory of types. Vernant appears to follow this line, introducing first the hierarchy of types (pp. 290–2) and then intro-

ducing the orders as a further refinement. However, Russell's ramified theory is better seen as beginning with the orders, and dividing the orders into different types. When he gets to the actual construction of the theory, this is in fact what Vernant does (pp. 293–4).

One of the difficulties which is glossed over is the priority of propositions versus propositional functions in Russell's account of the theory. Vernant suggests that the theory of types applies first to propositional functions (p. 290), but in "Mathematical Logic as Based on the Theory of Types" the distinction of orders is first applied to propositions, and propositional functions are considered derivative abstractions from the propositions. In *Principia*, though, Russell holds that propositions (or expressions which appear to indicate them) are "incomplete symbols", and he appears to allow an ontology of propositional functions. The issue is important, as it involves the ontological commitments of the theory of logical types, a topic which is of great interest to Vernant. Vernant adopts the line Russell later espoused that the theory of types is only a theory of symbolism (p. 305) and that the ontology of *Principia* includes only individuals, not classes, not propositions, and also not propositional functions (Chap. 5). This section of the book is well done. Vernant treats the vicious circle principle, the theory of types, and Russell's attempts to square the limitations placed on the variables with his view of logic as a completely general science whose variables should range over everything. Vernant sees the limitations on the possible values of a variable as the outcome of a pattern of thought which began with the theory of denoting of the *Principles*. His discussion of these issues is extremely valuable, although I have a feeling Russell was less clear about these issues than Vernant lets on.

The book closes with an interesting discussion of Russell's methodology, particularly the "regressive method" Russell described in his 1907 paper, "The Regressive Method of Discovering the Premises of Mathematics" (in *Essays in Analysis* [1973]). Vernant is sensitive to the fact that Russell did not think that the propositions of mathematics were analytic, nor that the first principles were self-evident, but that they were justified in part by their consequences, in a hypothetico-deductive manner (Vernant, pp. 445–6). Russell's changing attitudes toward the method of solving the contradictions and the proper ontology within which to conduct his programme are best understood in the context of this regressive method. Vernant is, I believe, a little too quick to criticize Russell here.

Vernant sees Russell's logicist project (particularly the "*autofondation*") tumbling with the need for such axioms as that of reducibility, infinity and choice. In this criticism he is not new, following a long line beginning with Wittgenstein and Ramsey, with the most recent probably being Hylton

²³ Now in *Papers 4*.

(*Russell, Idealism, and the Emergence of Analytic Philosophy*). However, Vernant should have made it clearer that the standard by which these axioms are judged to be problematic is not that of the "regressive method", but of a view of logic which is quite alien to that. Hylton suggests that the regressive method was tailor-made to justify the axiom of reducibility²⁴ and suggests that it blurs the distinction between logic and other subjects and somehow conflicts with the plan of the *Principles*. Vernant sees the logicist project collapsing because of the epistemological uncertainty which enters in with this method (p. 447). Two remarks are in order. First, both Vernant and Hylton are careful to note that Russell did not see his logicist project as grounding the certainty of mathematics, or showing that mathematics was analytic as many people who criticize these axioms have taken him to be doing. Secondly, an examination of Russell's logical manuscripts from the time of *Principles* up to "What Is Logic?" (1912)²⁵ reveals Russell's using this regressive method throughout, even before the formulation of the theory of types.²⁶

Russell himself, though, was troubled by these axioms, and only admitted the "axiom of infinity" as the antecedent of conditionals, never actually accepting it as an axiom. His doubts concerning the axiom of reducibility, at least after 1912, are no doubt in part a response to Wittgenstein's criticisms which stemmed from a position quite alien to the "regressive method". Russell gradually came to accept much of Wittgenstein's attitude toward logic, and Vernant sees the modifications ("the final amendment") to the second edition of *Principia* as the end of a long story of the development of Russell's mathematical philosophy. Like Hylton, I have tended to think that Russell changed his focus away from the logicist project toward epistemological matters during this time. Thus I have seen the amendments made to the second edition of *Principia* not as the ultimate development of the project, but as part of another project, and one that does not fit very well with the logicism which began with the *Principles*.

²⁴ Peter Hylton, *Russell, Idealism, and the Emergence of Analytic Philosophy* (Oxford: Clarendon P., 1990), p. 322.

²⁵ In *Papers* 6.

²⁶ There is a very clear statement of this method on page 630 of "Les Paradoxes de la logique", *Revue de métaphysique et de morale*, 14 (1906).
