

RUSSELL'S MARGINALIA IN HIS COPY OF BRADLEY'S *PRINCIPLES OF LOGIC*

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I. ORIGIN AND DESCRIPTION OF THE MARGINALIA

There has been interest recently in Russell's earliest philosophical work, work he did while he was under the influence of the British neo-Hegelian movement. Among the philosophers who influenced him during the period, the most important was F. H. Bradley. Hitherto, however, little attention has been paid to his study of Bradley's *Principles of Logic*, which first appeared in 1883.² Most philosophers who study Bradley read his *Logic* in the light of *Appearance and Reality*, which appeared ten years later. Russell, in contrast, read *Appearance and Reality* in the light of the *Logic*. The *Logic*, in fact, tended to dominate his understanding not only of Bradley, but of the neo-

¹ We would like to thank Ken Blackwell for his careful editorial work on this paper.

² Russell's copy of *The Principles of Logic* is the first edition (London: Kegan Paul, Trench, 1883) and is inscribed in his hand on the half-title: "B Russell | Trin. Coll. Camb. | June 1893". The inside of the front board bears the bookplate of Bertrand and Alys Russell; this very exemplar of their bookplate is reproduced in *Russell*, no. 19 (autumn 1975): 18. At an unknown date, the book was apparently loaned to G. H. Hardy. It was acquired by the Russell Archives in 1976 from Ian Hacking, who bought it at Galloway and Porter's sale of Hardy's library.

In 1922 Bradley brought out a corrected and expanded edition in two volumes (London: Oxford U. P.). Since it is the later edition that is now most widely available, page references will be given to both editions. Page numbers in square brackets are to the second edition. If only a single number is given, the reference is the same in both editions. All page references will be to Bradley's *Logic* unless otherwise indicated.

Hegelian movement as a whole, at least in so far as it had views on logic.

Russell kept a comprehensive list of the books and papers that he read from February 1891 until March 1902, in a small bound book titled "What Shall I Read?"³ which was presented to Russell by his maternal grandmother, Lady Stanley of Alderley. Although this list cannot be regarded as a complete record for the time covered, it does indicate that Russell read *The Principles of Logic* on two occasions. "What Shall I Read?" reveals that Russell first read Bradley's *Logic* in September 1893 and reread it in January 1898. These readings frame Russell's period as a neo-Hegelian. In 1893, Russell is just beginning his exploration of this position. Bradley's *Logic* had been recommended to Russell in September 1892 by the Oxford philosopher Harold Joachim. Joachim was an idealist who had been strongly influenced by Bradley (they were both at Merton College). He was also the brother-in-law of Russell's Uncle Rollo, and Russell, who at the time was studying mathematics at Cambridge but contemplating switching to philosophy, asked his advice about what to read. Joachim sent him a long list of mainly historical works but recommended Bradley, among works on logic, as "First rate—but very hard".⁴ The following summer, Russell started the formal study of philosophy and, although logic was not among the subjects he studied, he put Bradley's *Logic* high on his list of books to read. It was in fact the second important neo-Hegelian work that he read.⁵ By 1898, when the problems with neo-Hegelianism were piling up, Russell turned again to Bradley hoping for a plausible solution.

A critical examination of Russell's marginalia in his copy of *The Principles of Logic* reveals some clues as to when the various comments and observations were made. It seems that many of the inscriptions were made on Russell's first reading of the book in 1893. Comments expressing either doubt about what Bradley meant (e.g. 1, 7, 15) or surprise that

³ "What Shall I Read?", *Papers* 1: 345–65.

⁴ Joachim to Russell, 23 Sept. 1892. See N. Griffin, "Joachim's Early Advice to Russell on Studying Philosophy", *Russell*, n.s. 7 (1987): 119–23.

⁵ The first was McTaggart's privately printed pamphlet, *A Further Determination of the Absolute* (1893). There McTaggart credits Bradley's article, "Reality and Thought" (*Mind*, o.s. 13 [1888]: 370–82), for having inspired him with the idea for his project. On two other occasions in his booklet McTaggart refers to points made in Bradley's *Logic* to support the claims he is arguing. McTaggart no doubt recommended Bradley to Russell, who studied *The Principles of Logic* one month after reading McTaggart's pamphlet.

he meant what he did (e.g. 4, 8) were most likely made in 1893. Eccentric comments such as the one about the soul and the cabbage (53) can also likely be dated to the first reading, since it depends upon a failure to distinguish numerical and qualitative identity, a distinction Russell would not have ignored in 1898. In 1898, as he struggled to free his dialectic of the sciences from contradictions, Russell became enmeshed in problems concerning the individuation and identification of the items needed for the special sciences. During this second reading he likely commented on Bradley's notion that all ideas and all meanings are universal (17, 18).

A third reading of the *Logic* must have occurred at a still later date as on two occasions Russell makes use of formal logic (34, 35) which was not available to him until after his study of Peano late in 1900. Furthermore, marginalia 39 and 40 must also be linked to a later reading as it was not until after 1899 that Russell came to reject the conventional view that infinite series were contradictory. As to when this third reading of Bradley took place, there is no strong evidence suggesting a particular date. Perhaps in 1905, when he was writing "Necessity and Possibility", Russell may have wanted Bradley's account of modality. A further possibility may be in 1910 when Russell and Bradley exchanged views in *Mind*. Perhaps Russell never read the whole of *The Principles of Logic* a third time. It is certainly possible that he occasionally turned to parts of Bradley's book for insights into particular issues and problems, adding comments when he did so.

The following table will indicate to which reading specific comments belong. Only the marginalia that can be dated with some degree of certainty are listed.

FIRST READING: 1893	SECOND READING: 1898	THIRD READING: UNCERTAIN DATE
1, 2, 4, 7, 8, 12, 15, 53	6?, 13?, 17, 18, 19, 20, 21?, 44, 45	34, 35, 39, 40

There are 193 pencil markings by Russell in his copy of Bradley's *Principles of Logic*. Of these, 131 consist of underlinings or vertical lines in the margin to draw attention to certain passages. Book 1 of the *Logic* is entitled "Judgment", and Russell took a greater interest in this segment

of the book. There are 124 marginalia in "Judgment", and Russell made verbal comments 36 times. This stands in contrast to the 69 markings found in Book II, "Inference", in which Russell commented sixteen times. This difference is especially evident when one notes that the second Book is lengthier than the first.

Russell's marginalia in *The Principles of Logic* have been fully recorded, and the complete transcript is available in the Bertrand Russell Archives at McMaster University. This transcript includes every marking, however innocuous, that Russell made in the margins of the book. An abridged table of the marginalia noting every verbal comment will be found in section III. The marginalia are numbered in bold type to facilitate referencing. Russell on occasion deleted or even erased marginalia. These, too, have been noted wherever possible.

II. SOME TOPICS IN THE MARGINALIA

Psychologism

It is important to realize that, prior to reading Bradley, Russell's knowledge of logic would have been based on reading authors like Mill and Jevons.⁶ These authors, following a long ideational tradition from Condillac and Locke onwards, treated judgment psychologically as the combination of ideas by the mind. Now Mill (*Logic*, Bk. I, Chap. 5), unlike many of his predecessors in the tradition, had been careful to distinguish judgment, thus conceived, from logic—thereby aiding the important nineteenth-century separation of psychological concerns from logical and philosophical ones. For Mill, a theory of judgment was part of psychology, not of logic.⁷

Bradley, by contrast, placed judgment firmly at the centre of logic. The first of the two Books of his *Principles of Logic* is devoted to judgment—by beginning thus, he says, "we may at least hope to touch the centre of the subject" (p. 1). For Bradley, however, judgment was not a

purely psychological matter. Bradley's theory of judgment is complex, and a detailed exposition of it is beyond the scope of this paper. Two levels of analysis have to be distinguished. At the mundane level, at which we judge, say, that snow is white, we ascribe a property (or as Bradley puts it, an adjective) to snow, a reality independent of the judgment. At the higher level, however, an "ideal content" (p. 10), which is the entire earlier judgment that snow is white, is ascribed to Reality (or the Absolute) itself, which, at this level, is the single subject of all judgment.⁸

Russell's first reading of Bradley was evidently compromised for a time by his familiarity with standard psychologistic treatments of judgment. As a result, it took some time for him to realize that judgment, for Bradley, was not a purely psychological matter. Thus, for example, when Bradley denies that every judgment has two ideas (p. 12 [II]), Russell insists that there are two: "the wandering adjective⁹ and Reality" (4; see also 7). It is not until he reaches page 49 that Russell realizes that Bradley's Reality, as it occurs as the subject of a judgment, is not an idea.

Russell, interestingly, seems to have no difficulty in appreciating Bradley's other innovation in the theory of judgment: that, ultimately, all judgments have the same subject, the Absolute. This is despite the fact that Bradley is far less explicit about this part of his doctrine in the early pages of the *Logic* than he is about his view that the constituents of judgment are not necessarily ideas. For example, Bradley's first general characterization of judgment is the following: "Judgment ... is the act which refers an ideal content ... to a reality beyond the act" (p. 10; italics added). Russell notes this as a definition, and passes on without further comment (2). By the following page, however, where Bradley says that in judgment an adjective is united with "a real substantive", Russell challenges "a" and writes "the would express his meaning" (3). What is surprising about this is that Bradley, so far, has said nothing that

⁸ Bradley makes his position clearer in one of the Terminal Essays appended to the second edition of the *Logic*. There he distinguishes between ordinary judgments, where the subject is what he calls "a selected reality", i.e., "a limited aspect and portion of the universe" (p. 629), and the higher level, where the subject is Reality (or the Absolute) itself.

⁹ Bradley's adjectives (properties) "wander" (or, more often, "float") from subject to subject.

⁶ J. S. Mill, *A System of Logic* (London: Parker, 1843); W. S. Jevons, *Elementary Lessons in Logic* (London: Macmillan, 1870). Russell studied both books closely before he went up to Cambridge.

⁷ The same was true of Jevons, though he is less explicit. Cf. *Elementary Lessons*, p. 12. See also *The Principles of Science: a Treatise on Logic and Scientific Method* (London: Macmillan, 1887; 1st ed., 1874), p. 4.

would permit one to suppose that, ultimately, the Absolute was the only subject of judgments. Indeed the Bradleian doctrine of the Absolute is hardly evident anywhere in the first edition of the *Logic*. One must suppose, therefore, that either this comment of Russell's was written on one of his subsequent readings of the book, or else that he came to the *Logic* already aware of this view of Bradley's.

Universals, reference, and identity

In attempting to separate logic from psychology, Bradley distinguishes between an idea considered as a particular "psychical existent", what he sometimes calls a psychic fact, and an idea considered *as an sign*, as something which stands for something else. The former is the concern of psychology, the latter of logic. Bradley argues that ideas in the logical sense are always universal. They can never in themselves stand for a unique particular, since, however precise we attempt to make them, they never exactly correspond to any given particular (pp. 35–8). By the same token, words fail to refer uniquely. Each word has a meaning which is universal, and the same difficulties in achieving unique reference recur. Descriptions, appeal to spatio-temporal location, the use of proper names or indexicals and demonstratives are all insufficient, according to Bradley, to guarantee a unique reference.

Bradley found this consequence less disturbing than one might suppose. For him, in the last analysis, all reference is reference to the Absolute, so uniqueness is ultimately achieved by the all-encompassing singularity of the Absolute itself. The failure of more local measures simply indicates our need to rely on the Absolute. Russell, who was a pluralist even in his idealist days, was in no position to take this way out. In fact, much of his work in 1896–98 concerned precisely the difficulties of individuating and identifying the elements that analysis revealed to be basic to the various special sciences. It is no coincidence that Russell wrote more marginalia on these issues in the *Logic* than on any other single topic. He was hardly in a position to recognize the importance of these problems in 1893, so it is likely that most of his marginalia on these parts of the *Logic* were written on his second reading of the book. By that time, problems of identity and individuation in his system of the sciences were at the top of his agenda. The "antinomy of the point" in *An Essay on the Foundations of Geometry* provides the format for subsequent antinomies generated by the other special sciences. The antinomy

arose from the fact that all points were (qualitatively) identical, though each was (numerically) different on account of its relations to other points. This becomes a genuine antinomy in conjunction with Russell's theory of internal relations, which required that each relation inhered in the intrinsic qualities of its relata.¹⁰

Despite his interest in what Bradley had to say on these issues, Russell was not inclined to follow him into monism.¹¹ Within the idealist philosophy that Russell shared at this time with Bradley, monism entailed a severe epistemological penalty. Both Russell and Bradley held that thought was essentially relational, so that a relationless Absolute such as Bradley countenanced would be inaccessible to judgment. Judgments at the "mundane" level, of course, remained possible, and accurate so far as they went, but they didn't go as far as the Absolute. Again, these were consequences Bradley countenanced with more equanimity than Russell could muster.¹² Russell, who had been lured to Hegelianism by McTaggart's promise of a further determination of the Absolute, was not prepared to accept that the Absolute was the one thing which could not be further determined.

It is not surprising, therefore, that Russell should take issue with Bradley on unique reference. Russell complains that Bradley confuses "universal" with "vague" (6) and "ambiguous" (13). In the same vein, he later wonders whether Bradley thinks all ideas and all meanings are universal (17, 18). These last two marginalia clearly form a pair and date from 1898; 6 and 13 perhaps belong to the same reading. The reference in 18 to Ward's article¹³ is illuminating, since Ward there distinguishes between resemblance (which Russell calls "identity of adjective") and

¹⁰ For the antinomy of the point see *An Essay on the Foundations of Geometry* (Cambridge: at the U.P., 1897), pp. 188–9, and N. Griffin, *Russell's Idealist Apprenticeship* (Oxford: Clarendon P., 1991), pp. 185–90. Antinomies of this kind drove the dialectic of Russell's neo-Hegelian system of the sciences, but by 1898 Russell's hopes for a consistent, pluralist resolution of the system were running out. In 1898 he abandoned the theory of relations which gave rise to them.

¹¹ Some of his sharpest comments are made against passages in which Bradley starts to slide towards the Absolute, e.g., 22, 23, and perhaps also 24.

¹² There are hints of Russell's dissatisfaction with this kind of epistemic holism in his marginalia, e.g. in 28.

¹³ James Ward, "Psychology", *Encyclopaedia Britannica*, 9th ed. (Edinburgh: Black, 1886), 20: 37–85. Russell did not read Ward's article until August 1894, so neither marginalium could have been written on his first reading of the *Logic*.

“individual identity”, a distinction nowadays expressed as that between qualitative and numerical identity.¹⁴ As Russell notes, we can identify *a* as the same individual as *b* only because we recognize that properties of *a* are also properties of *b*, but this does not mean that their individuality consists in their shared content. The haecceism that this suggests appears in Russell’s attempts to individuate points of space in a paper written in March 1898.¹⁵

In part, these problems of individuation have to do with the irreducibly relational nature of identification. Bradley is famous for having dismissed relational statements as meaningless, though there is little sign of this line of thought in his *Logic*. Instead, he relies on more commonplace arguments. He argues, for example, that merely increasing the specificity of our descriptions is never in itself enough to guarantee uniqueness of reference. Moreover, even if the description is augmented in ways which permit us to identify an item in a series, e.g., as the first member, we still cannot distinguish that series from others like it (pp. 63–4).

Bradley admits that what is given is unique and that we come closest to uniquely identifying it by the use of demonstratives, especially “this”. Here, it seems, we abandon descriptions and attempt to refer directly to what is given in all its naked particularity. But even here, we always fail to express the uniqueness we intend (p. 65 [66]). As Bradley explains, in the section Russell marked as “fundamental” (21), when we use “this” to express a judgment we still employ meanings which, like all meanings, are universal.

Direct reference plays an important role in Russell’s mature philosophy. As is well known, he held that items with which we are acquainted can be named. Names, whose referents are thus given to us, which he called “logically proper names”, identify these referents directly, without intervening descriptions. His favourite examples of logically proper names were the demonstratives “this” and “that”. His marginalia reveal how early he was sympathetic to this idea.

¹⁴ This distinction is evident in later marginalia, e.g. 44 and 45. The curious marginalium (53) about the soul and the cabbage (which Russell later criticized and then crossed out) depends upon ignoring this distinction, and presumably belongs to his first reading.

¹⁵ “On the Constituents of Space and Their Mutual Relations”, in *Papers* 2: 311–21. Marginalium 19, on space and time as principles of individuation, relates to the same issues.

There is one point in the marginalia in which Russell goes beyond his later published writings on logically proper names and comes close to some contemporary views on direct reference. In criticising Bradley’s claim that indexical expressions like “this”, “here”, and “now” “are symbols whose meaning ... covers innumerable instances” (p. 49), Russell replies that they do refer uniquely if used as genuine demonstratives, i.e. “if spoken, & accompanied by a gesture which is part of the symbol” (14). This is very likely the first time he appealed to logically proper names in order to secure direct reference. Further, the idea that the use of demonstratives should include a gesture as part of the symbol is an important one which does not, to our knowledge, occur elsewhere in Russell. Had it done so, it might have saved Russell’s theory of logically proper names from much misunderstanding and some criticism by linguistic philosophers.¹⁶

Universal propositions

There was one matter in logic on which Russell continued to give Bradley credit long after he had ceased to be in any way a Bradleian. This concerned the logical form of propositions. In traditional logic, all propositions were treated as having subject–predicate form. This was a doctrine that Bradley wisely rejected in the *Logic* for many good reasons. In particular, Bradley rejected the subject–predicate analysis for universal (A-form) propositions. Quite what account Bradley wanted to put in its place is not entirely clear (cf. p. 82), but at times he does come quite close to the modern view that they are universally quantified conditionals (p. 47), though in the absence of quantifiers and variables he was hardly in a position to give exactly the modern account. It is certain that reading Bradley’s *Logic* saved Russell from having an undue respect for subject–predicate form, and in the case of universal propositions he did acknowledge Bradley’s contribution.¹⁷

¹⁶ The idea that demonstratives include a gesture was taken up much later by David Kaplan, “Demonstratives”, in J. Almog, H. Wettstein and J. Perry, eds., *Themes from Kaplan* (Oxford: Oxford U. P., 1989), pp. 481–614.

¹⁷ See, e.g., “On Denoting”, *Papers* 4: 416; “Mathematical Logic as Based upon the Theory of Types”, *LK*, p. 70; “The Philosophy of Logical Atomism”, *Papers* 8: 208. For more detail on Bradley’s influence on Russell on this point, cf. N. Griffin, “Bradley’s Contribution to the Development of Logic”, in James Bradley, ed., *Philosophy after F. H. Bradley* (Bristol: Thoemmes, 1996), pp. 197–202.

Russell marked the passages in the *Logic* in which Bradley came closest to the modern view (9, 10)—though, unfortunately, it is not possible to date these marginalia.¹⁸ Wisely Russell balks at Bradley's claim that "All animals are mortal" is "about *mere* hypothesis" (p. 46 [47], italics added; 9). Marginalium 10 is more interesting. When Russell qualifies Bradley's statement that "No truth can state fact" to "no universal truth can state fact", he not only supplies a qualification essential for the truth of Bradley's claim, but apparently accepts that there are neither general facts nor conditional facts. These are views that reappear later in his philosophy.¹⁹

Necessity and conditionals

Bradley held the view that necessity was always conditional: something was necessary because of something else (p. 183 [199]). This view puzzled Russell (34), as well it might, for it seems hardly possible that Bradley intended (as Russell put it) that q was necessary, provided only that " $p \supset q$ " was true.²⁰ Russell wonders whether p also must be true. But Bradley denies this (p. 185 [201]): for Bradley, q is necessary whether it is necessarily true or necessarily false. It is no doubt a strange notion of necessity in which every consequence of a true proposition is necessarily true and every consequence of a false proposition necessarily false. But in the passage from which this view comes, it is confounded with an even stranger one: namely, that all necessary propositions are of the form "if p then q " (35), a doctrine which has odd consequences for the theory of deduction. Linguistically, it is easy to confuse the two positions, since "if p then necessarily q " can be read both ways (depending on whether "necessarily" qualifies "if ... then ..." or " q "). It is much harder to confuse the two theories, though Bradley seems to make little effort to distinguish them.

Russell's use of formal notation in 34 and 35 means that these

¹⁸ Russell had embraced a conditional account (though not the modern one) of A-form propositions by 1898 (cf. "An Analysis of Mathematical Reasoning", *Papers* 2: 172).

¹⁹ Cf. *PM* 1: 44–5; "The Philosophy of Logical Atomism", *Papers* 8: 208.

²⁰ Russell's use of the material conditional is, of course, an anachronism. Bradley may well have had a much stronger connective in mind—it is certain he wasn't thinking of material implication! Russell's use of " \supset " may, however, be no more than shorthand—an allowable expedient within the margins of a book.

marginalia could not have been written before 1901, which is when the concept of material implication first occurs in his work. By the same token, the next marginalium must have been written *before* he had the notion of material implication. He asserts that no consequences can be drawn from a contradiction (36). Notoriously, a contradiction materially implies any proposition. Before the principles of classical logic, including the notorious paradoxes of material and strict implication, were laid down, a variety of views were current on matters like this. Russell here presumably relies upon a conception of negation on which negating a proposition "cancels" it. On such a cancellation view, a contradiction voids itself, leaving no judgment from which consequences can be drawn. This view of negation is to be found in Boethius, in Abelard, and in Berkeley's *The Analyst* and is natural for contemporary connexivist logics.²¹ What is odd is to find Russell appealing to it at the end of the nineteenth century, as if it were standard logical doctrine. So far as we know, cancellation negation does not occur in any of the logicians Russell is known to have studied at this time.²² He had certainly not read Boethius or Abelard. The one author on this list from whom he may have got it is Berkeley. Although there is no record of his reading *The Analyst*, his view of the calculus after his mathematical education at Cambridge was in many ways similar to Berkeley's, though this, in itself, is less than conclusive evidence that he studied *The Analyst*. Even if he did, it remains puzzling that he should accept Berkeley's opinion on this point, against prevailing contrary views.

Some points of special concern

One important point of difficulty in Russell's philosophy after he'd abandoned neo-Hegelianism was that of accounting for the unity of complexes. In *The Principles of Mathematics* this was identical with the problem of accounting for the unity of propositions, in which form it has received some discussion by commentators. Bradley pressed him on

²¹ See Graham Priest, "Negation as Cancellation, and Connexive Logic" (unpublished typescript). Connexive logics typically embrace such non-classical principles as $\neg(\neg A \vdash A)$, $\neg((A \vdash B) \& (\neg A \vdash B))$ and their close relatives.

²² This includes Bradley, Bosanquet, and Jevons. Nor is cancellation negation to be found in the writings of the Cambridge logician, W. E. Johnson, whose views Russell might have been expected to be aware of.

the issue in articles in *Mind*,²³ which he subsequently collected in *Essays on Truth and Reality*.²⁴ Writing to Bradley to thank him for a copy of the book, Russell acknowledged the difficulty of the problem and admitted that he remained dissatisfied with the solutions he had found so far.²⁵

One of his marginalia (33) relates specifically to this problem. Though the question at issue is the unity of Caesar (considered as a bundle of attributes), it transfers easily to the familiar form which concerns the unity of propositions. For propositions, the bond which relates the attributes of Caesar becomes the copula which relates the terms of the proposition; and the question of whether the bond is one of the attributes becomes the question of whether the copula is one of the terms. This was a matter on which Russell reversed himself. In *The Principles of Mathematics* (pp. 44–5, 95n.) the copula was not one of the terms, in *Theory of Knowledge* (*Papers* 7: 80) it was. There is a general discussion of the unity of complexes in an unpublished manuscript, "On Functions" (1904).²⁶ There, complexes are identified by their constituents together with their mode of combination, which is not itself a constituent.²⁷ It comes as no surprise to find that, although Russell was not satisfied with his own solutions to this problem, he was not impressed by Bradley's.

The most unusual of Russell's marginalia is his long comment on page 48 [49] of the *Logic* (12; see also 47), where Bradley maintains that logic must treat judgments which are rejected as invalid (i.e. false) on metaphysical grounds. Russell rejects this on the extraordinary ground that logic is concerned only with ideal (and therefore true) judgments and thus not with any judgments which "metaphysics shows to be logically faulty". (One wonders how even metaphysics could show them to be *logically* faulty, if they fell outside the scope of logic.) Russell assumes

that Bradley had arrived at his theory of judgment by an induction, presumably on the judgments people actually make—a naive concession to psychologism which was never part of Bradley's method. The view of logic which Russell hints at in this note is certainly a bizarre one, and one wonders where he got it. Clearly this marginalium was written on his first reading.

III. TEXT OF THE MARGINALIA

Russell employed his standard manuscript abbreviations in composing his marginalia. Since their meaning is not doubtful, they have been expanded silently here. Square brackets in the Marginalium column are Russell's own. Angle brackets enclose our editorial comments. All underlining is Russell's own.

PASSAGE IN <i>Logic</i> (+ ANY UNDERLINING BY BR)	MARGINALIUM
1. BOOK I. JUDGMENT, p. 5 §6. We might say that, in the end, there are no signs save ideas, but what I here wish to insist on, is that, for logic at least, all ideas are signs. Each we know exists <u>as a psychical fact</u> , and with particular qualities and relations.	only so?
2. p. 10 Judgment proper is the act which refers an ideal content (recognized as such) to a reality beyond the act. This sounds perhaps much harder than it is.	definition
3. pp. 10–11 [10] In the act of assertion we transfer this adjective to, and unite it with, <u>a</u> real substantive.	<u>the</u> would express his meaning.
4. p. 12 [11] (i) It is not true that every judgment has two ideas. We may say on the contrary that all have but one.	On your theory there <u>are</u> two ideas, the wandering adjective & Reality. [but cf. pp. 49–50, where it appears that Reality is not an idea]

²³ "On Appearance, Error and Contradiction", *Mind*, n.s. 19 (1910): 153–85; "Reply to Mr. Russell's Explanation", *Mind*, n.s. 20 (1911): 74–6.

²⁴ Oxford: Clarendon P., 1914.

²⁵ Russell to F. H. Bradley, 30 January 1914, RA.

²⁶ Now published in *Papers* 4: 98–110.

²⁷ See N. Griffin, "Terms, Relations, Complexes", in A. D. Irvine and G. A. Wedeking, *Russell and Analytic Philosophy* (Toronto: U. of Toronto P., 1993), pp. 159–92, for a general discussion of these issues. The notion of the logical form of a complex is, of course, tied up with the notion of a mode of combination.

PASSAGE IN <i>Logic</i> (+ ANY UNDERLINING BY BR)	MARGINALIUM
<p>5. p. 14 [13] §12. I pass now (ii) to the other side of this error, the doctrine that in judgment one idea is subject, and that the judgment refers another to this. In the next chapter this view will be <u>finally disposed of</u>, but, by way of anticipation, we may notice here two points. (a) In "wolf eating lamb" the relation is the same, whether I affirm, or deny, or doubt, or ask.</p>	?
<p>6. p. 39 [37] You may call it, if you please, mere impotence of our imagination, or you may call it that idealizing function of the mind which is the essence of intelligence, still the fact remains that never at any stage can any fact be retained without some mutilation, some removal of that detail which makes it particular.</p>	"Universal" here is equivalent to "vague"
<p>7. p. 43 [43-4] A fact is individual, an idea is universal; a fact is substantial, <u>an idea is adjectival</u>; a fact is self-existent, an idea is symbolical.</p>	always? This hangs together with the view that Reality as the subject of judgment is not an idea. See pp. 49-50 & p. 66.
<p>8. p. 44 [45] The individual is so far from being merely particular that, in contrast with its own internal diversity, it is a true universal (cf. Chap. VI.). Nor is this a paradox. We are accustomed to speak of, and believe in, realities which exist in more than one moment of time or portion of space. Any such reality would be an identity which appears and remains the same under differences; and it therefore would be a real universal.*</p>	Is the universal here a purely adjectival universal? It <u>seems</u> otherwise.
<p>9. p. 46 [47] We mean, "Whatever is an animal will die," but that is the same as <i>If anything is an animal then</i></p>	

PASSAGE IN <i>Logic</i> (+ ANY UNDERLINING BY BR)	MARGINALIUM
<p>(cont.) it is mortal. The assertion really is about <u>mere hypothesis</u>; it is not about fact.</p>	not <u>mere</u>
<p>10. p. 47 [48] The universal judgment is thus always hypothetical. It says "Given one thing you will <i>then</i> have another," and it says no more. <u>No truth can state fact.</u></p>	No universal truth
<p>11. p. 48 Let us make a search and keep to this question, Is there nowhere to be found a categorical judgment? And it seems we can find one.</p>	One indeed! said the Dormouse. <Russell erased this comment, which is from <i>Alice's Adventures in Wonderland</i> , Ch. 7.>
<p>12. p. 48 [49] The third class (iii), on the other hand, have to do with a reality which is never a sensible event in time. "God is a spirit," "The soul is a substance." We may think what we like of the validity of these judgments, and may or may not decline to recognize them in metaphysics. But in logic they certainly must have a place.</p>	Validity is not indifferent to Logic. Logic seeks the ideal of judgment, & cannot expect to find this in judgments which metaphysics may show to be logically faulty. A dialectic from falser to truer ^{<*>} views of judgment should also be a dialectic from falser to truer judgments. The true nature of judgment is not to be collected from an induction, & is not necessarily true of <u>all</u> judgments, but only of <u>true</u> judgments. <*Originally Russell had written "truer to falser", but he inserted a

be found a categorical judgment? And it seems we can find one. Universal judgments were merely hypothetical, because they stated, not individual substantives, but connections of adjectives. But in singular judgments the case is otherwise. Where the subject, of which you affirm categorically, is one individual, or a set of individuals, your truth expresses fact. There is here no mere adjective and no hypothesis.

These judgments are divisible into three great classes. And the distinction will hereafter be of great importance. (i) We have first those judgments which make an assertion about that which I now perceive, or feel, or about some portion of it. "I have a toothache," "There is a wolf," "That dough is broken." In these we simply analyze the given, and may therefore call them by the name of *Analytic judgments of sense*.* Then (ii) we have *Synthetic judgments of sense*, which state either some fact of time or space, or again some quality of the matter given, which I do not here and now directly perceive. "This road leads to London," "Yesterday it rained," "To-morrow there will be full moon." They are synthetic because they extend the given through an ideal construction, and they all, as we shall see, involve an inference. The third class (iii), on the other hand, have to do with a reality which is never a sensible event in time. "God is a spirit," "The soul is a substance." We may think what we like of the validity of these judgments, and may or may not decline to recognize them in metaphysics. But in logic they certainly must have a place.

§ 8. But, if judgment is the union of two ideas, we have not so escaped. And this is a point we should clearly recognize. Ideas are universal, and, no matter what it is that we try to say and dimly mean, what we really express and succeed in asserting, is nothing individual. For take the analytic judgment of sense. The fact given us is singular, it is quite unique; but our terms are all general, and state a truth which may apply as well to many other cases. In "I have a toothache" both the I and the toothache are mere

* These analytic and synthetic judgments must not for one moment be confounded with Kant's. Every possible judgment, we shall see hereafter, is both analytic and synthetic. Most, if not all, judgments of sense are synthetic in the sense of transcending the given.

Validity of an analytic judgment is based on the fact that the subject and predicate are contained in each other. In a synthetic judgment, the subject and predicate are not contained in each other. The fact given us is singular, but the terms are general. In "I have a toothache" both the I and the toothache are mere

generalities. The actual toothache is not any other toothache, and the actual I is myself as having this very toothache. But the truth I assert has been and will be true of all other toothaches of my altering self. Nay "I have a toothache," is as true of another's toothache as of my own, and may be met by the assertion, "Not so but I have one." It is in vain that we add to the original assertion "this," "here," and "now," for they are all universals. They are symbols whose meaning extends to and covers innumerable instances.

Thus the judgment will be true of any case whatsoever of a certain sort; but if so, it can not be true of the reality; for that is unique, and is a fact not a sort. "That dough is broken," but so are many others, and we do not say which. "This road leads to London" may be said just as well of a hundred other roads. "To-morrow it will be full moon," does not tell us what to-morrow. Hereafter it will constantly be true that, on the day after this day, there will be a full moon. And so, falling in all cases to state the actual fact, we state something else instead. What is true of all does not express this one. The assertion sticks for ever in the adjectives; it does not reach the substantive. And adjectives unsupported float in the air: their junction with reality is supposed and not asserted. So long as judgments are confined to ideas, their reference to fact is a mere implication. It is presupposed outside the assertion, which is not strictly true until we qualify it by a suppressed condition. As it stands, it both falls as a singular proposition, and is false if you take it as a strict universal (cf. § 62 foll.).

§ 9. But judgment, as we saw in the foregoing Chapter, is not confined to ideas, and can not by any means consist in their synthesis. The necessity for two ideas is a mere delusion, and, if before we judged we had had to wait for them, we certainly should never have judged at all. And the necessity for the copula is a sheer superstition. Judgments can exist without any copula and with but one idea.

In the simplest judgment an idea is referred to what is given in perception, and it is identified therewith as one of its adjectives. There is no need for an idea to appear as the subject, and, even when it so appears, we must distinguish the

In an analytic judgment, the subject and predicate are contained in each other.

In a synthetic judgment, the subject and predicate are not contained in each other.

Page 48 from Bradley's *Logic*. Marginalia 11 and 12 are both on the page.

Marginalia 13, 14 and the start of 15 are on this page from Bradley's *Logic*.

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	transpose symbol to correct his statement to "falsus to truer" >
13. p. 49 The <i>actual</i> toothache is not any other toothache, and the <i>actual</i> I is myself as having this very toothache.	is an ambiguous term necessarily universal?
14. p. 49 They are symbols whose meaning extends to and covers innumerable instances.	not if spoken, & accompanied by a gesture, which is part of the symbol.
15. pp. 49–50 [50] There is no need for an idea to appear as the subject, and, even when it so appears, we must distinguish the fact from grammatical show. It is <u>present reality</u> which is the actual subject, and the genuine substantive of the ideal content.	? is it not <u>the idea</u> of present reality? This question seems fundamental.
16. p. 58 It not only distinguishes a part from the environment, but it also characterizes and qualifies it.	cf. §8
17. p. 59 [59–60] If there <i>is</i> an idea conveyed by the name, whenever it is used, then it surely means something, or, in the language which pleases you, it must be "connotative." But if, on the other hand, it conveys <i>no</i> idea, it would appear to be some kind of interjection. If you say that, like "this" and "here," it is merely the ideal equivalent of pointing, then at once it assuredly <i>has</i> a meaning, but unfortunately that meaning is a vague universal.	The point to be argued is, whether all ideas are purely adjectival, or whether all identity is merely identity of content; but this point is not argued. [See p. 156]
18. p. 61 The <u>meaning</u> of such a name is universal, and its use implies a real universality, an identity which transcends particular moments. For, unless the	But is all meaning, or every universal, adjectival? Identity of adjective

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(<i>cont.</i>) person were recognized as distinct, he would hardly get a name of his own, and his recognition depends on his remaining the same throughout change of context. We could not recognize anything unless it possessed an attribute, or attributes, which from time to time we are able to identify. The individual remains the same amid that change of appearance which we predicate as its quality. And this implies that it has real identity. Its proper name is the sign of a universal, of an ideal content which actually <i>is</i> in the real world.	is of course necessary for recognition, but is not <u>therefore</u> what we recognize. Cf. Ward's Psychology p. 81.
19. p. 63 [63–4]. <i>A second vertical line is added parallel to the first one covering the second sentence.</i> §21. We must get rid of the erroneous notion (if we have it) that space and time are "principles of individuation," in the sense that a temporal or spatial exclusion will confer uniqueness upon any content. It is an illusion to suppose that, by speaking of "events," we get down to real and solid particulars, and leave the airy region of universal adjectives. For the question arises, What space and time do we really mean, and how can we express it so as not to express what is as much something else? It is true that, in the idea of a series of time or complex of space, uniqueness is in one sense involved; for the parts exclude one another reciprocally. But they do not exclude, unless the series is taken as one continuous whole, and the relations between its members are thus fixed by the unity of the series. Apart from this unity, a point on its recurrence could not be distinguished from the point as first given. And elsewhere we might ask, how far such an unity is itself the negation of mere exclusivity.	Important.
20. p. 65 [66] No amount of thisness which an event possesses will exclude the existence of <u>self-same</u> events in	do you mean that thisness suffices for ident-

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(cont.) other like series.	ity? If so, the view throws light on your denial of the Self. Cf. p. 72.
21. p. 68 Between the fact and the idea of the "this" in judgment, there can be no practical difference. The idea of this would be falsely used, unless what it marks were actually presented. But in that case we should be trying to use a sign, when we have before us the fact which is signified. We can use the idea so far as to recognize the fact before us as a fact which is "this;" but such a use does not go beyond the given. It affirms of the subject a predicate without which the subject disappears. It implies discrimination within the fact in which, since the aspect discriminated is not separable from the given, that given with its aspect still remains as the subject. So that the addition of the idea adds nothing to the subject. And if again it were possible to import the idea from the content of <i>another</i> fact, the operation would be uncalled for and quite inoperative.	This § is fundamental. <Russell added quotation marks around the word "this" in the second sentence.>
22. p. 71 §30. The endeavour to find the completeness of the real, which <u>we feel</u> can not exist except as an individual....	Is this a dogma, or derived from metaphysics?
23. p. 71 [72] §31. The real then itself transcends the presentation, and invites us to follow it beyond that which is given.	This has not been proved hitherto.
24. p. 72 §32. But such continuity, and the consequent extension of the "this" as given, depend, like every other ideal construction, on identity. An inference always, we shall see hereafter, stands on the identity of indiscernibles. Sameness of	<There is a 2nd vertical line in the margin beside the 3rd sentence opposite. In the margin beside the 2nd line

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(cont.) quality proves real sameness (vid. Book II. Part I. Chap. VI.). And the identity here has a double form. (i) In the first place the symbolical content must have "thisness." (ii) In the second place it must share some point with the "this."	Russell added: "Dogma". Underneath this there appears to be an entire passage that was erased and is now illegible except for the final 3 words: "objection seems fatal.">
25. p. 73 It is thus by inference that we transcend the given through synthetic judgments, and our following Books must explain more clearly the nature of inference, and the <u>enormous</u> assumption on which it reposes.	I should think so!
26. p. 79 True indeed they may be if truth means a natural and inevitable way of representing the real. But if by their truth we understand more than this; if we say that the reality <i>is</i> as it appears in our ideal construction, and that actually there <i>exists</i> a series of facts past present and future—I am afraid that truth, if we came to examine it, would change into falsehood. It would be false if measured by the test of perception, and it may be, if tried by another standard, it would be false still.	Implying distinction between Reality & existence.
27. p. 79 The judgments exist, and logic can do nothing else but recognize them.	?
28. p. 129 [136] We must answer, for the present, that it rests on our impotence. There is no great principle on which we can stand. We can not find any opposite of <i>b</i> or opposite of <i>c</i> which is not also an opposite of <i>A</i> ; and we boldly assume that, because we find none, therefore there is none. The conclusion from impotence may itself seem	How about "omniscience"? (p. 130)

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(<i>cont.</i>) impotent, but, as we shall hereafter see, there remains some doubt if it may not in the end be taken as the ground and the sole ground we have for believing anything (Book III. II. Chapter III).	
29. p. 135 [145] For if anything is individual it is self-same throughout, and in all diversity must maintain its <u>character</u> .	?
30. p. 160 [173] If you have ideas of smaller wholes, enclosed in and subordinated to larger wholes, will it there be true that the wider the synthesis the emptier it becomes?	e.g. Has the stomach more content than the whole body?
31. p. 165 [178] The natural and the true interpretation of "Dogs are mammals" is that dog and mammal are different attributes, and that these differences co-exist within the same things; or again, that, though the things are certainly the same, for all that they possess two different attributes, dog and mammal. But this natural interpretation involves the abandonment of the theory of inclusion within the predicate.	In this case we ought to have "Mammals are dogs".
32. p. 170 [182-3] It is an elementary mistake to suppose that number confers particularity and destroys intension. And the error reveals a deep foundation of bad metaphysics. <u>Number is surely nothing but an attribute.</u>	?
33. p. 178 [191] In "Caesar is sick," Caesar is not affirmed to be nothing but sick: <u>he is a common bond of many attributes, and is therefore universal.</u>	What sort of bond? Is a bond itself an attribute? If not, the universal is not defined by content.

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34. p. 183 [199] (<i>b</i>) It is easy to give the general sense in which we use the term <i>necessity</i> . A thing is necessary if it is taken not simply in and by itself, but by virtue of something else and because of something else. Necessity carries with it the idea of mediation, of dependency, of inadequacy to maintain an isolated position and to stand and act alone and self-supported. A thing is not necessary when it simply <i>is</i> ; it is necessary when it is, or is said to be, <i>because of</i> something else.	If $p \supset q$, q is necessary. (p need not be true?) [NO, p. 185]
35. p. 185 [200-1] §10. I admit it is not the same thing to affirm "If M is P then S is P ," and "Since M is P therefore S is P ." And the difference is obvious. In the latter case the antecedent is a fact, and the consequent is a fact: they are both categorical (Chap. II. §71). In the former case the antecedent may be false and the consequent impossible. But the necessity in each case is one and the same. $S-P$ <i>must</i> be true, if you take $M-P$, and take $S-M$, and draw the conclusion. That is all the necessity it is possible to find. The knowledge that $S-M$ $M-P$ are both true, and that $S-P$ is a statement which holds of fact, falls outside the necessity and does not increase it. The hypothetical result becomes categorical by an <i>implied</i> addition. And the hypothetical <i>connection</i> may not even then become categorical. The bond of necessity is a logical passage, and to say that this logical passage itself exists in fact demands an assumption which can not be hazarded in the face of objections. In logic we must be content to say that, if the premises are categorical, the result is categorical. We can not add that this result is <i>necessary</i> , unless for a moment we treat the <i>data</i> as hypotheses, and mean no more than <i>If</i> $S-M$ $M-P$ are given, then $S-P$ must <i>follow</i> .	Bradley is not clear as to whether necessity attaches to $p \supset q$ or to q .

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36. p. 186 [201] "If two were three then four must be six" presents us with a truth which is compulsory.	No! If the hypothesis is logically self-contradictory, no consequence can be validly drawn. <A squiggly line down the centre of this passage appears to delete it.>
37. p. 209 [224] "It is even chances that the soul is nothing but a function of the body:" the probability is $\frac{1}{2}$. "It is one to two that God is a person:" the probability is $\frac{1}{3}$. "It is one to ninety-nine that the will is free:" the probability is $\frac{1}{100}$.	I doubt if such instances are legitimate. See p. 48. ²⁸
38. p. 209 [225] Admit that the case is highly improbable, it still is possible that in the mind of some man the grounds, present for and against such judgments as these, might be reduced to a common denominator.	This is a <u>bare</u> possibility.
39. p. 219 [234] Since an infinite sum is an impossibility, the infinite series can not possibly be thrown.	why?
40. p. 219 [234] There is a difference surely between ideals which as such do not exist, because they are abstractions, and ideals which are downright self-contradictions.	but why is an infinite-series self-contradictory
41. BOOK II. INFERENCE, p. 226 [246] (i) A is to the right of B, B is to the right of C, therefore A is to the right of C.	The premises here are insufficient: if e.g. A, B,

²⁸ Bradley cites these instances as counter-examples to the frequency interpretation of probability. Russell said little about probability until much later in his career. From this we can probably conclude that he accepted the frequency interpretation.

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	C are points on a circle the conclusion may be untrue. ²⁹
42. p. 246 [268] The attempt to manufacture them would lead to the search for a completed infinity; for the number of special relations has no end, and the possible connections in time, space, and degree are indefinite and inexhaustible.	But all might conform to a finite number of types like conics or quadratics.
43. p. 262 [287] ... I can only reply that I am waiting, and have been waiting for years, to be told what is meant by an "exact likeness."	X
44. p. 262 [287] But what can this mean? In the case of some twins it might be right to punish one for the other, and we should no longer care to identify criminals.	On the contrary, this is an objection to your principle that "sameness of quality proves real sameness." See p. 72.
45. p. 264 [289] I shall be told that spaces and times are indiscernible and yet are not identical. But this objection rests on a complete mistake. As spaces or times of a certain character A and B surely are identical; as different elements within the same series A and B are surely <i>not</i> indiscernible.	They are indiscernible as far as content goes.
46. p. 371 [400] §9. Now this observation has important consequences, for it points to the conclusion that, in considering number, we have no right to strike out the qualitative side. If the confused feeling of difference in degree between wholes came first, and these wholes were then after-	How is Psychology to prove such a conclusion?

²⁹ Though this point could have occurred to anyone, questions of cyclical order were of special interest to Russell in 1898 in connection with his rethinking of his views on projective geometry. See "Note on Order", *Papers* 2: 341-58.

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(cont.) wards analyzed into parts, and these parts were then once again reduced to equivalent units—if this was the psychological process, as I think we may agree it clearly must have been—then I venture to argue that this shows we are wrong, if we take quantities to consist in nothing but units, somehow taken together and barely co-existing.	
47. p. 377 [406] No doubt we may question the validity of this inference, but I do not see how we can deny its existence.	Invalid inferences are irrelevant to Logic.
48. p. 378 [406] True there is no distinction unless things are first different; but <u>for us</u> there can be no difference which does not follow on distinction.	[Italics]
49. p. 385 [414] But the earliest form of disjunctive reasoning dispenses with such a preliminary statement [that the disjuncts exhaust all possibilities]. Incompatible suggestions with respect to A come before the mind, and the suggestion which survives in that ideal struggle is accepted as fact. Thus we go direct to the assertion without any declaration that our previous denial has exhausted the subject.	But such inference is surely hopelessly vicious.
50. p. 401 [436] Reasoning thus depends on the identity of a content inside a mental experiment with that content outside.	what does content outside a mental experiment mean?
51. p. 415 [453] But suppose we start with possibilities not given as strict alternatives.	Such inference seems to me invalid
52. p. 421 [459] Take for instance, "A is <u>not equal</u> to B," and where is the third term?	"unequal" Bradley means.

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53. p. 421 [459] A and B are perceived to be unequal, but inequality presupposes that both have quantity.	But is not the soul not equal to a cabbage? [Not in any intelligible sense] <Russell's comments were deleted by a squiggly line down the middle.>
54. pp. 439–40 [479–80] The answer which before (Chap. III. §§ 12–18) we gave in the negative, seems now threatened with reversal. Inference so far has been found reducible to a double process of synthesis and analysis; and it seems that such a process exists also in judgment. Must we not then say that, as reasoning implies judgment, so judgment implies reasoning? We can not say this, and a distinction remains which it is impossible to break down. Inference is an experiment performed on a <i>datum</i> , which <i>datum</i> appropriates the result of the experiment. But in those judgments of perception, which we have been just discussing, there is properly no <i>datum</i> . I do not mean that, like the Deity of our childhood, they create their world from nothing at all, and exert their activity on a void externality or their own inner emptiness. What I mean is, that the basis, from which they start, and on which they act, is <i>for the intellect</i> nothing. It is a sensuous whole which is merely felt and which is not idealized. It is not anything which, as it is, could come before an understanding; and hence we can not take it as the starting-point of inference, unless we are ready to use that term in a somewhat loose sense. We needs must begin our voyage of reasoning by working on something which is felt and not thought. The alteration of this original material, which makes it first an object for the intellect, is thus not yet inference, because the	<There is a second vertical line added for emphasis which spans: "... which <i>datum</i> appropriates the result ... that term in a somewhat loose sense."> Important.

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<p>(cont.) start has not been made from an ideal content. Before reasoning exists, there must come an operation which serves to transform this crude material; and this operation is both analytical and synthetical. But it is not inference; for, though its result is intellectual, its premise, so to speak, is merely sensuous.</p> <p>Thus our primitive judgment falls short of inference in two main points. It is doubtful first (i) if the operation performed is not purely capricious. Psychologically, of course, it does not come by accident; but regarded logically it looks like chance. We have no rational ground we can produce, in order to justify our result. This is the first point; and secondly (ii) the stuff, upon which the act is directed, is not intellectual.</p>	
<p>55. p. 446 [486] §25. Beside the functions of analysis and synthesis we found that reasoning employed a third principle. The leap of transition from the possible to the real did not seem to fall under either of these heads.</p>	<p>I do not believe in this principle.</p>
<p>56. p. 499 [552] Inference, if valid, in the end must be valid on a certain hypothesis. The conclusion will follow, given a supposition.</p>	<p>In this case inference uses itself to prove its own validity & argument is circular.</p>
<p>57. p. 509 [562] “2+4-1 makes the integer five, and two units apart from that whole integer are two, <i>therefore</i> 4-1 has the quality of five, or is at least a part of the cause of that quality.”</p>	<p>Good</p>