RUSSELL'S EARLY MATHEMATICAL PHILOSOPHY

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Francisco A. Rodríguez-Consuegra. *The Mathematical Philosophy of Bertrand Russell: Origins and Development.* Preface by I. Grattan-Guinness. Basel, Boston, Berlin: Birkhäuser Verlag, 1991. Pp. xiv, 236. US$100.00.

One area of Russell scholarship has long remained neglected: the development of Russell's philosophical thought between the publication of *An Essay on the Foundations of Geometry* (1897) and *The Principles of Mathematics* (1903). Not only is this area interesting in its own right—Russell's logicism had its origins in this period—but an understanding of it is required to see the context from which Russell's later philosophy arose. N. Griffin's *Russell's Idealist Apprenticeship* (1991) does provide a thorough discussion of this development, but it considers it up to 1900. P. Hylton's *Russell, Idealism, and the Emergence of Analytic Philosophy* (1990) does look at the development, but it focuses on Russell's early idealism and later rejection of it and does not consider in any detail his logical and mathematical work of the period 1897–1903. The volume under review examines this work.

As much of Russell's early logical and mathematical work was never published, Rodríguez-Consuegra rightly devotes much attention to the unpublished manuscripts made available at the Bertrand Russell Archives and recently put together in *The Collected Papers of Bertrand Russell*, volumes 2, 3, 4 and 6. One of Rodríguez-Consuegra's aims in the book is to "show the genuine roots of Russell's mathematical philosophy" (p. 2) and to do so requires consideration of those who influenced Russell in this regard. Thus, more than a third of the book is occupied with Russell's nineteenth-century antecedents and includes an admirable study of Peano and his school.

In Chapter 1, "The Methodological and Logicist Background", the author reviews the relevant parts of the work of those—except for Peano and his school—who helped shape Russell's early mathematical philosophy, both technically and philosophically. For instance, proto-logicist elements in the thought of Boole and Peirce are identified. Dedekind's definitions of the mathematical concepts ideal, cut, chain, irrational, continuity, and infinity, Cantor's reduction of arithmetic operations to set-theoretic ones, and his theory of the transfinite, are examined with an eye to Russell's later assimilation of these achievements. Rodríguez-Consuegra offers an interesting discussion of Whitehead's view that mathematics and logic arise from a common basis and only distinguish themselves one from another at their more articulated regions. The one part of the book that covers an area covered by Hylton's book is a section of Chapter 1 that treats Bradley and Moore. The two treatments are rather dissimilar: for instance, whereas Hylton centres attention on the British Idealist view that analysis is falsification, Rodríguez-Consuegra claims that, according to Bradley, it is only by means of analysis that we can identify the logical form of a proposition and the constituents that instance it.

Rodríguez-Consuegra first considers Russell's early work proper in Chapter 2. Three unpublished manuscripts are examined in detail: "Analysis of Mathematical Reasoning" (1898), "Fundamental Ideas and Axioms of Mathematics" (1899), and the early draft (1899–1900) of *The Principles of Mathematics.* These represent Russell's principal efforts to develop a mathematical philosophy—a philosophical foundation for mathematics—before he met Peano at the Paris Congress of 1900. In the course of the examination, the author draws attention to the similarities between this early work of Russell and that of his antecedents considered in Chapter 1. We learn that some of Russell's positions of the period are rather incongruous with his later ones: for instance, numbers are indefinables, not items to be understood in other terms; and, echoing Whitehead, both logic and mathematics emerge from a common ground in which the whole/part relation is central. We also learn that the basis for the theory of quantity and magnitude which is the subject of Part III of *Principles* is already contained in the 1899 manuscript and, thus, was arrived at before the Paris Congress.

The work of Peano and the achievements of his associates Burali-Forti, Padoa, Pieri, and Vailati are taken up in Chapter 3. The chapter serves two purposes. First, it is part of a project, completed in the next chapter, to determine the extent to which Peano and his school influenced Russell and, more narrowly, the extent to which Russell may actually have borrowed from them. Secondly, the chapter studies Peano's work in its own right in order to refute some commonplaces about it.

Both purposes are well served. For instance, the author considers the commonplace that Frege and Peirce independently discovered the quantifier and that Peano acquired it from Frege. By devoting attention to the relevant texts and correspondence of Frege, Peano, Peirce, and Schröder, he argues (§3.1.5) that Peano likely discovered it on his own and, thus, he provides a significant re-evaluation of Peano's position in the history of logic.

Concerning the first purpose, recall that Russell says that Peano's "exposition has the inestimable merit of showing that all Arithmetic can be developed from three fundamental notions (in addition to those of general Logic) ..." (*PoM,* §120), *viz.,* zero, successor, and natural number. Rodríguez-Consuegra points out that, what's more, Peano himself in his *Formulaire de mathématiques* (1901) indicates that one can offer nominal definitions (specifications by means of equivalence classes) of these three notions as well as of the cardinal number of a class in terms of the notions of "general Logic". Rodríguez-Consuegra also observes that Burali-Forti had already arrived at nominal definitions of the notions zero, successor, and natural number in terms of those of general logic in his "Sur les différentes méthodes logiques pour la définition du nombre réel" (1900)—independently of Frege. In this paper, Burali-Forti also concluded that this way of construing these notions is superior to both definition by abstraction and definition by postulates, thus anticipating Russell's criticism of Peano's use of definition by abstraction in *Principles* (Chapter XI).

In this chapter the author also arrives at the conclusion that Pieri was the one of Peano's school who discovered the method for transforming definitions by abstraction into nominal definitions and that Burali-Forti was the first to apply the method to arithmetic: for his definition of natural number in his "Sur les différentes méthodes logiques pour la définition du nombre réel". One of the principal themes of the book is that this method was central to Russell's logicism. It was "the most important recourse that made Russell's logicism possible" (p. 131).

The chapter serves its first purpose in another obvious way. It contains a discussion of the work done by Peano, Pieri, and Vailati to axiomatize geometry—including projective geometry—to which Russell's accounts in *Principles* are much indebted.

The fourth chapter examines the work that Russell carried out after his first contact with Peano at the Paris Congress. It considers Russell's initial reaction to this contact by looking at his published and unpublished manuscripts, correspondence, and notes that he added to manuscripts written before the Congress. There is some discussion of Russell's new logic. However, the most significant part of the chapter devotes itself to evaluating the extent to which his development of the foundations of arithmetic is owing to Peano and his school.

The received view is that shortly after attending the Paris Congress, Russell arrived at his definition of the cardinal number of a class, independently of Frege, as the class of all classes similar to that class, and that this first appeared in his "Sur la logique des relations" (1901; translated as "The

Logic of Relations"). Indeed, Russell says so much in autobiographical remarks.

In Chapter 3 we saw that Burali-Forti had already arrived at nominal definitions of the notions zero, successor, and natural number in terms belonging to general logic in his "Sur les différentes méthodes logiques pour la définition du nombre réel" and that Peano indicated his own definitions of these three notions as well as of the cardinal number of a class in the *Formulaire.* Now, Rodríguez-Consuegra notes that H. C. Kennedy has claimed[1] that Peano's definition of the last notion was prompted by Russell's; even though the *Formulaire* was published before "Sur la logique des relations", Russell's paper was known to Peano beforehand. However, by appealing to earlier versions of Russell's paper, the author carefully argues that, on the contrary, Russell probably wrote his definition only after having read the papers of Burali-Forti and Peano.

The fifth and last chapter concerns itself with philosophical and methodological problems in Russell's thought that were identified in the course of the preceding chapters. It contains an interesting discussion of the origin and evolution of Russell's logicism. We find that the texts that Russell composed shortly after the Congress of 1900 do not express this position and that the first complete exposition of it appears in "Recent Work on the Principles of Mathematics" (1901; edited and reprinted as "Mathematics and the Metaphysicians" in *ML* [1918]). Rodríguez-Consuegra observes that most of the logicistic ideas expressed in this piece are already to be found in the work of Burali-Forti, Peano, and Pieri.

This chapter is the most philosophical of the book, and in it one encounters several difficulties. For instance, the author claims (p. 198 and elsewhere) that by the time Russell is writing *Principia Mathematica* he construes relations as extensionally individuated as opposed to intensionally individuated. Moreover, Russell identifies such relations with propositional functions of two arguments while abandoning classes as entities. It might be more accurate to say that in *Principia* Russell intensionally individuates propositional functions and treats both classes and relations as "logical fictions" by contextually defining class symbols in *20 and relation symbols in *21. So there is neither the extensionality nor the disanalogy between classes and relations that Rodríguez-Consuegra suggests.

Rodríguez-Consuegra also seems to claim that certain of Russell's definitions of mathematical notions *eliminate* them—e.g., "Logicist definitions ...

[1] "What Russell Learned from Peano", *Notre Dame Journal of Formal Logic,* 14 (1973): 367–72.

presented certain fundamental *concepts* of mathematics (e.g., the concept of number) near to an ontological disappearance" (p. 208)—without explaining how. Actually, Russell's definitions of the cardinal number of a class and of inductive cardinal do not eliminate these items at all. If we spell out such definitions, we find that they really contain assertions of the existence and uniqueness of the items in question whose justification implicitly appeals to comprehension axioms—whose power is effected in *Principia* by, among other things, the circumflex notation and rules of substitution. Thus, as Quine has suggested, such definitions are better called *specifications.*

On page 215, Rodríguez-Consuegra reminds us that a term according to Russell in *Principles* is anything that may be an object of thought, that may occur in a proposition, or that may be a logical subject (*PoM,* §47). Then he claims that Russell contradicts this characterization by dividing terms into two types: things and concepts. As is well known, there are many problems with the ontology of *Principles* some of which involve its notion of term. It would have been helpful if Rodríguez-Consuegra explained precisely the problem he has in mind—that is, in what the contraction consists.

One general criticism of the volume is that the author uses many terms of art which are construed differently by different authors—e.g., analysis, constructive definition, elimination, logicism, platonism, and subject–predicate proposition—without devoting any time to say what he himself intends by them. As a result, various passages are not as clear as they might be. We read of "logicist arithmetic" put forward by non-logicists. At some points, Rodríguez-Consuegra uses "subject–predicate proposition" to mean simple propositions consisting of the ascription of a singulary predicate to an individual. At others, he uses it to mean propositions whose constituents fall into different logical categories as opposed to being completely type-free. Yet, no connection is ever explicitly established between the two uses. In addition to this criticism, some may find that the author's prose in general does not come across very cleanly, although one should take into account the fact that English is not his native tongue.[2]

In *The Mathematical Philosophy of Bertrand Russell: Origins and Development,* Rodríguez-Consuegra shows that he has a deep understanding of Russell's early published and unpublished writings and a comprehensive knowledge of his larger philosophical development. The volume should be of interest to scholars of Russell's early thought.

[2] *Typographical Errors:* There are many spelling errors. "Analize" occurs repeatedly. In several places (e.g., pp. 156, 183), "recourse" should be "resource". On pages 106–7, part of a paragraph is printed twice. By contrast, text is missing between the last line of page 107 and the first of page 108.