PART VII OF *THE PRINCIPLES OF MATHEMATICS*

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This paper continues a series of studies on the relations between the manuscript of *The Principles of Mathematics* and the published text.[1] Here I examine the manuscript of Part VII on Matter and Motion. As previous studies in this series have shown, the published text of *Principles* combines materials written over a period of three or four years (1899–1902) during which Russell's views were rapidly evolving. The result is a text in which there are a number of discernible temporal layers. The text of Part VII is perhaps the most striking example of this phenomenon. It consists almost entirely of an old layer, which was Part VII of the 1899–1900 version of *Principles*, and a new layer, which was most probably written in May 1902, just before Russell sent the book off to Cambridge University Press.[2]

Russell's own recollections of the composition of Part VII are therefore, perhaps predictably, inconsistent. As we know, Russell wrote the manuscripts of Parts III–VI of *Principles* in the fall of 1900. In the

[1] Kenneth Blackwell, "Part I of *The Principles of Mathematics*", *Russell*, n.s. 4 (1984): 171–88; Michael Byrd, "Part II of *The Principles of Mathematics*", *Russell*, n.s. 7 (1987): 60–70; Byrd, "Part V of *The Principles of Mathematics*", *Russell*, n.s. 14 (1994): 47–86; Byrd, "Parts III–IV of *The Principles of Mathematics*", *Russell*, n.s. 16 (1996): 145–69; Byrd, "Part VI of *The Principles of Mathematics*", *Russell*, n.s. 19 (1999): 39–61.

[2] This view of the composition of Part VII is advanced by Ivor Grattan-Guinness in "How Did Russell Write *The Principles of Mathematics* (1903)?", *Russell*, n.s. 16 (1996): 101–27.

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Autobiography, he says that he also wrote Parts I, II, and VII "at that time" but that he had to rewrite them later (*Auto.*, I: 145). Recent scholarship shows convincingly, I think, that no manuscript of Parts I and II was written in the fall of 1900.[3] I will argue that the same is true of Part VII; parts of it were written in early 1900, prior to Russell's study of Peano, and parts were written considerably later, in the spring of 1902. In correspondence with Jourdain from 1910, Russell's recollection is different; he says that Parts I and II are "wholly later, May 1902" and that Part VII is "largely earlier". This is almost exactly right, as regards Part VII. About 50 of the 79 pages comprising Part VII predate the work done in the fall of 1900.[4]

This essay is accompanied by a collation of the manuscript with the published text. In contrast to other Parts of the text, for example, Parts I and V, the extent of the alteration between manuscript and the published text is minor. There are about 200 words of altered text for the 79-page manuscript. By comparison, there are some 3,500 words for the 200-page manuscript of Part V. I think that this contrast is explained by the fact that Russell did a substantial amount of work on Part VII in the spring of 1902. The older parts of the manuscript include many crossed-out lines and newly inserted words. There are, in addition, several pages, inserted in the old material, that recognizably postdate Russell's 1900 views in both terminology and doctrine. Finally, two chapters, 56 ("Definition of a Dynamical World") and 58 ("Absolute and Relative Motion"), seem to have been added at the later date.

The list of variants is given at the end of the essay. It is constructed in the same manner as similar lists for earlier papers in this series. The list is read as follows. At the left is a number such as 466: 12, which is read "Page 466, line 12 from the top". This is followed by the reading to be found in the first impression of the published text of *The Principles of Mathematics*. This is followed by a square bracket, then the

[3] For example, Byrd, "Part V", pp. 52–6; "Parts III–IV", pp. 151–3, and references cited therein.

[4] Some pages from earlier manuscripts also found their way into the Fall 1900 manuscripts of Parts III–VI. This is usually manifested by the occurrence of double numbers on the folios. For example, Chapter 20 ("The Range of Quantity") is largely taken from the 1899–1900 manuscript of *Principles*. See *Papers*, 3: 50–64.

corresponding reading from the printer's copy of the manuscript. Editorial braces enclose my comments.

Section 1 of this paper is devoted to a close study of the manuscript text itself. I try to establish that the text has the temporal layers indicated above; thanks to the work of Gregory H. Moore, the editor of Volume 3 of Russell's *Collected Papers*, this can be done quite precisely. In section 2, I address a question that Russell raises in Chapter 54 ("Motion") about the connection between existence, time, and change: must something exist in order to occupy a time? The manuscript and published text vacillate about the answer to the question and its importance. I will attempt to identify the forces that produce this instability. In section 3, I turn to a discussion of what Russell sometimes calls "the antinomy of causality": "Each element has an effect, but no effect apart from the whole" (*Papers*, 2: 271). In the text of *Principles*, this problem is discussed in connection with Newton's Third Law of Motion; in particular, it is concerned with the attribution of component accelerations as effects of certain forces. In the Preface to *Principles*, Russell says that this "difficulty" was one of two problems in Dynamics, which he had encountered six years before, and which had led him to a re-examination of the foundations of geometry and mathematics (*PoM*, p. xvii). I will examine why this problem seemed significant to Russell and what solution he proposed in Part VII.

1. THE MANUSCRIPT TEXT

All but the final three leaves of the manuscript of Part VII have an "M" in their upper left-hand corner; this indicates the subject of this part, Matter and Motion. As Gregory Moore points out (*Papers*, 3: 763), some of these have small periods ("M.") after them, while others do not. A variety of different kinds of evidence, presented below, support the conclusion, drawn by Moore, that the leaves marked "M." were part of the 1899–1900 version of *Principles*, and that the leaves marked "M" were not.

Like the manuscript of Parts I and II, and unlike that of Parts III–VI, the leaves of Part VII contain section numbers and printer's markings. The first section of Part VII is numbered "434"; this has been crossed out and replaced by "436"; subsequent sections are numbered

consecutively beginning with "435". Thus, the numbering of the sections is off by three from that of the published text. Russell added §§299–301 in Chapter 38 on transfinite ordinals to the text after its submission to Cambridge University Press.[5] Two leaves contain dates: folio 30, annotated "M.", bears the date "June 1900" in its upper left-hand corner; folio 76, annotated "M", is dated "May 23, 1902". This is the day on which Russell said that he completed the manuscript of *Principles*. He wrote Lucy Donnelly: "I finished today my magnum opus on the principles of mathematics, on which I have been engaged since 1897" (quoted in *Papers*, 3: xxxvi; *Auto.*, 1: 163).

The first 76 leaves are numbered consecutively. The final three leaves are numbered 1 to 3 and consist of Russell's closing summary of *Principles*. Since they refer to Appendix B, it is reasonable to think that they were written in the Fall of 1902. There are two leaves with "a" numbers: 11a and 16a. There are many pages with double numbers. They begin at folio 39, the initial leaf of the chapter on Newton's Laws of Motion. Folios 39–48, 50–5, 57–8, 69–75 contain a second set of crossed-out numbers; these numbers are, respectively, 35–44, 47–52, 53–4, and 55–9.[6] These leaves are all annotated "M." The leaves without double numbers—49, 56, and 59–68—are all annotated "M".

In these manuscripts, double-numbered leaves typically indicate that they have been extracted from some earlier manuscript of Russell's.[7] That is a reasonable presumption here, and the dating on folio 30 suggests, of course, that the material is from the 1899–1900 version of *Principles*, which Russell completed in the spring of 1900.

In the first three chapters of Part VII, no leaves have double numbers, and almost all the leaves are annotated "M." The exceptions are folios 1, 6, 11, 16a, 20–2, and 28. All leaves in Chapters 56 ("Definition

[5] Byrd, "Part V", pp. 49, 81.

[6] There are several more puzzling leaf numbers: folios 53 and 54 have three numbers. In addition to the pairs 53–4 and 50–1, there is also the pair 35–6, written between the other pairs. Russell might, at some point, have considered relocating these folios at the end of the chapter on Causality (Chap. 55). The subject matter is appropriate and it fits the numbering scheme; the last leaf of this chapter is folio 34. The other double-numbered leaf is 76; it also is numbered 69. It is the only such leaf that is annotated "M". I cannot make sense of this.

[7] For examples of this practice, see Byrd, "Part V", p. 48; "Parts III–IV", pp. 147–8; "Part VI", pp. 31–2.

of a Dynamical World") and 58 ("Absolute and Relative Motion") are annotated "M", and none have double numbers. The last leaf of the chapter on Causality (Chap. 55) is folio 34. The first double-numbered leaf is the initial leaf of Chapter 57, the chapter on Newton's Laws; its second number is 35. Folio 34 concludes with a short paragraph, which Russell crossed out in the final version. It reads: "The above very abstract discussion of causality will now be made more concrete by application to the Laws of Motion." Thus, in the 1899–1900 manuscript, Chapter 55 was immediately followed by Chapter 57.

So, it is clear that a considerable portion of the manuscript of Part VII is the manuscript of the 1899–1900 version of *Principles*. What one finds in *Papers* 3 as "Part VII" of the 1899–1900 version of *Principles* is extracted by Moore from the manuscript under discussion here.[8] It consists exclusively (and exhaustively) of the folios annotated "M.". Moore's reconstruction omits some material on these folios (e.g., the final paragraph on folio 34), especially interlinear insertions, and includes material that is crossed out on the manuscript. (The details are in the Textual Notes for this paper in *Papers*, 3: 775–7.) Moore's judgment as to what was originally in the 1899–1900 manuscript seems to me to be excellent.[9]

1A. THE OLD LAYER OF THE TEXT

The material in this earlier layer of text exhibits concerns and views characteristic of Russell's work *after* he and G. E. Moore abandoned idealism and adopted what Hylton calls "Platonic atomism", but *before*

[8] Moore's description of his procedure of extraction is found in the Textual Notes to Paper 1 of *Papers* 3 on page 763. He says that the 1899–1900 material "can be distinguished from printer's manuscript by differences of ink and handwriting, as well as by the fact that 'M.' is written in the upper left-hand corner of each folio, while no such period occurs after 'M' in the printer's manuscript."

[9] I have a minor quibble with Moore's reconstruction. On page 175 of *Papers* 3, he notes that folios 45–6 of the 1899–1900 version are missing. He also includes in brackets "<Chapter VI>"; in my judgement, there is no evidence of a missing chapter here. The chapter on Absolute and Relative Motion was not in the 1899–1900 version, and the double foliation suggests that the chapter on Hertz's Dynamics immediately succeeded the chapter on Newton's Laws. Further, Moore's suggested location for "Chapter VI" is in the middle of the discussion of the law of gravitation. I also doubt that there is a "Chapter III" as indicated on page 167.

Russell studied Peano's work. These ideas receive clear, published form in *The Philosophy of Leibniz*, which was also written in 1899–1900, and they are profitably examined by careful attention to this work.

The old layer of Chapter 53, "Matter", consists of all the published text except the first paragraph of the chapter, the second paragraph of §439, and the final section of the chapter, §441. In the 1899–1900 material, Russell's primary concern is to argue that matter is not to be characterized in terms of the traditional notion of substance, but instead in terms of certain of its characteristic relations to space and time. The traditional notion of substance is regarded by Russell as involving the view that every proposition has a subject and a predicate. Russell, of course, held this to be a ground for rejecting the traditional notion. These points are made clearly in *The Philosophy of Leibniz*, where Russell offers statements of number ("There are three men") and statements of relation as counterexamples to the thesis about subject–predicate form (*PL*, pp. 12–14).

In Chapter 54, "Motion", most of the material is from the old layer. The new material consists of §444, beginning with the word "Nevertheless", all of §445, the last paragraph of §446, and all of §447. There are two primary points of discussion in the old layer of text. First, Russell proposes and elaborates his reductive theory of change and motion. His proposal is that all there is to change is that an entity e is P at a time t_1 and is not P at another moment t_2. In particular, there is no such thing as a *state* of change or motion. This is what Russell means when he asserts, "Change, in the metaphysical sense, I do not admit at all" (*PoM*, p. 471).

Here, comparison with *Philosophy of Leibniz* is clarifying. According to Russell, Leibniz is an example of a philosopher who holds that a correct account of motion invokes *states* of motion. A state of motion is distinguished from a state of rest by the presence of force. Reversing Russell's own views on Dynamics, Leibniz holds that force is something real in things, whereas space and time are not.[10] Russell quotes

[10] Russell holds that "the notion of force is one which ought not to be introduced into the principles of Dynamics" (*PoM*, p. 474). Also, Russell regarded his own work as having removed the logical barriers to accepting the reality of absolute space and time. See, for example, *PoM*, Chap. 51, "Logical Arguments against Points". The main outlines of the argumentation in this chapter also date from late 1899 and early 1900, and are contained in "The Notion of Order and Absolute Position in Space and Time" (*Papers*, 3: 234–58), from which Russell extracted the presentation that he gave to the International Congress of Philosophy in August 1900.

Leibniz on this matter: "Force is something real, even in created substances; but space, time, and motion partake of the nature of mental entities and are true and real, not of themselves, but since they involve divine attributes" (*PL*, p. 88).

A central tenet of Russell's interpretation of Leibniz is that much of Leibniz's philosophy can be derived from a few premisses, most of which involve broadly logical issues. The first of these is the doctrine that every proposition is of subject–predicate form. Russell sees this view at work in the present case as well. According to Russell, force represents Leibniz's attempt to evade the *relations* apparently involved in motion. Force is an *intrinsic property* of individual objects; motion is a "real phenomenon" whose underlying reality is the force of individual objects (or monads) (*Leibniz*, pp. 83, 88).[11]

The second main topic of the old layer of Chapter 54 is the relation between change, time, and existence. Russell's initial characterization of change in this chapter involves no reference to existence. Consequently, it seems to allow for the idea that a non-existing, but possible, object might change. To take an example from this chapter, the stories in the *1,001 Nights* seem to involve change by Russell's definition. Nevertheless, Russell's own examples in this chapter give existence a crucial role. For example, he writes: "Thus we may say that a term changes, when it has a fixed relation to a collection of other terms, each of which exists at some part of time, while all do not exist at the same series of moments" (*PoM*, p. 470).

Russell attempts to address this matter, starting in §444, where he asks: "Can a term occupy a time without existing?" (*PoM*, p. 471). In the 1899–1900 manuscript, Russell's answer is clearly positive: "I think we must say that it can" (*Papers*, 3: 165). This, and subsequent remarks, in the manuscript are overwritten and replaced in the pub-

[11] There are further interesting contrasts that warrant discussion. According to Russell, Leibniz tries to use his views on the reality of force to resolve the problems of the continuum. Russell, on the other hand, argues that force is not to be a basic notion of Dynamics, since force is the supposed cause of acceleration, and acceleration is "a mere mathematical fiction" (*PoM*, p. 474). The latter claim is supposedly shown by Weierstrass's arithmetization of the continuum (*PoM*, p. 473).

lished text by a much less direct response: "At first sight, one is tempted to say that it can" (*PoM*, p. 471). I will address Russell's vacillation on this matter in section 2.

In Chapter 55, "Causality", the new material is §448 (the initial section of the chapter), §450, and the final paragraph of §452 (the last section of the chapter). In the old material, Russell begins with a discussion of the causal relation "as commonly understood". A crucial feature in his account is that the relata need not exist. This is said to be a basic characteristic of the causal relations dealt with in Rational Dynamics, and also in deliberation and choice. It can be non-trivially true that *A* causes *B*, even when no *A*'s or *B*'s exist. That is, there may be non-existing *A*'s and *B*'s that stand in the causal relation.[12] In *Philosophy of Leibniz*, Russell ascribes a similar view to Leibniz: "possible existents involve possible causes, and the connection between a possible cause and a possible effect is similar to that between an actual cause and an actual effect" (*PL*, p. 27).

Next, in the old layer of Chapter 55, Russell presents an "apparent" antinomy about the causation of particulars by particulars. The effects of two forces on a particle are commonly said to produce two component accelerations on the particle. But in fact neither component acceleration exists. Only the resultant acceleration does, and the component accelerations are not parts of the resultant acceleration. In *Philosophy of Leibniz*, this problem is posed as a serious and unsolved problem for all existing theories of Dynamics (*PL*, p. 98). By the time he writes the old layer of text, Russell has found a solution to this problem, a solution which he presents in Chapter 57, "Newton's Laws of Motion".

At the end of Chapter 55, Russell proposes his own, more general account of the causal relation. Russell argues later in Part VII that this more general account is required for a proper statement of the laws of Dynamics. Crucially, on this account, the causal relation may have *many* relata, not just *two*: "from a sufficient number of events at a sufficient number of moments, one or more new events at one or more new moments may be inferred" (*PoM*, p. 478).

[12] If Russell thinks that there are causal relations between non-existents and thinks that relata of causal relations have temporal location, then his views in this chapter require that he think that non-existents can have temporal location, thus settling the question posed in the previous chapter.

Here again, it is instructive to see Russell's view in the light of his interpretation of Leibniz. Russell contends that the denial of interaction between substances, characteristic of Leibniz's philosophy, reduces to the view that there are many independent causal series, not just one. Here, an independent causal series is identified as one in which the states of a given substance are caused, not by the whole preceding state of the universe, but rather by some *one* definite existent at a preceding moment, and in fact, by the previous states of *that very object* (*PL*, p. 48). Furthermore, Leibniz's contention that causality is essentially internal in this way is said to derive from the fact that Leibniz was compelled to replace the *relation* of causation by the *attribute* of activity. And again, this latter position is supposed to be a consequence of Leibniz's adherence to the subject–predicate view of propositions (*PL*, p. 45).

The new material in Chapter 57, "Newton's Laws of Motion", is quite limited, comprising just the last two paragraphs of §459, the last half of the second paragraph of §461 (beginning with the words "there should be"), and the last four sentences of the chapter (in §488, beginning with the words "The laws of motion"). In the initial half of the chapter, Russell is concerned to explain the meaning and import of Newton's laws, given his commitment to a formulation of these laws that does not treat force, velocity, and acceleration as primitive concepts. He argues that the general form which such laws should take is this: from the configuration of particles at *two* different times, the configuration of those particles at a *third* time can be inferred. The third time may be before, after or between the two original times. Furthermore, as a consequence of the law of gravitation, the positions of all particles are relevant to the specification of the relevant antecedent configurations. As Russell frequently puts it, "There is no independent system in the actual world short of the whole universe" (*PoM*, p. 496).

His analysis of Newton's laws leads Russell to negative conclusions about the notion of causality "as commonly understood". The common notion treats causation as essentially binary, and as involving succession. The notion central to Dynamics involves many events at three different times, and allows inference to earlier and intermediate states. Russell concludes that "on the whole, it is not worth while

preserving the word 'cause' ..." (*PoM*, p. 486). No such scepticism about the common notion of causation is visible in *Leibniz*.

Finally, in Chapter 57, Russell presents his resolution of the antinomy of causality. This consists in three claims, as I see it. (1) The *only* case where the antinomy arises in Dynamics is in the case of the ascription of component accelerations in the production of resultant accelerations. (2) The component accelerations are not physical facts, but are rather mathematical limits. (3) In a correct formulation of the laws of Dynamics, where accelerations play no role, there is no ascription of causality to single particles, only to configurations of them. I will return to Russell's proposed solution in section 3.

Chapter 58, "Hertz's Dynamics", was originally entitled "Other Suggested Laws of Motion". It is quite short, six manuscript leaves. The old layer of text includes the material from the beginning of the chapter, §470, through all but the last two sentences of §472. The remainder of the chapter was written later. The principal point of the old layer of the text is that Hertz's version of Newtonian dynamics still conforms to the basic points made earlier: the fundamental laws relate three configurations of particles; there is no independent system of particles short of the whole universe.

1B. THE NEW LAYER OF TEXT

We know that Russell rewrote Part I of *Principles* in May 1902. On May 16, he wrote to Alys: "I expect to have my book quite finished in another two months ..." (*SLBR*, 1: 234). Just a week later, the book was finished. On the 24th, Russell wrote to Alys: "Thee will be surprised and amused, after all my talk of two months, to hear that I finished my book yesterday. I found that a pile of old MS., which I had expected to have to re-write, required only a few additions and corrections, so I arrived at a sudden termination" (*ibid.*, 1: 236). I think it is clear that the "corrections" and "additions" to which Russell refers are the new layer of Part VII.[13]

As noted earlier, folio 76, which is the final folio of Part VII except

[13] I don't intend for this claim to rule out alterations to Parts II–VI in May 1902, although I can find no evidence that these parts were altered substantially at this time.

for the summary sections, is dated "May 23, 1902". This is a folio annotated "M". On folio 66, in the chapter on Absolute and Relative Motion,[14] Russell footnotes his review of Couturat's work on Leibniz's logic: "See my review of *Couturat*, La Logique de Leibniz, in Mind, October 1902." We know from correspondence that Russell completed this review in March 1902 (*Papers*, 4: 535). However, Russell's reference to this review is, in fact, incorrect. The review did not appear until 1903, and this is corrected in the published text. (See the List of Variants, 492: 44.) So Russell wrote the material in this section after March 1902, and prior to sending it to the printer in late May. The only time during this period in which Russell is known to have worked on *Principles* is during the first three weeks of May.[15]

The folios marked "M" also contain terminology that indicates that they were composed quite late in the writing of *Principles*. One striking example is on folio 6. Here Russell writes: "For us, however, it is sufficient to observe that all unities are propositions or propositional concepts, and that consequently nothing that exists is a unity" (*PoM*, p. 467). The notion of a "propositional concept" first appears in the May 1902 version of Part I. Russell introduces it in Chapter 3 to label the distinction between a proposition asserted ("*A* is greater that *B*") and a proposition unasserted ("*A*'s being greater than *B*"). The latter is said to be "a proposition considered as a complex concept". This distinction is then used to sharply pose Lewis Carroll's famous conundrum about the nature of rules of inference. This question is not considered in the May 1901 version of Part I.

A second example of late terminology occurs on folio 16a. Here Russell restates the reductive theory of change in terms of "propositional functions": "From the mathematical point of view, change arises from the fact that there are propositional functions which are true of some but not all moments of time ..." (*PoM*, p. 472). The notion of a propositional function first occurs in the May 1902 version of Part I of *Principles*, where it is treated as one of the fundamental notions of logic. In manuscripts preceding this one, Russell standardly used the notion of a "proposition containing vari-

[14] All folios in this chapter are annotated "M".

[15] The early part of this period is one of considerable marital turmoil for Russell and Alys. In mid-April, Alys went to Brighton for a "rest cure".

ables" to express roughly the same idea.

Let me turn now to the subject matter of the new layer of text. It is not surprising that the logicist treatment of Rational Dynamics is one main theme of this new layer.[16] At a number of places in Part VII, Russell puts forward the view that the concepts of Rational Dynamics, such as matter and motion, can be expressed in terms that use only the logical constants. For example, a material universe is characterized as a class of many-one relations, the domains of which relations are a common one-dimensional series and the ranges of which are a common three-dimensional series, satisfying the condition that the intersection of any two such relations is empty (*PoM*, p. 468). Russell's treatment of Dynamics parallels his treatment of geometry in Part VI. Just as he defined "Euclidean space" or "projective space" in purely logical terms by replacing primitive concepts such as "point" and "line" by class variables, in Part VII he defines a certain kind of "Dynamical World" by replacing basic concepts, such as "material atom", by logically specifiable analogues; in this case, a many-one relation of a certain kind.

Russell emphasizes that the definition of a Dynamic World does not require that there be any actually existing entities satisfying the structure so defined. Furthermore, there are coherent, incompatible definitions. Thus, according to Russell, Rational Dynamics ought to investigate non-Newtonian Dynamics, just as Geometry includes the study of non-Euclidean Geometry. Russell draws the same conclusion about Rational Dynamics that he drew in Part VI about Geometry: "The à priori truths involved in Dynamics are only those of Logic: as a system of deductive reasoning, Dynamics requires nothing further, while as a science of what exists, it requires experiment and observation" (*PoM*, p. 488).

Russell's treatment of Dynamics is thus broadly within the spirit of

[16] A logicist thesis about Rational Dynamics is stated at the following places in Part VII: §441, the final paragraph of §446, the last paragraph of §452, Chapter 56, the last half of §462, and the summary section, §473. All these are on folios labelled "M", with the exception of the last paragraph of §452. This paragraph occurs on folio 34, where Russell has crossed out the last paragraph of Chapter 55 ("Causality"), and inserted a new paragraph that introduces the newly written Chapter 56 ("Definition of a Dynamical World").

what Alberto Coffa called *conditional logicism*.[17] The purely mathematical part of geometry involves conditionals, the antecedents of which are the axioms of the appropriate branch of geometry, treated as defining a kind of space, and the consequents of which are the theorems of the relevant kind of geometry. Pure Dynamics is conceived similarly; its à priori component is a system of deductive reasoning. The premisses in this reasoning are the laws of motion, conceived as parts of a definition of a class of material universes. Russell is somewhat less clear about what the conclusions are in this case. One kind of conclusion that he certainly seems to have in mind consists of conditionals relating configurations of particles at certain times to their configuration at other times.

A standard criticism of conditional logicism is that it is too broadly applicable, and hence perhaps trivial. Here is how Landini has recently put this point:

> Whenever intuitions (empirical or otherwise) governing the subject matter of a given field seem needed for inferences, simply render a first-order axiomatization of the field and then all the inferences will thereby be "reducible" to pure logic. We have pure physics, pure biology, pure geography![18]

Coffa makes a similar point in "Russell and Kant", but he then mounts an historically based defence of Russell's conditional logicism in the case of Geometry.[19]

I think that a similar, historically based defence can be given of Russell's conditional logicism in the case of Dynamics. While "pure geography" would no doubt have seem peculiar to Russell, "pure physics" is an idea with which he would have been familiar. For Kant in fact deploys such a notion; in a footnote at B21 of the *Critique*, Kant writes:

> Many may still have doubts as regards pure natural science. We have only, however, to consider the various propositions to be found at the beginning of (empirical) physics, properly so called, those, for instance, relating to the

[17] Alberto Coffa, "Russell and Kant", *Synthese*, 46 (1981): 247–63.

[18] Gregory Landini, *Russell's Hidden Substitutional Theory* (London and New York: Oxford U.P, 1998), p. 20.

[19] Coffa, "Russell and Kant", pp. 252–5.

permanence of the quantity of matter, to inertia, to the equality of action and reaction etc., in order to be soon convinced that they constitute a *physica pura*, or *rationalis*, which well deserves, as an independent science, to be separately dealt with in its whole extent, be that narrow or wide.[20]

Specifically, as regards motion, Kant held that it gives rise to a significant body of à priori knowledge; at B49, he writes: "Thus our concept of time explains the possibility of that body of à priori synthetic knowledge which is exhibited in the general doctrine of motion...." Russell opposes this *Kantian* conception of pure physics according to which there are à priori, non-logical features of motion, and according to which Newton's laws are non-logical à priori truths. Russell's view is that *all* there is to "pure physics" is logic. In my view, it is clearly plausible in this case to give such a claim an eliminativist reading.

In the case of geometry, Coffa and Michael Friedman have both argued that Russell's criticism of Kant is directed at Kant's view that *reasoning* in geometry, while à priori correct, requires appeal to a non-logical element, construction in pure intuition.[21] Friedman also argues that features of *motion*, such as its continuity, play a role in Kant's interpretation of argumentation in calculus. Since the reasoning in mathematics is à priori, Kant holds that the correctness of this reasoning depends on à priori features of motion. Russell, on the contrary, holds that the arithmetization of calculus shows that motion plays no role in argumentation in calculus. In the new layer of text, Russell pointedly remarks that the claim that motion is continuous is "an entirely new assumption, having no kind of necessity" (*PoM*, p. 473). So, Russell's "pure Dynamics" is best seen as a well-grounded response to Kant's conception that "pure physics" is a substantial subject underlying significant mathematical reasoning.

Aside from the exposition of logicist ideas, the new layer of text consists of modest "additions" and "corrections". For example, at the end of §436, Russell adds a new paragraph in which he discusses the

[20] Kant, *Critique of Pure Reason*, trans. Norman Kemp Smith (New York: St. Martin's Press, 1965), p. 56.

[21] Michael Friedman, *Kant and the Exact Sciences* (Cambridge, Mass.: Harvard U.P., 1992), Chap. 1. Russell states this point clearly in §434 of *Principles*. Kant expresses the view in question at A715–17, B743–5, quoted in Friedman, pp. 56–7.

fact, emphasized in the May 1902 version of Part 1, that propositions are unities, not simply aggregates of their constituents. In §444, he inserts two new paragraphs, in which he attempts to clarify his view on whether an object can occupy a time without existing. In §461, he inserts a new half-paragraph correcting his mathematical statement of the idea that approximate calculation of local effects is possible in a universe governed by the law of gravitation.

Chapter 58, "Absolute and Relative Motion", is something of an anomaly. It belongs to the new layer of text, although its subject matter is appropriate to the old layer. In this chapter, Russell considers Newton's empirical arguments for the existence of absolute motion, as well as several nineteenth-century attempts to evade Newton's conclusions. Russell holds that Newton's arguments are unrefuted and that the evasions are flawed. These are views that Russell held while writing the 1899–1900 version of *Principles* (see *Papers*, 3: 143). It is thus unclear to me why this material was added only at the later date.

2. CHANGE, TIME, AND EXISTENCE

In the Preface to *Principles*, Russell says that his fundamental philosophical views are derived from Moore's. One such view is "the non-existential nature of propositions (except such as happen to assert existence) and their independence of any knowing mind" (*PoM*, p. xviii). The "existential theory of judgment", which Russell opposes, holds that every proposition is concerned with something that exists (*PoM*, pp. 449–50). At numerous places in Part VII, Russell uses his "non-existential" conception in connection with Rational Dynamics. In fact, Russell says that it is characteristic of Rational Dynamics that its propositions involve objects that do not exist. For example, in a statement from the old layer of text, Russell writes:

> In all Dynamics (as I shall prove later) we work with causal connections; yet, except when applied to concrete cases, our terms are not existents. Their non-existence is, in fact, the mark of what is called Rational Dynamics. (*PoM*, p. 475)

Similar ideas are found in the new layer of text (see *PoM*, p. 493).

Russell uses the non-existential account of propositions to do philo-

sophical work in Part VII. For example, in the chapter on causality, he notes the subjunctive character of standard causal claims. When it is said that *A* causes *B*, "neither *A* nor *B* need ever exist, though if *A* *should* exist at any moment, *B* *must* exist at a subsequent moment ..." (*PoM*, p. 475; emphasis mine). Russell's way of accommodating this subjunctive element is to allow the relevant conditionals to apply to concrete, but non-existing, objects. Russell also argues that, in decision-making, the alternatives that we consider must represent "valid", but non-existent, causal chains.

So, it is not surprising that when Russell comes to characterize basic notions, such as change and motion, he gives characterizations that do not assume existence. His reductive characterization of change illustrates this point. Russell writes:

> Change is the difference, in respect of truth or falsehood, between a proposition concerning an entity and time *T* and a proposition concerning the same entity and another time *T'*, provided the two propositions differ only by the fact that *T* occurs in one where *T'* occurs in the other. (*PoM*, p. 469)

Since, on the non-existential view of propositions, true propositions need not involve existent objects, this characterization *seems* clearly to allow that non-existent objects may change.

Russell addresses this matter directly in §444 of the published text. He asks: "Can a term occupy a time without existing?" (One assumes that Russell means "Can a term occupy a time *t* without existing at *t*?") Russell's account of causality, with its reliance on non-existent occurrences, demands a positive answer. However, Russell's answer in the published text is quite guarded and indirect. At the end of the section, he says: "non-existential occupation of a time, *if possible at all*, is radically different from the existential kind of occupation" (*PoM*, p. 472; emphasis mine). In the rest of this section, I want to examine Russell's response at this point, with the aim of clarifying what is behind this hedged conclusion.

First, it is worth noting that §444 contains both old and new material. The manuscript pages of this section are heavily edited, and there is a newly inserted leaf, folio 16a. This folio contains all but the first six sentences of the section. So much of what appears in the published text of §444 was written by Russell in May 1902, "correcting"

material to be found in the 1899–1900 manuscript of Part VII.

The "corrections" are significant. In the 1899–1900 manuscript, Russell's answer to the main question ("Can a term occupy a time without existing?") is straightforward, unhedged, and predictable: "I think that we must say that it can" (*Papers*, 3: 165). After giving an argument to which we shall turn shortly, Russell concludes the paragraph:

> And we can now abandon all reference to existence in the notion of change; there is change in *Waverley* and in the *1001 Nights*, though there is no existence. Only the occupation of time, not the existence is essential. (*Papers*, 3: 166)[22]

To see why Russell retreats from this direct answer, one needs to look at the consequences he draws from this answer in the 1899–1900 manuscript. There he writes:

> "*A* exists now" is, as it stands, a relation between *A*, existence, and the present time. But if *A* may occupy the present time without existing, that shows the proposition to be analyzable into "*A* is an existent" and "*A* is now", neither of which implies the other. (*Papers*, 3: 165)

In the margin of the manuscript is the remark, "This is wrong." What does Russell take to be wrong here? The published text contends that the conditional in the second sentence is false. Russell carefully argues that given the view that occupation of a time does not imply existence, the proposition "*A* exists now" is *not* analyzable as the conjunction of the propositions "*A* is an existent" and "*A* is now".[23] Russell's point is correct. Suppose that occupation of a time is separable from existence; then *A* may exist at some time and yet occupy other moments when it does not exist. So both "*A* is an existent" and "*A* is now" can be true, although "*A* exists now" is false. He thus draws precisely the opposite conclusion about analyzability from the one drawn in the 1899–1900 manuscript.

[22] In the manuscript, §444 (numbered 441) is on folios 16 and 17. All material after the first six sentences is crossed out, and folio 16a is inserted between folios 16 and 17.

[23] In the published text, Russell replaces "*A* is an existent" with "*A* exists," with the gloss that "exists" is to be understood tenselessly.

In the published text, Russell in fact draws the stronger conclusion that "*A* exists now" is not analyzable at all. It involves, he says, an "ultimate" three-term relation, one between *A*, existence, and the present time. By way of contrast, "*A* is now" involves a two-term relation between *A* and the present time. On this basis, Russell concludes that being at a time and existing at a time are "radically different".

Several points should be made about Russell's claims here. First, the fact that existential occupation cannot be analyzed in the simple way that Russell considers does not of itself show that it is unanalyzable. (We would need a clearer conception of the resources permissible in Russellian definitions to go further here.) Second, the fact that existential occupation is not analyzable in this way is not an argument against the intelligibility, or reality, of non-existential occupation. Russell's argument shows that the conditional connecting the intelligibility of non-existential occupation and the analyzability of existential occupation is false. So, there can be no inference on this basis from the unanalyzability of existential occupation to the incoherence of non-existential occupation. And Russell clearly does not withdraw his commitment to non-existential occupation, though his endorsement becomes restrained, in contrast to the 1899–1900 manuscript.

The unanalyzability of existential occupation leaves Russell with a problem that he addresses in the last paragraph of §444. The problem is that Russell thinks the *same* laws of Dynamics apply to both existing and non-existing objects. What form do these laws take? Russell holds that these laws say that from the configuration of particles at two times, we can infer the configuration of these particles at a third time. So, a simple-minded such law might be: if a particle p_1 occupies point s_1 at t_1 and particle p_2 occupies point s_2 at t_1, and if particle p_1 occupies point s_3 at t_2 and particle p_2 occupies point s_4 at t_2, then particle p_1 occupies point s_5 at t_3 and particle p_2 occupies point s_6 at t_3. This law is supposed to apply both to the case where the particles exist at the times in question and to the case where they do not. Further, given the nature of the causal relation, if these particles exist at the earlier times, then they must exist at the later times.

The problem is that, according to Russell, existential occupation is "radically different" from non-existential occupation; so how can the same laws apply to both? The law stated above appears to have the

quantificational form:

O. $(\forall p)(\forall q)((O(p, s_1, t_1) \,\&\, O(q, s_2, t_1) \,\&\, O(p, s_3, t_2) \,\&\, O(q, s_4, t_2)) \supset (O(p, s_5, t_3) \,\&\, O(q, s_6, t_3)))$

What predicate is "*O*"? It apparently has no explicit place for "existence"; so it seems that "*O*" is the non-existential spatio-temporal occupation relation; that is, the three-place relation, including a spatial relata, that corresponds to the two-place non-existential temporal occupation relation.[24]

How does this law apply to the case when the particles exist at the relevant spaces and times? In this case, according to Russell, we are dealing with an unanalyzable four-place relation between particles, points, instants and existence. Let us represent this relation by the four-place predicate "$EO(p, s, t, e)$", where "*e*" names existence. Then the existential analogue of the earlier law is:

EO. $(\forall p)(\forall q)((EO(p, s_1, t_1, e) \,\&\, EO(q, s_2, t_1, e) \,\&\, EO(p, s_3, t_2, e) \,\&\, EO(q, s_4, t_2, e)) \supset (EO(p, s_5, t_3, e) \,\&\, EO(q, s_6, t_3, e)))$

It is clear that O does not logically imply EO and that EO does not logically imply O.

This contrasts with what would have been the case if existential occupation had been analyzable in the way Russell originally stated. For in that case, the existential law would have had the form:

EO*. $(\forall p)(\forall q)((O(p, s_1, t_1) \,\&\, p \text{ is an existent} \,\&\, O(q, s_2, t_1) \,\&\, q \text{ is an existent} \,\&\, O(p, s_3, t_2) \,\&\, p \text{ is an existent} \,\&\, O(q, s_4, t_2) \,\&\, q \text{ is an existent}) \supset (O(p, s_5, t_3) \,\&\, p \text{ is an existent} \,\&\, O(q, s_6, t_3) \,\&\, q \text{ is an existent}))$

EO* follows by logic from O. The very same law O applies to both the existential and the non-existential cases.

In the last paragraph of §444 (written in May 1902), Russell tries to

[24] The problems to be described would also arise if we took the existential form of the law (EO) as basic, and tried to derive (O).

close the gap between O and EO. He writes:

> For existence, being a constant term, need not be mentioned, from a mathematical point of view, in defining the moments occupied by a term. From the mathematical point of view, change arises from the fact that there are propositional functions which are true of some but not all moments of time, and if these involve existence, that is a further point with which mathematics as such need not concern itself. (*PoM*, p. 472)

How does this succeed in restoring the proper relation between O and EO? It is true that "e" occurs as a constant term in the antecedent and consequent of EO. So, for certain purposes, we might legitimately rewrite EO as follows:

EO′. $(\forall p)(\forall q)((EO'(p, s_1, t_1) \,\&\, EO'(q, s_2, t_1) \,\&\, EO'(p, s_3, t_2) \,\&\, EO'(q, s_4, t_2)) \supset (EO'(p, s_5, t_3) \,\&\, EO'(q, s_6, t_3)))$

where the new three-place predicate $EO'(p, s, t)$ is defined as $EO(p, s, t, e)$. But this does not reduce EO′ (and hence EO) to O; for the predicate "EO'" is certainly not equivalent, by Russell's lights, to "O".

Russell says that, "from a *mathematical* point of view", existence need not be "mentioned". Russell's thought seems to be that if existence is "not mentioned", then we are left with non-existential spatiotemporal occupation. This picture would surely be appropriate if existential occupation had been analyzable; for then if we dropped "A is an existent" from the conjunction "A is now and A is an existent", we would be left with non-existential occupation. Given Russell's conclusion that existential occupation is unanalyzable, it is less clear how this result is supposed to follow. What is clear is that Russell's account only works if it is assumed that existential occupation entails non-existential occupation:

EO ⊃ O. $(\forall p)(\forall s)(\forall t)(EO(p, s, t, e) \supset O(p, s, t))$

This principle is not a *logical* principle, since it employs essentially a concept, *existence*, that is not, on Russell's account, a logical constant. It has all the marks of what Russell might call an "ultimate" metaphysical principle.

Given the principle (EO ⊃ O), Russell can make appreciable headway in uniting existential and non-existential instances of the laws of Dynamics. Starting from the law O; we can use (EO ⊃ O) to prove:

EO1. $(\forall p)(\forall q)((EO(p, s_1, t_1, e) \,\&\, EO(q, s_2, t_1, e) \,\&\, EO(p, s_3, t_2, e) \,\&\, EO(q, s_4, t_2, e)) \supset (O(p, s_5, t_3) \,\&\, O(q, s_6, t_3)))$

But this is not quite all that the existential form of the law asserts. In its consequent, the law EO asserts not just $O(p, s_5, t_3)$, but the stronger $EO(p, s_5, t_3, e)$. That EO makes the stronger claim is a consequence of a feature of the causal relation carefully noted by Russell. In the chapter on causality, Russell writes:

> A causal relation, we have seen, has no essential reference to existence, as to particular parts of time. But it has, none the less, some kind of connection with both. If one of its terms is among existents, so is the other; if one is non-existent, the other is also non-existent. (*PoM*, p. 476)

So when the relevant particles existentially occupy certain locations at certain times, then the causal law requires that they existentially occupy certain locations at a later time. However, the consequent of EO1 is weaker, according to Russell, than the consequent of EO. (Of course, positing the converse of (EO⊃O) would be unacceptable to Russell.) So, even granting the postulate (EO⊃O), Russell has not fully attained the desired unification.[25]

[25] The problems here seem to me to stem from (1) Russell's recognition that laws apply to non-actual, but possible, cases coupled with (2) Russell's attempt to represent possibility via the simple truth or falsity of propositions about these non-actual objects. (Note Russell's remarks about necessity at *PoM*, p. 454.) Nino Cocchiarella holds that *Principles* contains the resources for reconstruction of a possible worlds account of modality. (See his "Meinong Reconstructed versus Early Russell Reconstructed", in his *Logical Studies in Early Analytic Philosophy* [Columbus, Ohio: Ohio State U.P., 1987], p. 120.) If that were so, it would be easy to devise a world-relativized version of the laws of Dynamics that would resolve the problems here discussed.

3. THE ANTINOMY OF CAUSALITY

As noted earlier, Russell says, in the Preface to *Principles*, that the book had its origins in two problems that he had encountered in Dynamics. One of these involved absolute motion and the relational theory of space. The other is the problem that Russell sometimes refers to as the "antinomy of causality". The simple statement of the antinomy found in the Preface is this: "when a particle is subject to several forces, no one of the component accelerations actually occurs, but only the resultant acceleration of which they are not parts ..." (*PoM*, pp. xvi–xvii). In the Synoptic Table of Contents for his manuscript "The Fundamental Ideas and Axioms of Mathematics" (1899), Russell states the problem more abstractly as follows: "Each element has an effect, but no effect can be asserted apart from the whole. *Illustration* from the compounding of accelerations" (*Papers*, 2: 271; emphasis mine). In this final section, I will examine this antinomy, why Russell takes it to be important, and the solution that he offers to it in *Principles*.

Russell says that these two problems first impressed themselves on him six years earlier. This would be in 1896 or 1897. However, as Griffin notes, there is nothing in Russell's work, published or unpublished, from these years that is directly about this antinomy.[26] In later writing, Russell typically introduces the antinomy in connection with Newton's Third Law of Motion. But his discussions of the third law during 1896 and 1897 give no indication that its interpretation is philosophically problematic in some way.[27]

The first substantial discussion of this issue is in the unpublished note "On Causality as Used in Dynamics", which is dated "March 1898". It is thus written at the same time that Russell is beginning to work on the manuscript of "An Analysis of Mathematical Reasoning",

[26] Nicholas Griffin, *Russell's Idealist Apprenticeship* (Oxford: Clarendon P., 1991), pp. 221–3.

[27] See *Papers*, 2: 15, 90, for examples. The latter occurs in a note "Motion in a Plenum" from 1897. What Russell says is that, on the plenal theory of matter, "the further insight into the nature of force, afforded by the third law, seems to fail us here." One might see this as a recognition that the third law posits a kind of "causal atomism" that the plenal theory renders problematic.

which was written during the spring and early summer of 1898.[28] In this note, the problem is presented in its specialized form, as a problem about the interpretation of Newton's Third Law. Here is how Russell states the problem:

> Dynamics starts with the view that each particle of matter can be isolated, and has a behaviour natural to it when isolated—namely, uniform rectilinear motion. It then discovers that no particle *is* isolated, and it endeavours to state the effect of outside matter as the sum of effects of separable pieces; every particle, on this view, has on every other particle a definite effect, and the resultant behaviour of the particle in question is got by summing these partial effects. But this summation is algebraical; i.e. it results in a single effect of the same kind as the supposed partial effects, and not containing these partial effects as part of its content. No single one of the partial effects is validly predicable of it. (*Papers*, 2: 108)

It is significant, I think, that Russell does not, in this note, describe this as an "antinomy" or "paradox" in Dynamics. It is rather a "difficulty" that needs to be "carefully examined". Russell sets forth three approaches to the problem without unequivocally endorsing any of them.[29]

Russell's discussion of the second of the three approaches is, I think, especially revealing. This is a solution that Russell puts in the mouth of "the mathematicians". On this view, Russell's problem is simply the result of conflating *mathematical* and *logical* addition. The idea is that "the separate effects *are* simply added, ..., but the addition is mathematical, not logical" (*Papers*, 2: 119). How is this distinction supposed to help? The idea is that in mathematical addition, "we need not expect to find [the effects] a and b separately as marks of the effect, since they have been merged in a single quantity which is their sum" (*ibid.*). In the case of the addition of accelerations, for example,

[28] For the dating of "An Analysis of Mathematical Reasoning", see Griffin's discussion in *Papers*, 2: 155–61.

[29] In his introduction to this note, Griffin suggests that it is "most likely" that Russell is satisfied by the interpretation put forward in the last paragraph of the note. It is true that Russell does not set out objections to this interpretation as he does in the case of the earlier two approaches. But it also seems to me that the third interpretation is clearly open to the same objection that Russell makes to the interpretation proposed in the second paragraph of the note.

the addition is addition of vectors, and the vector sum does not contain the vector addends as parts or components. This is in contrast to the way that the logical sum (or union) of two classes contain the original classes as parts.

Russell does not dismiss this approach. He clearly accepts the distinction between mathematical and logical addition as tenable. In fact, he sees it as a manifestation of "the fundamental cleft" between mathematical reasoning and logical reasoning! (*Papers*, 2: 110). But Russell clearly does not see the distinction as sufficient of itself to resolve the issues raised. The problem is simply transformed into a question about the relation between a mathematical sum and the terms whose sum it is. The question is this: are the terms "preserved in the sum" in *any* way that will allow the attribution of separate effects to the separate particles?

Russell here anticipates and answers an objection to his treatment of this problem that is raised by Ernest Nagel in *The Structure of Science*. Nagel quotes Russell's initial presentation of the "antinomy of causality" in *Principles* (§451), and concludes that the problem is the product of a simple confusion about addition:

> However, all this argument shows is that by the component of a force (or of an acceleration) we do not mean anything like what we understand by a component or part of a length—the components of forces are not *spatial parts* of forces.[30]

Nagel's point is that it would be a mistake about the nature of vector addition to suppose that the component effects have to be *parts*, in some straightforward sense, of the resultant effect. But as the 1898 note shows, Russell understood this point perfectly well. He simply did not think that the problem was resolved by drawing this distinction. The problem is not simply that the component effects are not *parts* of the resultant effect; it is that it appears that the component effects are *nonexistent*.[31]

[30] Nagel, *The Structure of Science*, 2nd ed. (Indianapolis: Hackett, 1979), p. 386.

[31] Nagel seems to concede this point, but he then, in my view simply asserts that the result is unproblematic. He writes: "Moreover, no antinomy arises from the supposition that, on the one hand, the effect of each component force acting alone does not exist, while on the other hand the actual effect produced by the joint action of the components is the resultant of the partial effects. For the supposition simply expresses what is the case, in a language conforming to the antecedent *definition* of the addition and resolution of forces" (*The Structure of Science*, p. 387).

The "antinomy of causality" first appears in print in *Philosophy of Leibniz*. The status of the problem has now been elevated to that of a "principal difficulty", which "no existing theory of Dynamics can avoid" (*PL*, p. 98). Russell offers no indication of approaches to the problem, nor does he give any indication of his preferred mode of resolution. This is striking, since his writing of *Leibniz* is roughly contemporaneous with his drafting of the 1899–1900 manuscript. Moreover, Russell was not shy about stating his own philosophical opinions in *Leibniz*.[32]

I turn finally to the resolution that Russell offers to the problem in *Principles*. I shouldn't really say "the" resolution. For I think that there are two quite distinct aspects to Russell's approach. One is a "local" solution directed specifically to the interpretation of composition of forces in Dynamics. There is also the suggestion of a strategy to apply in the face of similar problems elsewhere.

The local solution depends on what Russell takes to be the consequences for Dynamics of the arithmetization of calculus. In a section from the new layer of text added at the end of Chapter 54, Russell writes:[33]

> It is to be observed that, in consequence of the denial of the infinitesimal, and in consequence of the allied purely technical view of the derivative of a function, we must entirely reject the notion of a *state* of motion.... There is no transition from place to place, no consecutive moment or consecutive position, and no such thing as velocity except in the sense of a real number which is the limit of a certain set of quotients. (*PoM*, p. 473)

[32] Russell's preferred solution in *Principles* depends on the contention that quantities represented by derivatives, such as instantaneous velocity, are to be thought of as limits of series, not as fractions dx/dt. However, in the 1899–1900 manuscript, Russell also held that the notion of a limit depends on the notion of the completed infinite and Cantor's transfinite cardinal numbers. But the latter are held by Russell to be incoherent. (See *Papers*, 3: 115, 123–5.) Perhaps this might explain Russell's reticence in *Leibniz*.

[33] This view is also expressed clearly in the old layer of text in the first paragraph of Chapter 57.

Russell holds that neither instantaneous velocity nor instantaneous acceleration are physical quantities, because they are standardly represented as mathematical limits. He infers that, in a proper statement of the laws of motion, there should be no reference to acceleration and velocity; instead, the only notions to be used are those that do express true physical quantities. Thus the only remaining notions are those that pertain to the masses of particles and their spatiotemporal configurations. So, the laws of Dynamics must be expressed solely in these terms. The resulting laws attribute no independent causal effects to particular particles; no law says that the spatial location of a single particle at a particular time implies that that particle, or another particle, will have a certain location at a later time. Rather the laws relate the configurations of many particles at two times to the configuration of many particles at a some other time. As Russell says, in such laws "nothing can be attributed to particular particles".[34]

This solution depends on Russell's views about velocity and acceleration. Consequently, this solution would not directly apply to other cases where we might be think that (*a*) physical laws attribute some sort of causal powers to individual objects, but (*b*) when several such objects interact in accordance with these laws, the resultant effect does not, in any obvious way, "contain" the effects of the causal powers of the individual objects. Laws governing attraction and repulsion of electrically charged particles, such as Coulomb's Law, are examples of this type.

I think that Russell also has a more "global" approach in mind, one that does not depend on the details of his analysis of the composition of forces in Dynamics. Russell sometimes presents his resolution of the problem as one in which he is replacing the idea that a causal law relates one particular to another by a more general one in which groups of particulars are related to other groups[35] (*PoM*, p. 477, last

[34] *PoM*, p. 487. I do not endorse Russell's argument here; I don't think that the fact that instantaneous accelerations can be represented mathematically as limits shows, or is even strong evidence for, the claim that they are not physically real quantities.

[35] Griffin gives a similar description of the global strategy in *Russell's Idealist Apprenticeship*: "In POM the problem is resolved by a radical revision of the concept of causality" (p. 223).

paragraph). The "particular to particular" conception regards dynamic situations atomistically: force A_1 causes acceleration B_1. The problem that then arises is that, in typical situations, there are many interacting forces A_2, A_3, ..., and many "component accelerations" B_2, B_3, ..., produced. But then the puzzle is that, by Newton's third law, none of B_2, B_3, ..., is really produced, but rather a resultant acceleration C. This certainly seems to make the initial "law", that force A_1 causes acceleration B_1, false. For force A_1 exists, but acceleration B_1 does not.

Russell's resolution is to replace the initial atomistic conception of causal laws with a "group to group" conception which takes into account the interaction effects of the initial forces. Now, of course, Russell does not state his resolution in terms of forces and accelerations. His description is in terms of *configurations* of groups of particles. But I think it important that the success of this resolution does not hinge on the elimination of forces, velocities, and accelerations. Russell's point could be implemented by reframing laws for forces and accelerations at the "group to group" level: such laws would relate collections of forces to collections of "resultant" accelerations.

This strategy is generalizable to other areas. If we are dealing with the interaction effects of several charged particles, we reframe the laws to a form designed to take into account the interactions. Similarly, this strategy appears to dictate that if we are dealing with the "total" effects of gravitational and electrical forces on particles, we should try to produce some kind of integrated law that describes the "resultant effect" of both kinds of forces.

Does this "global" strategy successfully address the antinomy from which the discussion began? Let me quote Russell's exposition of the antinomy in *Principles*:

> Thus the effects attributed to B and C are never produced, but a third term different from either is produced. This, we may say, is produced by B and C together, taken as one whole. But the effect which they produce as a whole *can only be discovered* by supposing each to produce a separate effect: if this were not supposed, it *would be impossible to obtain* two accelerations whose resultant is the actual acceleration. Thus we seem to reach an antinomy: the whole has no effect except that which results from the parts, but the effects of the parts are non-existent. (*PoM*, p. 477; emphasis mine)

The "global" strategy replaces simple, but false, "particular to particu-

lar", laws by "group to group" laws which correctly describe the actual properties of objects, but which will, in the end, be enormously complicated. In his statement of the puzzle, Russell emphasizes, correctly, the *epistemic* cost of the strategy. We *understand* and *explain* complex situations by seeing them as produced in some systematic way from simple ones. If, in the concern to have laws that state the facts as they actually are, we simply replace the simpler laws, the cost is that we no longer have access to this familiar kind of compositional way of understanding complex phenomena.

So, a full resolution of the problem would require addressing these epistemic concerns. Russell does recognize the need to do this; much of §461 concerns how the use of approximations might be justified in the case of the law of gravitation. But Russell's discussion concerns a very simple kind of approximation, and he does not address the problem in its full dimensions.

What is important, I think, is that Russell is here confronting a problem of substance. In *The Structure of Science*, Nagel says that Russell's point is "terminological at best" (p. 386). I disagree with this assessment. The point at issue concerns the tension between the factuality of the laws of nature and their use in explanation. This point is emphasized in work by Nancy Cartwright that is strikingly reminiscent of Russell's in the way it poses the problem, though not in the solution suggested. In her essay "Do the Laws of Physics State the Facts?", she considers the interaction of gravitation and electrical forces on a given particle. To the proposal that the combined effect of gravitational and electrical forces is simply the vector sum of the two forces, Cartwright gives the following response:

> The vector addition story is, I admit, a nice one. But it is just a metaphor. *We* add forces (or the numbers that represent forces) when we do calculations. Nature does not "add" forces. For the "component" forces are not there in any but a metaphorical sense, to be added; and the laws that say they are there must also be given a metaphorical reading.[36]

[36] Nancy Cartwright, *How the Laws of Physics Lie* (Oxford: Oxford U.P., 1983), p. 59. For what is in essence the same sort of problem in biology, see Elliott Sober and David Sloan Wilson, *Unto Others: the Evolution and Psychology of Unselfish Behaviour* (Cambridge, Mass.: Harvard U.P., 1998), esp. pp. 31–5.

Cartwright's resolution is not to replace the simpler laws by complex "super laws", but to give up the facticity of the fundamental laws:

> The lesson to be learned is that the laws that explain by the composition of causes fail to satisfy the facticity requirement. If the laws of physics are to explain how the phenomena are brought about, they cannot state the facts. (P. 73)

It is characteristic of Russell to have noticed, and seen the significance of, this problem.

VARIANTS BETWEEN *The Principles of Mathematics*, PART VII, AND ITS MS.

CHAPTER LIII. MATTER.

465: 3 **437.**] 436 <*BR had written* 434 *which was replaced by the copy-editor.*>
465: 36 class-concept] class-conception
466: 12 any sense other than of Chapter IV] any sense
466: 38 organic whole in the above sense] organic whole
468: 13 *Material unit*] Matter

CHAPTER LIV. MOTION.

469: 9–11 Most of the relevant modern philosophical literature will illustrate the truth of these remarks: the theories suggested usually repose on a common dogmatic basis, and can] A perusal of the relevant modern literature will show the truth of these remarks: all the theories suggested repose on a common dogmatic basis, and all can
469: 11 can be] all can be
469: 14 , as a rule,] at once
469: 16 unexplained.] unexplored <*surely the MS. reading is correct*>
469: 25 difference, in respect of truth or falsehood,] difference
469: 28–9 differ only by the fact that T occurs in the one where T' occurs in the other.] express something other than the mere relations of the entity to T and T'.
470: 37 would have] have
470: 38 would then consist] then consists
472: 43–4 an interval containing the given moment otherwise than as an end-point can be assigned, at any moment within which interval A] any number of other moments can be assigned, both before and after it, and as near it as we choose, at which A
473: 2–4 there are such intervals, but all have the said moment as an end-term, is one of transition from rest to motion or *vice versâ*.] neither of these can be done is a moment when A is in motion, provided A occupies *some* place at neighbouring moments on either side.
473: 8 some period] any period
473: 13 some terms] some

473: 36–8 physical facts (*i.e.* as properties belonging *at each instant* to a moving point, and not merely real numbers expressing limits of certain ratios)] physical facts

CHAPTER LV. CAUSALITY.

475: 8 acceleration.] accelerations.
475: 35 subsequent] following
476: 10 as to] or to <*Surely a misprint.*>
477: 11 involves either temporal distance, or magnitude] involves magnitude
477: 12 which last we agreed] which we agreed
477: 13 Thus if our measure is effected by means of distance,] But except in this respect,
478: 20 if, for any value of r,] if any
478: 34 e'_4] e'_n <*The ms. could have been misread; there are no other instances of* e'_4 *in the examples.*>
479: 19 Dynamics assume] Dynamics assumes

CHAPTER LVI. DEFINITION OF A DYNAMICAL WORLD.

481: 30–1 In the Dynamics applicable to the actual world, the specification of S requires the notion of mass.] <*Fn. added.*>

CHAPTER LVII. NEWTON'S LAWS OF MOTION.

483: 35–8 second integral. The above, however, is a very specialized form of the second law; in its general form, the function F may involve other coefficients than the masses, and velocities as well as positions.] second integral.
484: 30 as it] it <*From the first edition through subsequent impressions, there is a gap at the beginning of the line where "as" should have been printed.*>
485: 19–20 that, so far as gravitation is concerned,] that
485: 28 Chapter LV.] Chapter III.
485: 40 the motion of the ether] Electricity
486: 36 configurations whose distance in time is given, then] configurations, then
488: 1 space] sphere
488: 18–23 parts. If there are finite volumes containing an infinite number of particles, the notion of mass must be modified so as to apply no longer to single particles, but to infinite classes of particles. The density at a point will then be not the mass of that point, but the differential coefficient, at the point, of the mass with respect to the volume.] parts.
488: 35 of possible material universes] of material universes

CHAPTER LVIII. ABSOLUTE AND RELATIVE MOTION.

489: 31 Art. CV. Contrast Art. XXX.] <*Fn. added.*>
489: 32 *Denkens*, Leyden, 1890.] Denkens.
489: 33 *Entwickelung*, Leipzig, 1883. (Translated, London, 1902.)] Entwickelung.
489: 34 *Science*, London, 1892. (2nd edition, 1900.)] Science.
490: 16 acceleration] accelerations <*Also at 490: 17 and 33.*>
490: 39–40 an absolutely rigid "Body *Alpha*,"] a "Body *Alpha*," absolutely rigid and fixed,
491: 34 article] pamphlet
491: 42–4 (1896–7). For a later state-

M. June 1900

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§449. The examination of this difficulty, ~~we shall find,~~ will rudely shake our cherished prejudices concerning causation. The laws of motion, we shall find, actually contradict the received view, & demand a quite different & far more complicated view. In Dynamics „ We shall find (1) that the causal relation holds between events at three times, not at two; (2) that the whole state of the material universe at ~~each~~ [from] 3 the three times is necessary to the statement of a causal relation. ~~Between~~ In order to provide for this conclusion, Let us re-examine causality in a less conventional ~~spirit~~.

§449. Causality, generally, is ~~the~~ principle ~~that~~ in virtue of which, from a sufficient number of events at a sufficient number of moments, one or more events at one or more new moments can be inferred. Let us suppose, for example, that, by means of the principle, if we are given e_1 events at a time t_1, e_2 at a time t_2, ... e_n at a time t_n, then we can infer e_{n+1} events at a time t_{n+1}. If, then,

Fol. 30 of Part VII of the manuscript of *Principles*, pp. 477: 38–487: 6, showing its original date of June 1900 and Russell's revised section numbering.

ment of Mr Macaulay's views, see Art. *Motion, Laws of,* in the new volumes of the *Encycl. Brit.* (Vol. XXXI).] (1896–7).

492: 7 cannot] <*Here there is a non-alphabetic vertical mark that persists through all impressions.*>

492: 11–14 nothing. The axes can, in a sense, be defined by relation to matter, but not by a constant geometrical relation; and when we ask what property is changed by motion relative to such axes, the only possible answer is that the absolute position has changed.] nothing.

492: 39 relating] relatively <*A misprint, surely.* >

492: 44 See my article "Recent Work on Leibniz," in *Mind,* 1903.] See my review of *Couturat,* La Logique de Leibniz, in Mind, October 1902.

492: 45 *Entwickelung,* 1st edition, p. 216.] Entwickelung.

493: 26–7 * Cf. Art. "Nativism" in the *Dictionary of Philosophy and Psychology,* edited by Baldwin, Vol. II, 1902. <*Fn. added in proof. The article is by Moore.*>

CHAPTER LIX. HERTZ'S DYNAMICS.

496: 25–6 two universes which have the same causal laws as the actual universe] two universes

497: 2 cause and effect.] cause and effect. May 23. 1902.

497: 3–498: 16 **474.** We may ... established throughout.] <*Added on leaves date-stamped 27 January 1903 by Cambridge U.P. and headed:* [To be added at the end of Chap LIX as part of this chapter.]>

498: 2 not greater] not less

498: 3 From the existence of θ,] Thence,

498: 7 outside] inside