RUSSELL'S MODAL LOGIC?

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R ussell's discussions of modality hardly form a centrepiece of his philo-
sophical logic. He published no articles on this topic,[1] nor do any of his
books devote even a single chapter to it. Nonetheless, remarks on modality
appear—although with no great frequency—scattered throughout his writings,
spanning the entire period in which he was seriously engaged in logical matters.

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[1] "Non-Euclidean Geometry" (*Papers* 4: 482–5), first published in 1904, is arguably an exception.

In a 1905 manuscript only recently published, he argues that modal predicates are correctly applied not to propositions but to propositional functions.[2] More-over, in a roughly coeval review of Meinong, serialized in *Mind*, he expresses scepticism towards the view that "it is possible to distinguish, among true prop-ositions, some which are necessary from others which are mere facts" (*Papers* 4: 435). His views on modality receive further clarification in his 1913 Aristotelian Society lecture, "On the Notion of Cause"; but a more fully developed picture does not emerge until 1918–19. Two writings of that period, "The Philosophy of Logical Atomism" and *Introduction to Mathematical Philosophy*, state Russell's most considered thoughts on this topic. Even here, Russell does not devote an extensive amount of space to the subject. But their individual brevity notwith-standing, Russell's remarks, collectively, form a view of modality that is worth considering in detail, since it not only sheds light on his conceptions of logic and logical truth, but also provides an interesting, if incomplete, reworking of the traditional approach. Jan Dejnožka's new book attempts to provide an interpretation of these remarks. While Dejnožka is not the first Russell scholar to address them, his book is by far the most comprehensive discussion yet pub-lished.[3] Moreover, his interpretation of Russell, if correct, would require a radical reconception of the logic of *Principia Mathematica*. As I shall argue, Dejnožka's book falls far short of establishing anything so revolutionary.

As early as 1905, Russell has settled into what becomes his official view, that "there is no such comparative and superlative of truth as is implied by the notions of contingency and necessity" (*Papers* 4: 520). Later, in "On the Notion of Cause", he considers the doctrine that "That is necessary which is not only true, but would be true under all circumstances."[4] His response is definitive:

A proposition is simply true or false, and that ends the matter: there can be no question of "circumstances." "Charles I's head was cut off" is just as true in summer as in winter, on Sundays as on Mondays.   (*Papers* 6: 194)

The notions of necessity, possibility, etc., are therefore correctly applied to propositional functions, not to propositions. In particular, a propositional function $f(x)$ is *necessary* just in case it determines a true proposition for each value of x, *possible* just in case it determines a true proposition for at least one value of x, and *impossible* just in case there is no value of x for which it deter-mines a true proposition. Brief reflection will reveal that the property of being

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[2] "Necessity and Possibility" (*Papers* 4: 507–20). The suggestion also occurs in Russell's review of Meinong; see *Papers* 4: 436, 450.

[3] Which is not to imply that there are not striking omissions; see note 8.

[4] Taken from Baldwin's *Dictionary of Philosophy and Psychology*.

necessary, understood in the above sense, is precisely what the universal quantifier expresses, whereas the property of being possible is what existential quantification expresses. (Being impossible is expressed by the existential quantifier prefixed by the negation sign.) Most readers pause, however briefly, at these definitions. What recommends the use of "possible" as applying to a function that is, in Russell's words, "sometimes true"? One suspects that Russell is using the expression in a way that deviates from its standard use in philos-ophy. The suspicion is confirmed in "The Philosophy of Logical Atomism":

When I say a propositional function is possible, meaning there are cases in which it is true, I am consciously using the word "possible" in an unusual sense, because I want a single word for my fundamental idea, and cannot find any word in ordinary language that expresses what I mean.   (*Papers* 8: 223)

In fact, Russell does allow for a notion of possibility that is applicable to prop-ositions, but this is itself defined in terms of the propositional-function concep-tion. On Russell's usage, a proposition is never possible *simpliciter*: it is only possible with respect to a given constituent. Thus, when speaking of a proposition's being possible, he uses the locution "possible with respect to a constituent" as opposed to simply "possible".[5] Similarly, *mutatis mutandis*, for necessity.

As the quotation from "On the Notion of Cause" makes clear, Russell's antipathy to the traditional doctrine of necessity is based on the intuition that the truth of a proposition is not relative to a world or a circumstance. There really is not much by way of argument in this article, or elsewhere for that matter.[6] The idea that there might be alternative non-actual circumstances at which "Charles I's head was cut off" is false is simply rejected out of hand—seasons and days of the week not really being to the point. One is grad-ually led to the conclusion that Russell promotes his analysis not because it recapitulates the intuitive notion we have of possibility, but because it is the only account available within the constraints of the function-argument analysis of propositions he favours.

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[5] In fact, Russell does allow for a proposition's being possible *simpliciter*, in the following sense:

p is possible (full stop) just in case, for some constituent σ of p, p is possible with respect to σ (see *Papers* 6: 195; *Papers* 8: 222).

It is unclear whether the corresponding account of propositional *necessity* would be "for some constituent σ of p, p is necessary with respect to σ" or "for each constituent σ of p, p is necessary with respect to σ"; Russell doesn't go on to say, although the account of logical necessity put forward in Chapter 18 of *IMP* would suggest the latter formulation.

[6] See, for example, *Papers* 4, at p. 518, *Papers* 8, at p. 203, or *IMP*, at pp. 165–6.

To further complicate matters, Russell himself is independently committed to using *necessary* and *possible* as predicates of propositions. In a discussion of logical truth, he reminds us that:

Pure logic, and pure mathematics (which is the same thing), aims at being true, in Leibnizian phraseology, in all possible worlds, not only in this higgledy-piggledy job-lot of a world in which chance has imprisoned us. There is a certain lordliness which the logician should preserve: he must not condescend to derive arguments from the things he sees about him. (*IMP*, p. 192)

In the subsequent paragraph, logical necessity is explicitly equated with truth in all possible worlds. One might ask how the latter notion relates to the propositional function analysis of modality.[7] Or how there can be "no question of circumstances" for the logician, given the explicit reference to possible worlds. Russell provides no answer.[8]

The question how to reconcile Russell's apparently contradictory claims is difficult. Given that his use of "logically necessary" as a predicate of propositions appears less than thirty pages after he effectively denies the validity of such usage (*IMP*, pp. 165–6) it is not an option to suppose that Russell somehow either forgot his earlier position or (silently) revised it in the interim. The most likely interpretation is that Russell, in *Introduction to Mathematical Philosophy*, still maintains the view, advanced in "Necessity and Possibility", that there is no single, univocal conception of necessity, but a cluster of mutually related notions. This would not clear him of the charge of dishonesty when he claims, without qualification, that "there never was any clear account of what was added to truth by the conception of necessity" (*IMP*, p. 165). Nonetheless, this interpretation is fully compatible with his central point, which is that the failure to distinguish propositions from propositional functions has been "a disgrace to philosophy" and, in particular, an impediment to clear thinking about modal notions (p. 166). And there *is* the very real possibility that Russell is "speaking

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[7] For Russell, propositional functions take only actually existing individuals as arguments.

[8] There is an account of propositional necessity, curiously neglected by Dejnožka, that is first stated in "Non-Euclidean Geometry" and which reappears, in substantially the same form, in *IMP* and *PLA*. On this account, to say that a proposition *p* is possible is to say that *p* is not ruled out by what we know. For example, if (to use Russell's example) a non-Euclidean asserts that it is *possible* that physical space is non-Euclidean, he means that "we do not know whether space is Euclidean or non-Euclidean." Still, Russell adds: "I should myself maintain that, in an ultimate logical sense—i.e., when all reference to our ignorance is excluded—all propositions are merely true or false" (*Papers* 4: 482). This proposal provides a pragmatic account of modal predication—of when it is acceptable to *assert* that a proposition is possible. It does not depart from Russell's position that, strictly speaking, when *p* is false, the assertion that *p* is possible will also be false. Thus, it does not mitigate the tension between Russell's official doctrine and his practice of appealing to modal notions.

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with the vulgar" when he makes appeal to possible worlds. Such talk is only to aid speculation and plays no fundamental role in, for example, his characterization of logical truth.

It appears then, that, Russell is, if anything, hostile to the idea that modality plays a fundamental role in logic, his occasional indulgence in "possible worlds talk" notwithstanding. Dejnožka holds the very opposite. Not only does Russell embrace modality, he espouses a variety of modal logics. Dejnožka has us consider, for example the system MDL, described as "a formula for reinterpreting *Principia* as a modal theory" (p. 96).[9] The interpretation utilizes the familiar definitions (p. 3):

$$f(x) \text{ is necessary } =_{df} f(x) \text{ is always true;}$$
$$f(x) \text{ is possible } =_{df} f(x) \text{ is sometimes true;}$$
$$f(x) \text{ is impossible } =_{df} f(x) \text{ is never true.}$$

This is, of course, not quite right: we should be defining the first-order quantifiers in terms of modal notions, not the other way round. (Otherwise, we are interpreting the modal operators, not the quantifiers.) Nonetheless, the intention is clear: first-order logic is a modal logic when we interpret quantification in terms of the relevant modal notions.

While Dejnožka insists that "Russell's intentions concerning MDL are serious" and that Russell "intends to use MDL to help analyze possible worlds talk" (p. 30), the support he gives for these provocative claims is simply that Russell indulges in "possible worlds talk" in a variety of disparate contexts. Granted, Russell is apparently (and, as I maintain, *only* apparently) of two minds concerning modal discourse, but the challenge then is not simply to ignore or gloss over, as Dejnožka does, those contexts in which Russell is critical of modal notions, but to attempt to make them compatible with those other contexts in which Russell appears to be more tolerant of modality.

Yet, even if we momentarily discount the weight of Russell's numerous anti-modality excursions, MDL presents us with some significant puzzles. First, given that, as the definitions make clear, Russell is using expressions such as "necessarily" and "possibly" in an artificial manner, it is hard to see how MDL provides us with anything that differs from the standard interpretation of *Principia*. Recall that Russell defines "*f(x)* is possible" as "*f(x)* is sometimes true." But Russell also defines existential quantification in terms of a propositional function's being possible. So, it would appear that "possible" and "sometimes true" express precisely the same property of propositional functions. And,

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[9] All references will be to the volume under review unless otherwise indicated.

indeed, Russell uses the two predicates interchangeably:

When you take any propositional function and assert of it that it is possible, *that it is sometimes true*, that gives you the fundamental meaning of "existence". (*Papers* 8: 204; emphasis my own)

Thus, it seems that the interpretation of *Principia* on offer cannot differ from the standard interpretation. The modal terminology Russell utilizes to introduce existential quantification adds nothing to the familiar notion, of a propositional function's being "sometimes true".

On the one hand, Dejnožka seems to agree with this conclusion, since he claims that "For Russell, neither existence nor possibility is a fundamental notion." They both reduce to the same "fundamental ... idea." But rather than seeing that this fact precludes a modal reading of *Principia*, Dejnožka takes it to provide evidence of one:

Thus those who deny that *Principia* is a modal logic because possibility is nothing for Russell are placed in the position of having to deny that *Principia* is an existential logic. For existence is equally nothing for Russell. It would be folly to argue that Russell has an existential logic but no modal logic because Russell "banishes" modality yet somehow "admits" existence. For Russell, the idea that existence is a property of things "is rubbish". Existence is not "going about" in the world any more than possibility is. (P. 113; citations omitted)

This fails to address the obvious question, which is why we should think that being a modal logic in Russell's sense is not just what we always thought *Principia* was—a logic in which existential quantification expresses the property of being "sometimes true". To respond by saying "That's just what being possible is, viz., being sometimes true" is either to say something false (that's *not* what it is to be possible) or trivial (that's what Russell, on his own testimony, stipulated "being possible" to mean—so what?).

In addition, any further analogies between first-order quantifiers and modal operators suggested by MDL must be made with extreme caution. While investigations into the parallels between modal and first-order logic have shown that the Lewis system S5 can be interpreted as a system of first-order logic, it must be emphasized that only a fragment of first-order logic can serve as the interpretation, viz., monadic quantification theory.[10] Since the fragment in question is, well, a *fragment* of first-order logic, it is hard to see how this interpretation,

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[10] There are restrictions that apply: the formalization allows for only one variable, which, when it occurs free, always receives the same assignment. For details, see Chapters 1 and 4 of A. N. Prior and Kit Fine, *Worlds, Times and Selves* (Amherst: U. of Massachusetts P., 1977).

twice cited by Dejnožka (pp. 64, 109), would support his claim to have provided a "formula" to "reinterpret" *Principia*.

I have not taken into account any logics besides MDL that Dejnožka attributes to Russell. The textual basis for these attributions is slender, in each case resting solely on informal remarks of Russell. The methodological basis for the attributions is also slightly suspect:

I shall impute a modal logic to Russell if either of two tests is met: (i) *it is more reasonable than not* to paraphrase Russell's thinking into the modal logic, or (ii) *it is more reasonable than not* to suppose that Russell would have substantially assented to the modal logic as a paraphrase of his thought. (P. 61; emphasis added)

This is just vacuous: reasonable to whom, and for what purpose? These questions are left hanging. Dejnožka provides a battery of quotations that catch Russell making reference to possible worlds and, without much ado, decides, for each individual quotation, which existing modal system can best accommodate it. Of course, Russell, like most reflective persons, makes use of the subjunctive and, occasionally, speaks of non-actual situations ("It would be a better world if ..."). And, like most of us, he is committed neither to possible worlds, still less to a modal logic.[11, 12]

There is much in Russell's writings on modality that Dejnožka fails to grapple with, and that is what is singularly disappointing about this book. While many aspects of Russell's thinking on modality are here brought to light—often for the first time—very little is treated with the level of caution, or the depth, that the subject matter demands. To take just one example: Dejnožka cites Russell's crisp response to C. I. Lewis on strict implication (p. 56). This is Russell, still at the height of his logical powers, responding to the intuitions about the implication relation that would give rise to modern modal logic (*IMP*, pp. 153–4). Yet, Dejnožka's treatment of this passage is cursory, even dismissive.[13] And that is a pattern the book never really breaks. The quo-

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[11] I am not disputing that the question, which system of modal logic best captures Russell's characterization of logical truth (say, *ca. IMP*), is of philosophical interest. For what it's worth, I do think it is of interest—just not historical interest.

[12] Similar worries attach to the claim that Russell espouses a notion of relevant implication—the topic of Chapter 9. Citing Russell's discussion of the principle of induction in *The Problems of Philosophy*, Dejnožka writes: "Whether we agree with him or not, Russell is committed to there being some inner connection between universals concerning regularities in the past and universals concerning events in the future. This seems enough to make the 1912 Russell a relevance logician in the broad positive sense" (p. 138). It is impossible to take this seriously.

[13] It is claimed, misleadingly, that Russell rejects Lewis's notion of strict implication "for a rather technical reason". In fact, Russell's rejection is based on his conviction that the relation of strict implication is unnecessary for mathematical logic and, consequently, "on general grounds of econ-

tations from Russell are often too complex—and sometimes just too immediate-
ly baffling, given their overall context—to fit into Dejnožka's picture. It is work
enough to have them, collectively, form a picture of any sort; but the task of the
historian is to see whether they do, and, if not, to let us know the bad news.
Dejnožka's book is not, then, the work that Russell's fascinating, if elusive,
body of doctrine deserves, but an almost wilful misreading—yet one that
Russell's words, uprooted and mangled though we find them here, sublimely
resist.

omy, ought not to be admitted into our apparatus of fundamental notions" (*IMP*, p. 154). The
reasoning leading to the rejection of this notion is thus not technical in nature, but is both methodo-
logical, turning on considerations of theoretical simplicity, and, more importantly, philosophical,
turning on considerations concerning the nature of logic.