REPLY TO URQUHART

I. Grattan-Guinness
Mathematics / Middlesex U.
43 St. Leonard's Road, Bengeo, Herts. SG14 2JW, UK
i.grattan-guinness@mdx.ac.uk

I am very grateful to Alasdair Urquhart for reviewing my history of mathematical logic, and finding the book to be generally welcome.[1] Several of his criticisms are duly noted, and I hope that a chance for corrections will arise in the future. There were indeed more typographical errors than expected: the proof-reading and indexing had to be accelerated to meet the deadline of a medical operation. But the apparent implication that many formulae are faulty does not seem to be justified; corrections here would be particularly welcome. Incidentally, a book apparently starved of symbolism contains over 60 numbered formulae in the sections dealing with *Principia Mathematica*, and over 50 in the chapter on the Peanists—maybe modest, but far more than in most other comparable literature. The Frege formula was not reproduced on page 181 because I did not want to spend space on describing the details and their context, which were not the issue at hand.

Mention of Frege leads to a point of discord. I distinguish the German mathematician Frege from the English construct whom I call "Frege′", who was made up by analytic philosophers as a father-figure and often overrides studies of Frege. For Urquhart I indexed them separately "rather confusingly"; but for me the (standard) confusion is to *mix* the two together. Frege was a Platonist concerned with the contents of thoughts as such, especially in order to ground a novel philosophy of *arithmetic* (and an uncertain amount of mathematical analysis), to be distinguished from his philosophy of geometry. By contrast, Frege′ is a philosopher of (only the English) language, deeply concerned with meaning and putting forward a philosophy of *mathematics*. The distinction was

[1] Review of I. Grattan-Guinness, *The Search for Mathematical Roots, 1870–1940: Logics, Set Theories and the Foundations of Mathematics from Cantor through Russell to Gödel*, in *Russell*, n.s. 21 (2001): 91–4.

russell: the Journal of Bertrand Russell Studies
The Bertrand Russell Research Centre, McMaster U.
n.s. 21 (winter 2001–02): 155–6
ISSN 0036-01631

made to combat the conflation of two such radically different stances; it is not likely to make much impact, but no apologies are offered for the effort.

Another of Urquhart's main criticisms is of the weakness of Chapter 9; so little on Hilbert's metamathematics, for example, and even less on Brouwer's intuitionism. Yes indeed; and these would be severe comments about a *general* history of the foundations of mathematics of that time. However, the book is explicitly presented as concerned with logicism and its attendant background, theories and consequences (pp. 7–8). The limitations of Chapter 9 are emphasized on pages 410 and 507, and (apart from undoubted internal weaknesses) arise from *the weakness of logicism itself* during the period covered: its eclipse, especially by metamathematics, is mentioned on pages 483, 523 and 563–4.

The final remark concerns Boole's restriction of the definability of the union of two classes to disjoint pairs. I cited Hailperin mainly for his clear explanation of the differences between Boole's algebra of logic and modern Boolean algebra. Hailperin has indeed also concocted an ingenious theory of signed multisets to vindicate Boole's practices; but surely it is a modern reading of Boole's work rather than part of its history. I know of no evidence that Boole himself consciously invented multiset theory within his logic; on the contrary, in his debate with Jevons (which I published some years ago and use on pp. 59–60), not only did Boole fail so to construe the union $x+x$ of the class x with itself, but surely he denied this reading of the union precisely to *avoid* multipartship (compare my p. 53). Jevons did claim the legitimacy of $x+x$, and he proposed the equation $x+x=x$; but he allowed only single parthood of individuals within x to the union. Again in a manuscript on the related topic of forming concepts, "the formula '*X*s and *Y*s' does not express an intelligible concept unless the symbols connected by *and* be interpreted to signify classes of things wholly distinct."[2] The emergence of multiset theory still appears to belong to C. S. Peirce cryptically in the 1870s (*Roots*, p. 146) and to Kempe explicitly in the following decade (pp. 137–40).

[2] G. Boole, *Selected Manuscripts on Logic and Its Philosophy*, ed. I. Grattan-Guinness and G. Bornet (Basel: Birkhäuser, 1997).