THE DEVELOPMENT OF RUSSELL'S DIAGRAMS FOR JUDGMENT

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In his 1918 lectures, *The Philosophy of Logical Atomism*, Russell discusses the impossibility of drawing a diagram or "map-in-space" of the form of belief (judgment). In this paper, I argue that an examination of diagrams appended to Russell's *Theory of Knowledge* shows him already anticipating this symbolizing difficulty in 1913 and—in the midst of attempting to adopt Wittgenstein's doctrine of propositional bipolarity—jettisoning attempts to diagram the form of belief.

INTRODUCTION

In his 1918 lectures, *The Philosophy of Logical Atomism*, Russell alludes to problems with a theory of symbolism, having already noted that it is impossible to draw a "map-in-space" (a diagram) of the form of belief (*Papers* 8: 198). In this paper I suggest that two diagrams occurring in Russell's work from 1913 anticipate his 1918 discussion of the problems of symbolizing belief. The diagrams in question occur in notes (called "Props") discovered with and appended to Russell's 1913 manuscript, *Theory of Knowledge*. These sketches, and these notes, clearly show

¹ In the following discussion I refer to the documents by the letters *A–H*. *A* is used to indicate the diagram that occurs in *Theory of Knowledge* at page 118; *B* is the draft appended as B.2; *C* signifies the sketches appended as A.4; *D* is Russell's map-in-space at *Papers* 8: 198; *E* and *F* are the two diagrams occurring in Appendix B.1 (i.e. in "Props"); *G* is Wittgenstein's diagram (in his *Notebooks 1914–1916*, 2nd ed., ed. G. H. von Wright and G. E. M. Anscombe [Oxford: Basil Blackwell, 1979], p. 95); and *H* is the illustration of bipolarity at *Papers* 8: 185.

russell: the Journal of Bertrand Russell Studies The Bertrand Russell Research Centre, McMaster U. Russell struggling to accommodate his own view of belief to Wittgenstein's conception of propositions.²

In order to trace the trajectory from Russell's troubles in these notes to his discussion of the problem of symbolizing belief statements in *The* Philosophy of Logical Atomism, it is necessary to determine exactly which diagrams to include in these notes. This seemingly straightforward issue is complicated by the fact that "Props"—which contains a doctrine of belief unlike that contained in Theory of Knowledge in omitting form as an object joined to other items of the belief—has physically attached to it another set of sketches of belief in which Russell omits form as an ingredient.3 I therefore consider whether these sketches (I call them C) depict Russell's multiple-relation theory of judgment at a stage prior to and discarded before the version presented in "Props", as the editors think, or represent the doctrine advanced in "Props" itself. While there is little reason to doubt parts of their reconstruction, it is more difficult to decide whether the editors are correct in deciding that the diagrams in C illustrate Russell's multiple-relation theory of belief at a stage prior to the form-containing version of that doctrine presented in *Theory of Knowl*edge. After I present the doctrine of belief in Theory of Knowledge, the editors' view of the relevant diagrams and my own reconstruction, I turn to the connection between the issues Russell touches on in "Props" and his 1918 discussion of symbols for statements of belief. My starting-point is a brief discussion of the evolution of Russell's theory of belief prior to 1913.

The years 1906, 1910, 1912, and 1913 mark distinct stages in the development of Russell's multiple-relation doctrine of belief. Introduced in "On the Nature of Truth" (1906), the doctrine amounts to viewing belief no longer as "one idea with a complex object" but rather as comprised "of several related ideas" (corresponding, if true, to a complex of objects). Russell sees in thus reducing beliefs to their constituent ideas

² "Props" occurs as appendix item B.1 in *Papers* 7: 195–200. The editors argue that "Props" appears "to be an attempt to take account of Wittgenstein's new theory of propositions" (p. 195).

For different interpretations of "Props" see, especially, Stephen Sommerville, "Types, Categories and Significance" (unpublished PH.D. thesis, McMaster U., 1979); and Herbert Hochberg, "Propositions, Truth and Belief: the Wittgenstein–Russell Dispute", *Theoria*, 66 (2000): 36.

³ Labelled A.4 in the appendix to *Papers* 7: 186–7.

^{4 &}quot;[I]n the event of the objects not standing in the corresponding relation, there will

and eliminating them as single entities a means of escaping the consequence, accepted in his 1903 *Principles of Mathematics*, that a false belief must correspond to a subsisting entity (the objective proposition). At the same time, the technique⁵ provides him with a way to avoid "paradoxes analogous to that of the liar, e.g., ... the man who believes that all of his beliefs are mistaken." Admitting that a theory of belief as a relation among *ideas* is "very likely open to fatal objections" Russell ultimately excised the theory from the paper.⁷

In subsequent versions of the theory of belief, things replace ideas as constituents of a *judging*-fact. In particular, judgments like "Othello believes that Desdemona loves Cassio" are thought to include a subject (Othello), a mental relation (*believes*), objects (Desdemona, Cassio), and a subordinate relation (*loves*). Thus in his 1911 essay "Knowledge by Acquaintance and Knowledge by Description" Russell writes: "At the time when I judge, there is a certain complex whose terms are myself and A and love and B, and whose relating relation is *judging*" (ML, p. 220; *Papers* 6: 154).9

be no objective complex corresponding to the belief, and the belief is belief in nothing, though it is not 'thinking of nothing', because it is thinking of the objects of the ideas which constitute the belief" ("The Monistic Theory of Truth", *Philosophical Essays*, 2nd ed. [London: Allen and Unwin, 1966], p. 46).

⁵ I cannot here address the complex history of the theory of incomplete symbols.

⁶ Russell, "On the Nature of Truth", *Proceedings of the Aristotelian Society*, 7 (1906–07): 46. In 1910 Russell reprinted the first two sections of this 1906 paper in *Philosophical Essays*, 2nd ed. (London: Allen and Unwin, 1966) under the title "The Monistic Theory of Truth" and eliminated the section on belief. At the same time (*ibid.*, pp. 147–59, and *Papers* 6), he published a different version of the multiple-relation theory in a mostly new paper titled "On the Nature of Truth and Falsehood" (also in *Papers* 6). This is Russell's first public adoption of the multiple-relation theory.

7 "On the Nature of Truth", pp. 46-7. In a footnote Russell acknowledges difficul-

ties defining correspondence-truth.

⁸ In part this shift follows from reasoning that "my judging obviously consists in my believing that there is a relation between actual objects, …, not in there being in fact a relation between my ideas of these objects" (*Papers* 7: 140). Notice that if in *judging* I must relate to objects (or fail to know *what* I am judging), naive objects of common sense (James, Othello, etc.) cannot serve as constituents in *judging* for they may not exist. In Russell's post-1906 versions of the theory of belief, then, only objects whose existence is indubitable—data of direct experience (sense-data and universals)—ultimately compose *judging*-facts. I omit here the history behind Russell's acceptance of sense-data.

⁹ In "Knowledge by Acquaintance and Knowledge by Description" Russell refers to his 1910 theory of judgment in *Philosophical Essays* and mistakenly cites the title as "The

But if belief is just a nexus of things, how it is possible to distinguish between cases of belief comprised of the same constituents and differing in sense? (How, e.g., are we to distinguish James' judgment that the pen is to the left of the cup from his judgment that the cup is to the left of the pen?) In 1910 Russell addresses this problem by arguing that "the relation [e.g., is to left of] as it enters into the judgment must have a 'sense'..." In correspondence a year later Russell withdraws that account and places the burden of distinguishing senses on the polyadic relation of *judging*: "[T]he judging alone", he says, "may arrange the terms in the order Mind, A, r, B, as opposed to Mind, B, r, A" and "sense ... must not appear in the r..." The 1912 *Problems of Philosophy* incorporates this adjustment and resolves the sense-problem by saying that the relation of judging puts things in order (PP_2 , p. 127). The theory goes through a final stage—or stages—in 1913, as I illustrate in the following.

THE 1913 MULTIPLE-RELATION THEORY AND ITS MAP-IN-SPACE

In the 1913 *Theory of Knowledge* Russell continues to claim, as in 1910–12, that a cognitive fact (he focuses on *understanding*) comprises a particular subject, a multiple relation of *understanding*, objects, and a subordinate relation (*loves*), but adds to this list the form of the complex (e.g., xRy), and then symbolizes the resulting five-termed complex: $U\{S, A, B, similarity, R(x, y)\}$. Russell depicts the understanding-fact in the following

Nature of Truth". "The Nature of Truth" is the title of an earlier, different paper on the theory of judgment published in 1906 in *Mind* as well as the title of yet another paper Russell read to the Jowett Society in 1905.

In 1917 Russell added the following comment in "Knowledge by Acquaintance and Knowledge by Description" to the reference in it to his 1910 paper: "I have been persuaded by Mr. Wittgenstein that this theory is somewhat unduly simple, but the modification which I believe it to require does not affect the above argument" (*Papers* 6: 154 n.2).

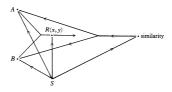
¹⁰ "On the Nature of Truth and Falsehood", *Philosophical Essays*, p. 158; *Papers* 6: 124.

124.

11 Citation taken from Griffin, "Russell's Multiple-Relation Theory of Judgment", Philosophical Studies, 47 (1985): 220. See also G. F. Stout, Studies in Philosophy and Psychology (London: Macmillan, 1930), pp. 239–57; reprinted from Proceedings of the Aristotelian Society, 11 (1910–11): 181ff.

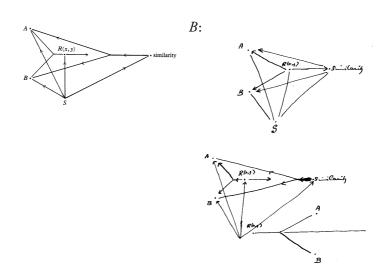
diagram (a possibility he rejects in his discussion of the form of the belief-fact in his 1918 lectures¹²):





In the diagram above we see Russell's representation of a subject's belief that two terms (A, B) are in a relation of *similarity* having a dyadic form R(x, y). When the *Theory of Knowledge* manuscript was discovered among Russell's papers, other diagrams of judgment not contained in the text were found bundled with it and plainly belong to the work. One of these, which occurs as an appendix to *Theory of Knowledge*, has a patent resemblance to A. I label it B and present it here beside A:¹³

A:



¹² Russell, *Papers* 7: 118.

¹³ What I call B is appendix item B.2 in Papers 7: 200.

Two sketches occur in B—or three, if the lower one represents both the diagram of the belief, and, protruding from the lower right corner, the complex that is believed. If (I return later to what I think Russell intended this third feature to signify.) If we ignore that right-most extension, the bottom sketch in B is identical to the diagram A contained in the text and resembles it in a way that the top sketch in B does not: in the A-diagram and the lower B-sketch the arrow proceeding eastward from the form B is short of meeting up with the relation *similarity*, while they actually meet up in the top sketch in B. Perhaps this indicates that the top sketch in B is earlier than the bottom one. The shifting representation of the connection of form to the relation may indicate that Russell wishes to show that the form is on a different level of abstraction than the relation and its terms, or else perhaps he does not wish the diagram to present the existence of a complex in which the terms actually are similar and occur in that form.

Despite these differences, Russell's use of form and his selection of the relation *similarity* in the sketches of belief in the *Theory of Knowledge* text as well as in *B* lead me to agree with the editors that Russell probably used *B* to draft *A*, the diagram given in *Theory of Knowledge*. It is less clear whether the same can be said of a third diagram, drawn on the verso of the last page of the set of notes titled "Props". ¹⁵ I present it here as *C*:

¹⁴ In B, the appearance of R(x, y) twice in the bottom diagram makes little sense if we treat it as a single picture rather than two. I believe that the mysterious "arm" seeming to protrude eastward in the lower sketch of judgment in B is a sketch of the relation of *similarity*, and its form, either carelessly overwritten by the sketch for judgment (and not really extending from it at all) or intended to be seen three-dimensionally as behind the sketch of judgment, which is to be seen as projected onto it. In any event the "arm" seems to me to have been sketched first.

¹⁵ What I label C is appended as A.4 in Papers 7: 186–7.

$$C$$
:



Two sketches occur within C. As the editors note, in the top sketch Russell depicts an arrow leading from a point A to a point B, the letter "L" occurs in script beneath it (presumably to indicate the relation is to the left of), and the direction of the arrow indicates the "sense" of the relation (*Papers* 7: 187). In the bottom sketch Russell uses the letter "S" for the judging subject and off to the left displays the letter "J". The bottom sketch depicts a relation leading from A to B with the letter "L" overwritten with "R", and shows, they think (and I agree), the relational complex appearing in the top sketch incorporated into the relation of judging. It is worth pointing out (as the editors do not) that in the lower sketch the relation R occurs as a term and in the upper sketch it actually relates the two items. Russell often indicates a term in a judging complex by x; counting the x's in the two C-sketches shows the difference in the status of L in the complex and in the judgment of that complex. The difficulty of the status of the relation is one Russell acknowledges in 1918 to be a flaw in his theory of judgment (*Papers* 8: 199).

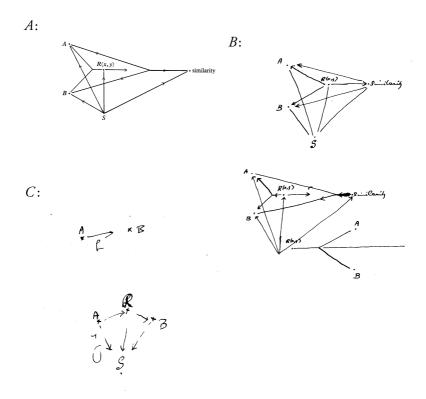
Like the cases discussed so far, the C-diagram appears to be a sketch of *judging* (in which the relation occurs as a term); it differs from A and B, however, in using a relation other than *similarity* and in omitting form as a constituent. What doctrine does C illustrate and where does it belong in a sequence of Russell's developing view of propositions asserting belief? According to the editors, C is an early version of B, itself a draft of the diagram A appearing in the text: "This diagram [C] may have been replaced by the one which appears in the text [A] and a draft of which is displayed as [B]" (*Papers* 7: 187). The reason the editors give

for this reconstruction is the absence of form in the bottom diagram in C compared to the diagrams in A and B which do contain form as an ingredient along with terms, relations and the judge. Speaking about draft B, for instance, they write that it "includes the form of the relation and thus makes a significant contrast with" C.¹⁶ The editors thus seem to believe that the bottom diagram in C represents the multiple-relation theory at an early stage in which form is absent and to take the sequence of diagrams as evidence of Russell adding form to his official (1912) theory during the writing of *Theory of Knowledge*.

The editors neglect to discuss an alternative reconstruction of the provenance of the *C*-diagrams, namely, that *C* depicts Russell's doctrine of judgment as presented in the notes called "Props", a version of his theory of judgment (influenced by Wittgenstein) in which form is again absent. Is it possible that *C* belongs with "Props" and that Russell sketched it as an illustration of problems in *that* theory? After all, as the editors acknowledge, "Props" postdates the form-containing version of the doctrine of judgment in *Theory of Knowledge*, and in "Props" Russell rejects this earlier role of form.¹⁷ I do not think this sequence is likely and agree with the editors on their view of the provenance of *C*, but the reasons for assimilating the sketches in *C* to the doctrine of judgment in *Theory of Knowledge*, and not the later theory in "Props", are less than obvious and are worth discussing. To assist the reader, I will reproduce *A*, *B*, and *C* together below:

¹⁶ *Ibid.*; at page 200 the editors repeat the contrast with A.4.

¹⁷ Perhaps it postdates the text by very little: the first page of "Props" appears on the verso of a rejected page of the *Theory of Knowledge* manuscript (concerning Meinong) and this rejected page can be confidently dated to the third week in May. The editors refrain from asserting a date for the notes but suggest that they may have been written as early as the third week in May, that is, around the time Russell was working on his chapter on understanding in Part Two of *Theory of Knowledge (Papers* 7: 197). Blackwell judges that it was written in May, that is, "very soon after Wittgenstein's second visit" in "Wittgenstein's Impact on Russell's Theory of Belief" (unpublished M.A. thesis, McMaster U., 1974), p. 85. It is worth noting that according to Blackwell (*ibid.*) Russell did not keep for re-use used sheets of paper for long periods of time. As "Props" ends with a page that has on its verso the diagrams in *C*, and the editors think that *C* is older than "Props", they apparently think that Russell wrote the page of text on Meinong, rejected it and saved the sheet, wrote *C*, rejected it and saved the sheet, and then wrote "Props" on their opposite sides.



To begin with, certain features of these diagrams can be dismissed as weighing in favour of neither reconstruction of the provenance of C. For example, that C is physically linked to the last page of "Props" decides in favour of neither view as long as we cannot determine which pages (or sides of pages) were written first. Likewise, on the editors' account (and mine) Russell's use of lower-case x's and y's is consistent throughout this phase of his thought and in contrast to his use of upper-case letters in the two diagrams (E and E) that occur in "Props". What little support this consistency might give to grouping E0 with the earlier theory of judgment rather than the one in "Props", is offset by the knowledge that throughout *Theory of Knowledge* Russell regularly shifts his notation in exactly this way.

Better support comes from an analysis of what it is Russell thinks

these diagrams can do. If the editors are right to say that the direction of the arrow in the upper sketch in C indicates the "sense" of the relation, then it is worth noting that sense seems to be expressed in the judging complex too, (eastward-pointing horizontal arrows occur there as well), that is, "sense" is part of the judging relation. In *Theory of Knowledge* Russell denies that relations have sense and that a judging relation can put the items judged in order and thus it seems that C precedes his making that alteration to his theory.

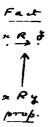
Furthermore, the sketches in C, B, and A attempt to represent judging as a spatial complex in which the judge occurs in a relation to what is believed. This is not the case, I shall argue, with the diagrams in "Props". In C, the upper sketch of the complex represents the relata of to the left of as one to the left of the other, while the diagram of judging presents these items as similarly occurring, though now in a spatial complex that includes the subject or judge. Turning to B, specifically to the similaritycomplex extending (I surmise) out from behind the (lower) diagram for judgment, we see that the relata (A, B) are shown one above the other, positioned in a vertical plane divided by *similarity*, indicated by a horizontal line. (Perhaps Russell uses this device because he thinks of the form of the relation *similarity* as cutting the plane between A and B into two identical parts: A-similar-B and B-similar-A.) The spatial quality is carried over into the judgment-diagram(s) in B: this is easier to see if we imagine the B-diagram(s) of judgment without the addition of form (occurring as in the stage of thought represented in *C*). For in it we still have *similarity* and two terms (A, B) on a vertical plane, but the subject has been added.

In sketching C through B Russell clearly wishes to indicate a belief fact as a spatial complex. A brief survey of issues and diagrams in "Props" suggests, however, that these notes break from any attempt to represent belief spatially. If the diagrams in "Props" do represent a shift away from a spatial conception of belief, then, it seems reasonable to join the editors in treating the theory in C as occurring before rather than after the official doctrine of the *Theory of Knowledge* text. More importantly, it means that in "Props" we can locate the origin of Russell's claim in his 1918 lectures that any attempt to depict judging spatially must fail. To show this, it is necessary to briefly discuss the theory of belief in "Props".

THE PROBLEM OF SYMBOLIZING BELIEF

In "Props" Russell explains *understanding* as a kind of perception, specifically, as perception of a "neutral fact". Russell elsewhere argues that perception (e.g., *seeing that a is R to b*) must be of a *fact* and that in perception of facts some understanding of the form of a proposition is involved; and I take these ideas to lie behind his present view of *understanding*.) A diagram, *E*, illustrates the relation:

E:



On the one hand, what we understand (the neutral fact) is further supposed to be a constituent of either a negative or a positive fact, i.e. contained in it along with its terms and relation. While this containment—and the notion of a neutral or negative fact—is obscure, I take Russell to mean, for example, that a positive or negative fact xRy contains x, y, R "neutrally" as well as the relating or non-relating of x, y, R that determines its negative or positive status. On the other hand, the neutral fact comprises the *judging*-fact. What differentiates *understanding* or *perceiving* from *judging* is that the former involves a negative or positive "direction" to the fact:

¹⁸ In *Theory of Knowledge*, the negative judgment $\sim aRb$ is reduced to disbelief in aRb; in his 1918 lectures Russell rejects such a move in the context of defending negative facts, saying a proposition must be verified by a fact, not explicated by the experience of disbelief (*Papers* 8: 187). Reasons such as these may lie behind his adoption of negative facts in "Props".

¹⁹ Russell writes: "It *looks* as if there actually were always a relation of x and R and y whenever they form *either* of the two complexes, and as if this were perceived in understanding. If there is such a neutral fact, it ought to be a *constituent* of the positive or negative fact. It will provide a meaning for *possibility*" (*Papers* 7: 194–5).

Judgment involves the *neutral* fact, not the positive or negative fact. The neutral fact has a relation to a positive fact, or to a negative fact. Judgment asserts *one* of these. It [i.e. judgment when it asserts a neutral fact in one of these ways] will still be a neutral relation, but its terms will not be the same as in my old theory. The neutral fact replaces the *form*. Call [a] neutral fact "positively directed" when it corresponds to a positive fact", "negatively directed" when it corresponds to a negative fact. (*Papers* 7: 196–7)

On this new version of Russell's theory of belief, a *judging*-fact comprises no form (and perhaps no separate items) but only a neutral fact directed towards or away from some fact. The theory is accompanied by the following diagram, F:

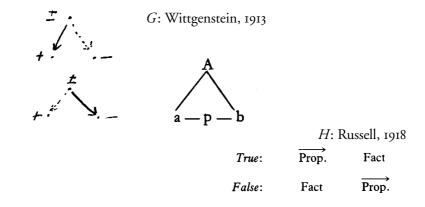




Russell depicts judging as a neutral fact in a positive or negative direction to a (positive or negative) fact, and uses a broken line to indicate the possibility of directing a neutral fact with one direction or polarity to the opposite kind of fact (*ibid.*). Notice that in *E* and *F* the subject is absent from the depiction. In fact, it is striking that in "Props" Russell entirely neglects to mention the subject or depict it in any symbol or sketch.²⁰ I do not suggest that Russell is eliminating the judge as an entity from his metaphysics (a move he only makes much later on), but that he no longer depicts judgment at all, only bipolarity. In this he is influenced by Wittgenstein. For the sake of comparison, Wittgenstein's own diagram of bipolarity is presented here as *G*, sandwiched between Russell's 1913 and 1918 depictions of bipolarity:

²⁰ Perhaps to avoid saying that belief is a relation of a subject to some thing, when Russell talks about the proposition in understanding pointing to the (neutral) fact he uses the awkward locution "when we understand, *what is happening points*" (*ibid.*; emphasis added).

F: Russell, 1913



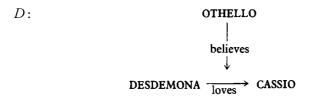
Russell's 1913 sketches correspond to Wittgenstein's "a-p-b" and suggest, rather than actually draw, the subject A who judges. Indeed, in depicting bipolarity without the subject he is more scrupulous than Wittgenstein, whose influence he shows and who in his 1913 notes objects that the subject or judge must be related to the two poles of a proposition and that "[t]his is obviously not a relation in the ordinary sense." I take that to suggest Russell has understood Wittgenstein's point. The sketches in F correspond, moreover, to Russell's 1918 depiction of the essence of a proposition as corresponding in two ways to a fact, shown above as H (Papers 8: 185). In short, these sketches (F-H) constitute a series.

Additional features of "Props" suggest that at the same time he adopts bipolarity, Russell is taking the additional step, as in his 1918 lectures, of jettisoning the assumption that it is possible to make a map in space of the form of belief. Consider again the diagrams in C, B, and A and how they belong together. Not only do they contain a subject, but they are intended to show the relation among the terms in the complex of judging. In "Props", though Russell gives a picture of perception and bipolarity, he makes no attempt to provide any sketch of a mental relation among a subject and other terms. His omissions show, I suspect, his

²¹ Wittgenstein, Notebooks, p. 95.

new doubt either that in judging a judge is related to a fact, or that we can depict the logical form of belief by the spatial properties of a map or symbolism.

In 1918 both these ideas occur. Russell rejects the idea that the form of belief is that of a relation between a person and a proposition, even though certain ways of speaking like "I believe the proposition p" and "I believe that such and such" might suggest otherwise. Belief cannot be a relation to a fact, he notes, because there are no false facts (and after 1903 belief cannot be a relation to a proposition-thing either). He further argues that in a belief proposition such as Othello's belief that Desdemona loves Cassio, *loves* must function as a verb and can't be replaced by a substantive. Any map or diagram of belief will invariably depict the relation as existing, even when the belief is false, and will thus fail to capture the form of belief. The diagram Russell uses to point out the limits on diagrams (and symbolism) belongs naturally with those we have seen in the series A through C. I therefore represent it here as D.²²



On this "map" *loves* occurs as a relation; that is, the horizontal, eastward arrow binds Desdemona to Cassio in the relation *loves*. But if in actuality Othello's belief is false and *love* doesn't relate Desdemona and Cassio at all, then the "map" is misleading. In the 1918 lectures Russell remarks that:

[W]here and how [the theory of symbolism] is wrong is that in the symbol you have this relationship relating these two things and in the fact it doesn't really relate them. You cannot get in space an occurrence which is logically of the same form as belief. When I say "logically of the same form" I mean that one

²² There is only one vertical arrow of belief or judgment leading from Othello, the judge or subject, to what is judged, rather than one for each item judged as in A-C, because what is believed, Russell now sees, must occur in belief as a relational whole.

can be obtained from the other by replacing the constituents of the one by the new terms. If I say "Desdemona loves Cassio" that is of the same form as "A is to the right of B". Those are of the same form, and I say that nothing that occurs in space is of the same form as belief. I have got on here to a new sort of thing, a new beast for our zoo, not another member of our former species but a new species. The discovery of this fact is due to Mr. Wittgenstein. (*Papers* 8: 198–9)

In a portion of *Theory of Knowledge* revised after Wittgenstein's objections led Russell to abandon it, and written after "Props", Russell writes:

It can be shown²³ that a judgment, and generally all thought whose expression involves *propositions*, must be a fact of a different logical form from any of the series: subject-predicate facts, dual relations, triple relations, etc. In this way a difficult and interesting problem of pure logic arises, namely, the problem of enlarging the inventory of logical forms so as to include forms appropriate to the facts of epistemology. (*Papers* 7: 46)

The omission from "Props" of a map of the form of belief (and the omission of the subject) suggest that Russell is aware of the problem of describing the form of belief—and beginning to be aware that he can no longer depict belief as a four- or five-term relation.

²³ [Russell's note:] As I have come to know through unpublished work of my friend Mr. Ludwig Wittgenstein.