Discussion

THE GENESIS OF THE TRUTH-TABLE DEVICE

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It has been suggested that Russell and or Wittgenstein arrived at a truth-table device in or around 1912 [Shosky 1997], and that, since the history of its development is so complex, the best one can claim is that theirs may be the first identifiably ascribable example. However, Charles Peirce had, unbeknownst to most logicians of the time, already developed a truth table for binary connectives of his algebra of logic in 1902.

SHOSKY'S THESIS

In "Russell's Use of Truth Tables" [Shosky *1997*], John Shosky argues that Ludwig Wittgenstein, or Wittgenstein and Bertrand Russell together, developed truth tables; that by 1912, Russell knew about truth tables. Shosky does not impute the actual original discovery or development of truth tables directly either to Wittgenstein or Russell, either jointly or individually, but he allows the inference to be drawn that Wittgenstein, or Russell, or both together, developed truth tables in their familiar form nearly a decade before they appeared in Wittgenstein's [*1922*] *Tractatus* or Post's [*1921*] "Introduction to a General Theory of Elementary Propositions" (and first announced in [Post *1920*]). Shosky goes so far as to say that Whitehead and Russell "seemed to have an idea of truth tables in their explanation of material implication in *Principia Mathematica*" (but he fails to explain what this means), and he convincingly and correctly, I think, says that Russell used (what—in retrospect—can be called) a "modified truth table" in Lecture III of "The Philosophy of Logical Atomism" in early 1918 [Shosky *1997*, 11]. The reference is to Russell's explanation that "you have as a schema,

for "*p* or *q*", using "TT" for "*p* and *q* both true"
"TF" for "*p* true and *q* false", etc.,

TT	TF	FT	FF
T	T	T	F

where the bottom line states the truth or falsehood of "*p* or *q*". (*Papers* 8: 185–6)

Shosky's conception of a "modified truth table" is open to question. John Slater, the editor of [*Papers* 8: 350], referring to these lines, wrote in the annotations that: "This is as close as Russell comes in his writings to giving what are now called 'truth-tables'. Jan Łukasiewicz, E. L. Post (who used + and – rather than T and F), and Wittgenstein all published in 1920–21 writings in which truth-tables are made explicit." Shosky also says [*1997*, 11] that Russell "examined truth-table language" in his *Introduction to Mathematical Philosophy* (1919); but he fails to explain what this means, and seems to me now, as I look at the relevant passages of the *Introduction*, to be conflating his own distinction between a truth-table device and a truth-table technique. Where or how Russell and Wittgenstein may have hit upon the device is left unsaid by Shosky, although he makes it clear that it is only when material implication has been introduced into logic that truth tables can be developed in full (see especially [Shosky *1997*, 16–19]). Thus, Shosky [*1997*, 12] sees Frege's work as a crucial step in the development toward making truth tables possible.

Ivor Grattan-Guinness was more forceful in his attribution even than Shosky, asserting [Grattan-Guinness *1997*, 600] that the method of truth tables was "invented by Bertrand Russell with his pupil Ludwig Wittgenstein around 1914 and first publicized in the spring of that year by Russell at Harvard in a lecture course on logic."

Shosky [*1997*, 12], more moderate than Grattan-Guinness, argues that "It is far from clear that any one person should be given the title of 'inventor' of truth tables." And Richard Zach [*1999*, 357 n.3] reminds us that "Peirce, Wittgenstein, and Post are commonly credited with the truth-table method of determining propositional validity."

I would argue that, *if* any one person should can be granted that title, it should be Charles Peirce, and I will—while admitting that William Stanley Jevons and Peirce's student Christine Ladd-Franklin each deserve partial credit—momentarily state my reasons for awarding the laurels to Peirce. First, however, I would like to take a closer look at Shosky's treatment of the history of truth tables.

What is particularly surprising is that Shosky and Grattan-Guinness would credit Russell with the invention of truth tables when there is already a large literature pointing to Charles Peirce as the originator of the truth table; the scholars over the course of much of the twentieth century who have pointed to Peirce's work as anticipating, or even actually presenting, the first extant written and identifiable documentation we have of a truth table include, e.g. [Łukasiewicz & Tarski *1930*, 40 n.2], [Berry *1952*, 158], [Church *1956*, 162], [Fisch & Turquette *1966*, 71–2], [Hawkins *1975*, 137 n.9], [Clark *1997*], [Zellweger *1997*], and [Lane *1999*, 284], the latter adding, most unequivocally, that: "it has long been known that [Peirce] gave an example of a two-valued truth-table." Hawkins [*1971*; *1975a*, 111, 114 nn.45–7] also discussed Peirce's work on truth tables and distinguished the difference between the truth-table technique (truth-functional analysis) and the truth-table device, calling the latter the *truth-table method*.

The most important feature of Shosky's presentation is his distinction, already alluded too, between the truth-table *technique* and the truth-table *device* [Shosky *1997*, 13]. The truth-table technique is the logical analysis of the truth values of a proposition, i.e. the truth-functional analysis of propositions. The truth-table *device* is the presentation of this analysis in tabular or matrix form. Shosky traces the former from Philo through Boole to Frege; in particular, Wittgenstein's main source is Frege, and Post's sources come from Jevons, Venn, Schröder, Whitehead, Russell, and C. I. Lewis [Shosky *1997*, 12]. Shosky notes that Quine took Frege, Peirce, and Schröder to be the source of the "pattern of reasoning" that led to truth-table development, and that truth tables were developed from that pattern by Łukasiewicz, Post, and Wittgenstein. It is because of this complex history that he concludes [Shosky *1997*, 12] that it would be difficult to name one single individual as the inventor of the truth-table device.

The principal source for Shosky's conclusion that Russell and Wittgenstein had developed the truth-table device some time in or around 1912 is found first of all on the verso of a leaf of manuscript (RA1 220.011450), titled "Matter. The Problem Stated" (in [*Papers* 6: 98–9]), held by the Bertrand Russell Archives, on which are notations, identified to be in Wittgenstein's hand, in which several truth tables are written out, along with Russell's notation for the truth-table matrix for $\sim p$. It should be said, however, that, although the date of the manuscript itself can be given with fair accuracy, the evidence for the contemporaneous dating of the notations on the verso is speculative only, based upon the fact that we know when Russell presented the talk for which the manuscript in question is a contribution and that Wittgenstein attended that talk; from this it is surmised that Russell and Wittgenstein discussed the contents of the talk, possibly within a few months of the actual reading and in connection with an early draft of the talk for which this manuscript was prepared.

The other crucial source is the manuscript notes of T. S. Eliot of Russell's logic course at Harvard, given in April 1914, in particular the lecture of 4 April

1914, in which truth tables are clearly and unequivocally found.[1] According to [Shosky *1997*, 22], these notes are (apart from Wittgenstein's jottings of 1912 on the back of RA1 220.011450) "the first recorded, verifiable, cogent, and attributable truth tables in modern logic." Shosky [*1997*, 25] thinks that Russell here *may* "simply be utilizing a device learned earlier from Wittgenstein. The evidence from 1912 would support this thesis."

ALTERNATIVE THESIS

Absent concrete evidence, it is sheer speculation to conclude on the basis of the notations from 1912 that Wittgenstein first suggested the truth tables to Russell, writing them down for Russell, and to which Russell then added the table for $\sim p$, or that, on the contrary, Wittgenstein wrote down the tables from an explanation that Russell gave Wittgenstein. That is, even if the tables are known to be primarily in Wittgenstein's hand, we cannot from that alone determine whether Wittgenstein was explaining the device to Russell or Russell was explaining it to Wittgenstein. As a matter of fact, Shosky's history leaves several important questions open:

(1) Did Wittgenstein and Russell work out truth tables jointly in 1912?
(2) If not, did Wittgenstein explain the device to Russell, or Russell to Wittgenstein?
(3) Where and how might Russell or Wittgenstein have first arrived at the device?
(4) How do we account for the coincidence of the publication within so short a time of Łukasiewicz's account (1920), Wittgenstein's (1922), Post's (1920/1921) and (we might add), somewhat later, Zhegalkin's [*1927*] account for propositional logic and his [*1928–1929*] extension of it to first-order predicate logic?

The answer to the first three questions would require new archival discoveries of truth-table devices or prototype truth-table devices in the papers of either Russell or Wittgenstein. Without such supporting documents, it is obviously fruitless to attempt to assign priority to either Russell or Wittgenstein. We can suppose, however, that the answer to the second question could shed some light on the answer to the first, and possibly also to the third, and even the

[1] The logic course which Russell taught at Harvard, conducted during the Spring 1914 semester, was begun by Harry Todd Costello, Russell having been unable to arrive in time for the start of the semester; Russell's first lecture for the course occurred on 17 March (see [Lenzen *1971*, 4).

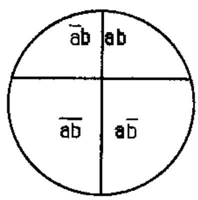
fourth, question. A corollary to the fourth question asks:

(5) Is there a common source for the nearly simultaneous work on truth tables by Łukasiewicz, Wittgenstein, Post, and Zhegalkin?

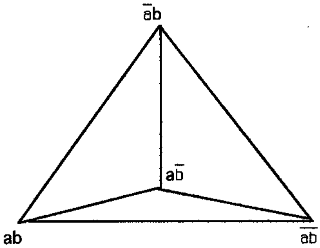
It would again be speculation, in the absence of answers to these questions, and especially to the first two, to suggest that Russell, rather than Wittgenstein, or for that matter Wittgenstein rather than Russell, deserves the bulk of the credit for formulating the truth-table device, and there is nothing in intellectual history that precludes the possibility of independent simultaneous discovery or invention. If, however, there is a source which may be common to Wittgenstein, Łukasiewicz, Post, and Zhegalkin, it is more likely to have been either Russell himself or someone else, with Russell in the latter case serving as a conduit. Again, we would be speculating in assigning the role to Russell, either as conduit or as originator. It would also be speculating to attempt to determine the identity of the person from whom Russell might have learned about truth tables, if Russell was mainly or merely a conduit. (If Wittgenstein were the originator, it is more likely that Russell would have been the conduit, rather than the reclusive Wittgenstein.) But if, in fact, Russell was a conduit for others who developed the truth-table device, the most likely medium would appear to be Russell's Harvard lecture and possibly Lecture III of "The Philosophy of Logical Atomism" in early 1918 or his *Introduction to Mathematical Philosophy* (1919), or both, and the most likely source Schröder's *Vorlesungen über die Algebra der Logik*, which Bocheński [*1970*, 333] points to as having the idea for such a decision procedure. But however close Schröder may be interpreted as coming to developing the idea, truth-table matrices do not themselves appear in the *Vorlesungen*, and it is not readily apparent that he makes the sharp distinction between the truth-table technique and the truth-table device that Shosky makes.

Abandoning speculation, let us examine the available facts.

The first and most salient fact is that the earliest recognizable and nearly complete development of the truth-table *device* is due to Charles Peirce and his students, in particular to his student Christine Ladd-Franklin (at that time still Christine Ladd). In the lecture notes for Peirce's course from the autumn of 1880 taken by Alan Marquand, we find Peirce using the diagram



to show the four combinations that two terms can take with respect to truth values—or which two objects can take with respect to existence (see editors' notes [Peirce *1989*, 569]). Peirce also suggested that the relations between these four combinations could be represented by the pyramid



(see editors' notes [Peirce *1989*, 569]). In her class notes, which contain some annotations by Peirce, Ladd constructed diagrams representing possible worlds for four combinations of the sets of truth values for two terms and then provided a tabular summary of her findings (see editors' notes [Peirce *1989*, 570]). In her paper "On the Algebra of Logic" [Ladd *1883*, 61], she pointed out that for n-many terms there were 2^n-many possible combinations of truth values, and she went on to provide a full-scale table for "the sixteen possible combinations of the universe with respect to two terms"; writing 0's and 1's for false and true and replacing the assignment of the truth-value false with the negation of the terms, she arrived at the table providing sixteen truth values for $\{\overline{ab}\}$, $\{a\bar{b}\}$, $\{\bar{a}b\}$ and $\{ab\}$ [Ladd *1883*, 62]. She also pointed out [Ladd *1883*, 63] that this table was borrowed from William Stanley Jevons's textbook *The Principles of Science* [Jevons *1874*; *1879*, 135], who, however, there rejected cases containing non-existent terms.

The next step was taken by Peirce himself. To determine whether this is so, let us begin by examining the assertions of George D. W. Berry on the matter.

Berry (in his [*1952*]) can, at first glance, be interpreted as supporting Shosky's contention that the first recognizable and ascribable example of a truth-table device is assignable to Wittgenstein and/or Russell. A closer reading of Berry's discussion, however, suggests that in fact he did acknowledge that Peirce did, indeed, "gave one example" of a truth-table matrix. Robert Lane, referring to [Berry *1952*, 158], [Fisch & Turquette *1966*, 71–2], [Łukasiewicz & Tarski *1930*, 40 n.2], and [Church *1956*, 162] in their citations of Peirce's [*1885*, 191; *1933a*, 213, 4.262], tells us [Lane *1999*, 284] that, "For many years, commentators have recognized that Peirce anticipated the truth-table method for deciding whether a wff is a tautology." Moreover, Lane [*1999*, 284], referring to [Peirce *1933a*; 4.262], adds that "it has long been known that [Peirce] gave an example of a two-valued truth-table." Lane explains that Berry [*1952*, 158] acknowledges this early appearance of the truth table. Peirce used the 1902 truth table, not to display the interpretations (or, as he himself said, the sets of values) on which a specific compound formula, consisting of three formulae, is true. He did not indicate the compound formula he had in mind. He seems to have intended the truth table to illustrate his claim that "a good many propositions concerning three quantities cannot be expressed" using propositional connectives. (Lane [*1999*, 304 n.4])

If it is true that "it has long been known that [Peirce] gave an example of a two-valued truth table", it is unclear how Shosky might possibly have missed it.

The citation from the 1885 article, "On the Algebra of Logic: a Contribution to the Philosophy of Notation" [Peirce *1885*, 191; *1933*, 3.387], in providing a definition of necessary truth and a technique for determining whether a complex proposition is necessarily true in terms of arranging its combinations of truth values, if read anachronistically, in effect gives what is unmistakenly an explanation for the use of an indirect truth table. In fact, it is a truth-functional analysis, and satisfies the condition for a truth-table technique, but yet not a truth-table device.

The question is whether Peirce ever, expressed in Shosky's terminology, arrived at a truth-table device or merely the truth-table technique. Berry [*1952*] held that Peirce developed the technique, but not the device. He referred [Berry *1952*, 158] to an untitled "paper written in 1902" and placed in Volume 4 of the Hartshorne and Weiss edition of Peirce's *Collected Papers* in which, he said, Peirce "explained the theoretical basis of truth tables and gave one example [Peirce *1933a*, 4.260–262]", although he thought that "Peirce apparently never devised a matrix or truth-table version of the propositional calculus". He went on to assert that Peirce "seems never to have thought of truth-tables as a means of defining connectives rather than as a technique for establishing truths."

In fact, in the work from 1902, Peirce displayed the following table for three terms, *x*, *y*, *z*, writing **v** for true and **f** for false:

x	*y*	*z*
V	**V**	**V**
V	**F**	**F**
F	**V**	**F**
F	**F**	**V**

Moreover, he noted there that, for a proposition having *n*-many terms, there would be 2^n-many sets of truth values. Shea Zellweger and the late William Glenn Clark have argued convincingly that Peirce developed both the truth-table technique and set forth an unmistakable truth-table device. The device can be found in a manuscript discovered by Shea Zellweger in 1983.

Clark announced Zellweger's discovery on 7 September 1989 at the Peirce

Sesquicentennial Congress (see [Clark *1989*]). The detailed analysis and reconstruction was presented by [Clark *1997*], and further developed by [Zellweger *1997*]. The pages containing Peirce's truth-table device were found in a manuscript at Harvard labelled MS 839:262.[2] It is interesting that already in the early 1960s the Italian historian of mathematics Ettore Carruccio (relying upon second-hand information from the incompletely documented reference [Vaccarino *1958*, 243]) was able to assert [Carruccio *1964*, 337–8] that logical operations were represented "by the logical matrices introduced by C. S. Peirce". What Vaccarino seems to have had in mind were probably Łukasiewicz's truth tables, as the examples he gave, using Polish notation, suggest. What is unclear therefore is whether Carruccio was crediting Peirce with creating the truth-table device or showing how matrix representations developed by Peirce could be used to present a truth-table device.

Peirce's table was originally part of the manuscript "The Simplest Mathematics" written in January 1902 ("Chapter III. The Simplest Mathematics (Logic III)", MS 431; in [Peirce *1849–1914*]); see [Clark *1997*, 308–9]). Here, Peirce presented *in matrix form* the truth values of the sixteen binary connectives of propositional logic:

CHARLES PEIRCE'S TABLE FOR THE 16 BINARY CONNECTIVES

1		2	3	4	5		6	7	8	9	10	11		12	13	14	15		16
F		F	F	F	T		T	T	T	F	F	F		F	T	T	T		T
F		F	F	T	F		T	F	F	T	T	F		T	F	T	T		T
F		F	T	F	F		F	T	F	T	F	T		T	T	F	T		T
F		T	F	F	F		F	F	T	F	T	T		T	T	T	F		T

The sources of Peirce's development of the truth-table device were (1) his work on truth-table technique and (2) his work on linear algebra and matrix theory.

The first source for Peirce's development of the truth-table device is found in the advances which he proposed to Boole's truth-functional treatment of propositions, in which, for example, Boole's [*1847*, 21] formulas

$$xy = 0$$

and

$$xy = 1,$$

[2] All MS designations are to the Robin catalogue numbers [Robin *1967*].

i.e.,

$$x(1 - y) = 0$$

which are the definitions respectively for "No x are y" and "All x are y", are redefined by Peirce [*1885*, 183] in terms of truth values {**v**, **f**}, so that, as Peirce notes

$$x = \mathbf{v}$$

and

$$x = \mathbf{f}$$

mean "[proposition] x is true" and "[proposition] x is false", respectively, and the formula

$$(x - \mathbf{f})(x - \mathbf{v}) = 0$$

is the definition of "x is either true or false". The truth-functional analysis of propositions was well known to Russell, of course. It can be found, for example, in his manuscript "On Fundamentals" (1905), where (at p. 371 of [*Papers* 4: 359–413], the "truth-value function" f is defined to be such that "if the truth values of its values are determinate when the truth values of the corresponding arguments are given." In the introduction to the first edition of *Principia Mathematica* [Whitehead & Russell *1910*, 72], Russell however assigns to Frege the credit for formulating the expression that "The *truth-value* of a proposition is its truth if it is true, and its falsehood if it is false."

Wittgenstein's truth table for implication (without Frege's assertion) appeared in both tabular form and linearly in the *Tractatus* [*1922*, 4.31, 4.442, 5.54], as

p	q	
T	T	T
F	T	T
T	F	
F	F	T

"(TT–T) (p, q)",
[...] or, more clearly,
"(TTFT) (p, q)"

with tautologies defined as "true for the entire possibilities of the truth values of the [component] elementary propositions", and contradictories as the converse, their truth values comprised only of falsehoods [*1922*, 4.46].

The second source for Peirce's development of the truth-table device is found in his work showing that all linear associative algebras are reducible to a matrix logic, in particular in his appendix to the 1881 edition of his father's *Linear Associative Algebra*. As I showed in [Anellis *1997*, 280–1], in his paper "Brief Description of the Algebra of Relations", Peirce [*1882*] studied dyadic and triadic relations. He provided a geometric interpretation, based upon matrices, according to which dyads are ordered pairs arrayed in squares or blocks, and triplets are ordered triples arrayed in cubes. It is argued there that dual relations can therefore be shown to be equivalent to absolute terms, and that a dual relation can be regarded as being equivalent to a triple relation, so that relations of tetradic order or greater are expressible in terms of relative products of triple relations. In this case, we write the dyadic and triadic relations as sums of all of the atoms of the blocks and cubes respectively. Thus, for example, a dual relative in an n–ary universe is a system of n^2 ordered pairs arranged in an $n \times n$ matrix. Letting A be an atom of the system and letting $A : B$ be an individual dual relative, Peirce obtained

$$A = A : A + A : B + A : C + \ldots + A : N$$

and

$$A : B = A : B : A + A : B : B + A : B : C + \ldots + A : B : N$$

to show how dual relatives may be rewritten as triple relatives. The arithmetic of such systems is well known, and left to the reader. A modern presentation of the details of the arithmetic of this system, particularly as developed in [Peirce *1882*], is presented by Irving M. Copi [Copilowish *1948*]. Indeed, Copi, in a lengthy historical discussion in [Copilowish *1948*, 193–6], stresses the roots of his work in the work of Cayley, but more particularly in [Peirce *1870*], and sets himself the task of developing a modern matrix logic for the algebra of relations using the notation of *Principia* rather than the Peirce–Schröder notation. He notes [Copilowish *1948*, 194] that Peirce's object appears to have been to introduce matrices "partly as an aid in his classification of relations, and partly for the sake of illustrations or examples," citing in particular [Peirce *1882*] and Peirce's manuscript "Nomenclature and Divisions of Dyadic Relations" (MS 538; *c. 1903*) to support his conjecture concerning Peirce's purposes. So Peirce (in [B. Peirce *1881*]; see notes and appendices) had developed a matrix logic in terms of which all of the algebras presented by Benjamin Peirce (as well as those presented by Cayley and Sylvester) can readily be expressed as special cases. Thus, for example, as Houser [*1989*, lii] notes, [Peirce *1882*] also argued that Sylvester's universal multiple algebra is just a special case or interpretation of his own logic of relatives (see [Sylvester *1884*], the published version of Sylvester's Johns

Hopkins University lectures to which Peirce was responding in [Peirce *1882*]).[3] Indeed, much of the work of both Benjamin and Charles Peirce on linear algebras may well have been inspired and initiated specifically in order to show that the algebra of relations has mathematical applications outside of logic. Associative algebra is included in the theory of matrices (although there is some disagreement about this point; compare, for example, [Brunning *1981*, 96–7; 137–8], and [Lenzen *1973*]; also see [Taber *1890*] and [Hawkes *1902*] for evaluations by Peirce's contemporaries). Peirce's colleague Henry Taber [Taber *1890*, 353] wrote that "Charles Peirce has shown that the whole theory of Linear Associative Algebra is included in the theory of matrices. He has also shown that every linear associative algebra has a relational form…." Some work in this direction was also undertaken by Whitehead [*1901*]. Moreover, Copi [Copilowish *1948*] has shown that, just as representable relation algebras are matrix algebras, so every proper relation algebra can be thought of as a Boolean matrix algebra. [Iliff *1997*] shows how Peirce's work in matrix representation contributed to Peirce's development of quantification theory.

In early 1906, Russell had also, as [Grattan-Guinness *1977*, 74–5] makes clear, taken a step of his own in connecting truth values with matrices in his substitutional theory of classes. Here, the matrix for the propositional function "p/a;$x!q$" represents the operation of substituting x for a in p, resulting in proposition q.[4] This substitutional procedure, says [Grattan-Guinness *1977*, 74], "left only propositions and their truth values, and individuals named by constants." But the truth-table device as found in the manuscript of 1912 to which Shosky refers is not fully evident here. Also in 1906, in the paper "The Theory of Implication" [*1906*], Russell provided an exposition, the most definitive of his expositions prior to the publication of *Principia*, of implication as

[3] The work trying to show that universal algebras are interpretations of matrix logic presented within the formalization of the calculus of relations was carried out in a series of papers, including "On the Application of Logical Analysis to Multiple Algebra" (1875) [Peirce *1986*, 177–9], "Notes on Associative Multiple Algebra" (MS 75; 1880–81), "Linear Associative Algebra Improvement in the Classification of Vids" (1873) [Peirce *1986*, 161–3], "Notes on the Fundamentals of Algebra" (1876) [Peirce *1986*, 186–7], "Notes on the Fundamentals of Algebra" (1876) [Peirce *1986*, 187–8), "Sketch of the Theory of Non-associative Multiplicator" (1876) [Peirce *1986*, 198–201], "Note on Grassmann's Calculus of Extension" (1877) [Peirce *1986*, 238–9], the undated manuscripts "Nilpotent Algebras" (MS 97), and "Nilpotent Algebras" (MS 80), as well as in the notes and appendices to [B. Peirce *1881*].

[4] The earliest indication to date that Russell was examining this possibility is found in unpublished writings dating from November 1905, and the first public presentation occurred in the paper "On the Substitutional Theory of Classes", which Russell read at a London Mathematical Society session in April 1906; see [Lackey *1972*, 15].

the most basic primitive of the class calculus and the propositional calculus. He characterized it in terms of what he accounted its most basic property: namely, that it is independent of the meaning or even truth of the terms, explaining that *provided* the antecedent is true (regardless of whether it actually is or not), the consequent must be true under implication, i.e. The True implies the True. In this paper he also suggested a Boolean interpretation in analysing the connective, replacing truth values False and True with Boolean values 0 and 1.

Next we turn to the fragments of Peirce's manuscript *Logic Notebook* (MS 339), in particular those dating from at least early 1909 if not somewhat earlier (in *Logic* (*Logic Notebook 1865–1909*); MS 339:440–441v, 443v), which had been examined by Max Fisch and Atwell Turquette, on the basis of which they concluded [Fisch & Turquette *1966*] that by 23 February 1909 Peirce was able to extend his truth-theoretic matrices to three-valued logic, thereby anticipating both Łukasiewicz and Post by a decade in developing the truth-table device for triadic logic and multiple-valued logics, respectively. [Fisch & Turquette *1966*, 72] note that both Łukasiewicz and Post perforce "worked without knowledge of Peirce's earlier results", and that consequently their contributions are not to be underrated. [Fisch & Turquette *1966*, 82–3] think that Peirce might have been led to work out a triadic logic by the debate between MacColl and Russell on the nature of implication, and that Peirce had corresponded with MacColl. But there is no evidence that MacColl would have had access to Peirce's triadic truth-table device, and still less that Russell would have had knowledge of it before 1915, when Peirce's "extant logical manuscripts", purchased by Harvard University from Peirce's widow in 1914, arrived at Harvard.

Finally, therefore, we simply cannot determine with any degree of certitude whether Russell learned, directly or indirectly, about the relevant work of Peirce on truth tables. There is simply no evidence that Russell, let alone Wittgenstein, knew of Peirce's manuscript of 1902. Therefore we can cannot exclude the possibility of independent discovery or formulation of the truth-table device by Russell and/or Wittgenstein. And we can only speculate upon any influence which Peirce's work might have had upon Russell in regard to the history of the truth-table device. In the absence of the relevant archival evidence, any such speculation would be futile and historically inappropriate. But the discovery by Zellweger of Peirce's manuscript of 1902 *does* permit us to unequivocally declare with certitude that *the earliest, the first recorded, verifiable, cogent, attributable and complete truth-table device in modern logic attaches to Peirce*, rather than to Wittgenstein's 1912 jottings and Eliot's notes on Russell's 1914 Harvard lectures.

Very often in history, including no doubt the history of logic, it is not always the discoverer or inventor of an idea, device, technique, or tool who is awarded the credit for the discovery or invention, but the one who disseminates knowledge of the novelty. The next job for the historian of logic pursuing the history of truth-functional logic would appear to be to attempt to ascertain, in so far as extant evidence permits, whether Russell learned of this aspect of Peirce's work, and, if so, when. And that will assuredly require archival excavations and discovery of materials not yet immediately available.

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