RUSSELL'S LOGICISM

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Stefano Donati. *I fondamenti della matematica nel logicismo di Bertrand Russell* [The Foundations of Mathematics in the Logicism of Bertrand Russell]. Florence: Atheneum, 2003. Pp. 988. €39.00.

The measure of Italian writings on Russell's logic and philosophy has been modest; but now we have the longest essay on his logic in any language. Many of the mathematical topics are treated, as well as the much better known philosophical ones; the standard of referencing of texts is high. Yet the book is also among the most perplexing histories of my acquaintance. Rather than following a synchronic treatment that tracks the many interacting ways in which Russell's theories changed over time, the author treats the subject diachronically, taking a topic or theme from the 1890s or 1900s onwards and sometimes up later writings, and then not always in chronological order. In addition, several significant topics and aspects are largely or wholly passed over.

After the introduction, Chapter 1 (pp. 41–108) describes Cantorian set theory, especially the treatment of finite and transfinite arithmetic. This is an important influence upon Russell that is often poorly treated by Russellians. Curiously, only one historical work in and around Cantor is cited.

After establishing this major figure, one would expect to read next about an even greater influence on Russell: Peano, and the growth of mathematical logic. Instead Chapter 2 (pp. 109–216) treats "The Russellian Foundation of [Finite] Arithmetic", where Peano features only for his axioms; there is little historical account of the chief originator of mathematical logic, and nothing on his important followers Pieri and Padoa. Russell's own definition of integers in terms of equivalent (well-ordered) classes is done in some detail, including comparisons with Frege and Peano's theories. Chapter 3 (pp. 217–322) treats Russell's "extension" to real and complex numbers (including the relation numbers), and some of their relationships to geometry. The second definition of real numbers (positive and negative) in *Principia Mathematica*, usually ignored, is duly noted.

Chapter 4 (pp. 323–404) deals with Russell's "crisis", and covers not only the paradoxes and his attempted solutions but also the axiom of choice. It is nice to see the latter topic given the space that it deserves; the author might have stressed more on page 375 that Russell seems to have slightly anticipated

Zermelo in its discovery.

Now come two jumbo chapters: 5 "Towards the Ramified Theory of Types" (pp. 405–640), and 6 on the definitive versions (pp. 641–860). The account begins with the first detailed discussion of Russell's *Principles of Mathematics* (1903), with Donati's book nearly half over. Chapter 5 then proceeds through the theory of denoting and goes up to the three possible solutions of the paradoxes that Russell was entertaining by 1906. One solution was the substitutional theory, which Russell pursued for some time; the author reviews both it and the differing opinions of its merits that have been aired by historians in recent times. (However, the paradox that Russell found within it is not considered.) The next chapter starts with the vicious-circle principle and examines the type theories in the 1908 paper and that (those?) given in *Principia*. An important limitation, indeed refutation, of logicism is missed on page 708; since only a finite number of types can be defined, the upper end of the sequence of Cantor's transfinite numbers is not definable, so that the territory of the paradoxes of the greatest cardinal and ordinal cannot even be approached. The chapter concludes with various "objections", such as queries over the dubious axioms. Zermelo's axiomatization of set theory comes here, though it is hardly an objection to Russell but an alternative approach to the foundations of set theory.

Finally comes Chapter 7, on "The Logicism of Russell" (pp. 861–944). Inevitably this matter has turned up earlier: the concern here is with its most general features, and with the book manuscript of 1913 on epistemology that Russell came to abandon. Also considered are some positions taken later by Gödel and Quine; the many (near) omissions include the early work (1925–35) of Chwistek, Quine and Carnap. Some aspects of Ramsey were handled among the earlier objectors to type theory, and others in connection with the axiom of choice.

The book ends with a survey of Italian translations of Russell's logical and philosophical writings. Apparently many of them are "untrustworthy". So unfortunately is this historical treatment overall, in that it is distorted or silent in several respects. For example, how can one properly describe Russell's foundation(s) of arithmetic *before* the type theories are in place? And why is so little said about Russell's reconstruction of Cantor's transfinite arithmetic, which was treated in some depth in Chapter 1? Again, no proper prehistory of mathematical logic is provided. The biographical side of Russell is confined to part of the introduction; for example, the tortuous publication history of *Principia* is completely ignored. The author has used Russell manuscripts, but only as published in the *Collected Papers*; and as Volume 5 is still not published, manuscripts from the important period 1906–08 are mentioned only from others' references to them. The bibliography is extensive; but very few items of historical literature after 1996 are cited, so that all sorts of pertinent things are missing. The last gap comes at the end: this massive and complicated book has no indexes at all.