

THE LOGIC OF *SINN*

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Kevin C. Klement. *Frege and the Logic of Sense and Reference*. London and New York: Routledge, 2002. Pp. xiii, 260. US\$85.00.

This is, I think, the best book on Frege I have ever read. It has a narrow focus, dealing only with Frege's distinction between *Sinn* and *Bedeutung*, but it covers it exceptionally thoroughly, adding considerably to our knowledge of what might have seemed to be an overworked topic and throwing, in the process, a flood of light on a number of other Fregean topics. Klement's book exemplifies what used to be the hallmark virtues of analytic philosophy—rigour, clarity, and attention to detail—and shows that, even on topics that have been deluged in ink, they pay off.

Frege's distinction between *Sinn* and *Bedeutung* has been widely hailed as his most important contribution to the philosophy of language, and, indeed, one of the greatest contributions to philosophy of language ever. Despite partial anticipations, notably in Mill's distinction between connotation and denotation,¹ Frege was the first modern philosopher to maintain that the distinction holds for all linguistic expressions and the first to distinguish sense from the psychological associations of the expression (which Frege labelled its "tone"). Despite Frege's systematic elaboration of propositional and predicate logic, he never attempted a similarly rigorous semantic theory. Nonetheless, his remarks on the *Sinn/Bedeutung* distinction suggest that he would have regarded it as a feasible project. He can indeed be thought to have set the stage for such a theory, without actually supplying one.

That Frege himself did not attempt a formal semantics based on the *Sinn/Bedeutung* distinction hardly constitutes dereliction of duty, but, as work on formal semantics burgeoned during the twentieth century, it becomes more surprising that the project attracted so little attention, especially since the *Sinn/*

¹ J. S. Mill. *A System of Logic* (1843), Bk. I, Chap. 2.

Bedeutung distinction retains wide intuitive appeal. At least one problem with the project was pointed out by Russell in a letter to Frege of 29 September 1902:² since Frege admits *Gedanken* as the *Sinne* of complete sentences, an analogue of the propositional paradox which Russell presents in Appendix B of *The Principles of Mathematics* (*PoM*, §500) should arise in the logic of *Sinn*.³ Frege, in the absence of a formal proof, refused to admit that this was so⁴—and, of course, in the absence of an explicit logic for *Sinn*, a formal proof was not forthcoming. Though he did not intend to be historically faithful to Frege, Alonzo Church took up the project of creating a viable logic of sense and reference in the 1940s,⁵ and it remained very much his project for the next 50 years. Church presented three different systems, which he called Alternatives 0, 1, and 2, with progressively more relaxed identity criteria for senses. On Alternative 2, which he favoured on account of its appropriateness for modal logic, two sentences, *A* and *B*, had the same sense if $A \equiv B$ was a thesis of the logic. This system had little relevance for Frege, since it was part of Frege's intention to use the notion of *Sinn* to capture what he called the "cognitive value" (*Erkenntniswert*) of an expression;⁶ plainly $A \equiv B$ may be a thesis even though *A* and *B* differ in cognitive value. Alternatives 0 and 1 attempt to capture a much stronger notion that Church calls synonymous isomorphism. They differ in that in Alternative 1 synonymous isomorphism is preserved under λ -conversion, while in Alternative 0 it is not; that is, in Alternative 1 " $(\lambda x Fx)a$ " and "*Fa*" are synonymously isomorphic while in Alternative 0 they are not.

Unfortunately, Church developed his theories within simple type theory, the theory which Russell had found himself forced to abandon on account of the propositional paradox he had presented to Frege in September 1902. It did not take long before Myhill proved a close relative of this paradox, usually now known as the Russell–Myhill paradox, in Church's Alternative 1—rigour has its perils!⁷ Church ultimately responded, as Russell had done, by ramifying his

² Frege, *Philosophical and Mathematical Correspondence*, ed. by G. Gabriel *et al.*, transl. H. Kaal (Chicago: U. of Chicago P., 1980), pp. 147–8.

³ There is an excellent account of the §500 paradox by André Fuhrmann, "Russell's Early Type Theory and the Paradox of Propositions", *Principia: revista internacional de epistemologia*, 5 (2001): "Russell Symposium", 19–42. Klement discusses the paradox in detail on pp. 176–88 and in "Russell's Paradox in Appendix B of the *Principles of Mathematics*: Was Frege's Response Adequate?", *History and Philosophy of Logic*, 22 (2001): 13–28.

⁴ Frege to Russell, 20 Oct. 1902, *Philosophical and Mathematical Correspondence*, pp. 149–50.

⁵ Alonzo Church, "A Formulation of the Logic of Sense and Denotation (Abstract)", *Journal of Symbolic Logic*, 11 (1946): 31; "A Formulation of the Logic of Sense and Denotation" in P. Henle, H. Kallen, and S. Langer, eds., *Structure, Method, and Meaning: Essays in Honor of Henry M. Sheffer* (New York: Liberal Arts Press, 1951), pp. 3–24.

⁶ Frege, "On *Sinn* and *Bedeutung*" (1891), in Michael Beaney, ed., *The Frege Reader* (Oxford: Blackwell, 1997), pp. 151–2.

⁷ John Myhill, "Problems Arising in the Formalization of Intensional Logic", *Logique et analyse*,

type theory.⁸ As Klement makes clear (pp. 117–24), Church's logic of sense and denotation differs in several important respects from Frege's views about *Sinn* and *Bedeutung*. In particular, Church allows functions to appear in subject position as if they were complete and the type restrictions he imposes (especially in his final ramified form of the theory) in order to avoid the paradoxes have no Fregean warrant (e.g., senses fall into different logical types according to the nature of their denotations—for Frege *Sinne* are objects independent of their *Bedeutungen*).

This last parenthetical claim about Frege requires qualification, for there is a widespread view (which I held myself until reading Klement's book) that the *Sinne* of function-expressions are themselves functions (sense-functions); for it would seem that, if they are not functions, then they are objects and then a *Gedanke* would consist merely of a string of objects without the necessary unity, which comes from an object's saturating a function. Against this view, Dummett⁹ has argued that, if the *Sinne* of function-expressions are sense-functions then the *Gedanke* expressed by the sentence " $F(a)$ " cannot be *composed*, as Frege says it is, of the *Sinn* of the function-expression and the *Sinn* of its argument-expression, for the value of a function for an argument is not composed of the function and the argument. Moreover, without the compositionality of *Gedanken* we are left with no explanation of how we can grasp the *Gedanken* expressed by sentences we have never heard before.

There seems, on this issue, a serious tension in Frege's philosophy which he did not seriously address. There is, in fact, no interpretation which is consistent with all the things he said on the topic. In a long and subtle examination, which pays close attention to Frege's texts, Klement comes to the conclusion that the *Sinne* of function-expressions are neither functions nor objects, but "a particular type of unsaturated entity in the realm of *Sinn*" (p. 74). They are peculiar in that the only objects that can complete them are *Sinne* (a fact which distinguishes them from functions) and, moreover, *Sinne* of different types according to the level of the original function-expression, since a *Gedanke* must always be the result of such a completion and *Gedanken* consist entirely of *Sinne* (p. 128). On this view we have to give up the view that object and function are an exhaustive dichotomy. In mitigation, Klement points out that in the places

1 (1958): 78–83. Other problems included analogues of the liar paradox.

⁸ Church, "A Revised Formulation of the Logic of Sense and Denotation. Alternative (1)", *Nous*, 27 (1993): 141–57. Between these two formulations, he tried a more Tarskian approach, ("Outline of a Revised Formulation of the Logic of Sense and Denotation", Parts I and II, *Nous*, 7 [1973]: 24–33; 8 [1974]: 135–56), which Klement describes on pp. 115–17.

⁹ Michael Dummett, *The Interpretation of Frege's Philosophy* (London: Duckworth, 1981), Chap. 13.

where Frege explicitly states that everything is either a function or an object he is talking about his own formal language, where indeed it is true (but only because Frege never did formulate his logic of *Sinn*). With or without mitigation, I think Klement must be right: of all the doctrines at issue, the exhaustive dichotomy of object and function seems to be the least entrenched in Frege's philosophy and the one that can be given up with least damage to the whole. It results, however, in additional notational complexities for the formal logic of sense that Klement goes on to develop (pp. 127–31).

After an introductory chapter outlining the need for adding a logic of *Sinn* and *Bedeutung* to Frege's logical system, Klement has two chapters of careful exegesis of Frege's writings. Not all issues are treated with the detail accorded to the vexed issue of the *Sinne* of function-expressions, but all are treated with a thorough knowledge of the texts and an extensive knowledge of the secondary literature as well. There is, for example, only a short discussion of the (equally vexed) issue of the context principle (pp. 77–83), but it is a perceptive one. Chapter 4 is an exposition of Church's intensional logic and its problems. All this leads up to Chapter 5, in which Klement lays out a genuinely Fregean logic of *Sinn* and *Bedeutung*, designed (unlike Church's) to capture as faithfully as possible Frege's own doctrines. One cannot but marvel at how carefully the system is constructed. The requirements are more than a little complicated: every expression expresses a *Sinn* and denotes a *Bedeutung*, there are objects and functions (of various levels) as well as the incomplete entities that are the *Sinne* of function-expressions, and constants and variables for each, as well as special constants to represent the *Sinne* of all the primitive signs of Frege's original syntax. For all this Klement deploys the typographic resources of the Roman, Gothic, Greek, Hebrew and Cyrillic alphabets and even then has to add asterisks for variables whose range is limited to complete *Sinn*. To the original nine axioms with which Klement formulates Frege's original function calculus and his calculus of value-ranges, he now adds twenty-eight axioms for the logic of *Sinn* and *Bedeutung* and an additional sixteen axioms to govern the special constants introduced for the *Sinne* of the logical symbols.

All this, of course, suggests a considerable labour, even for the reader.¹⁰ The pay-off comes in having the ability to demonstrate, for example, the validity of inferences such as that from "Aristotle was a philosopher" and "Jones believes that Aristotle was a philosopher" to "Jones believes something true" (pp. 154–5), which Blackburn and Code¹¹ set as a puzzle for Frege's theory. Problems with

¹⁰ Klement usefully lists the definitions, axioms, and rules of the system in an Appendix. It is pity he did not also include there a list of his typographical conventions and some of the special constants he uses. It would have saved the forgetful reader much thumbing back through the text.

¹¹ Simon Blackburn and Alan Code, "The Power of Russell's Criticism of Frege: 'On Denoting',

quantifying in are dealt with satisfactorily as well (pp. 156–7). Consider the inferences from, respectively,

(1) Gottlob believes that the morning star is a planet

and

(2) Gottlob believes that Vulcan is a planet

to

(3) There is something Gottlob believes to be a planet.

Those who like their quantifiers existentially loaded will be untroubled by the first inference but find the second problematic, for Vulcan (the planet which in pre-relativity days was thought to orbit between the Sun and Mercury) does not exist.¹² Without getting into the details of Klement's notation, (1) is transcribed by a formula which states that Gottlob believes the *Gedanke* which consists of the incomplete *Sinn* expressed by the function “ ξ is a planet” completed by the *Sinn* expressed by “the morning star”.¹³ This immediately implies that there is a *Sinn* such that, when it saturates the *Sinn* expressed by “ ξ is a planet”, it yields a *Gedanke* believed by Gottlob. But we can't get from this to the claim, made by (3), that there is an object which the *Gedanke* believed by Gottlob is about unless we have the additional assumption that the *Sinn* which saturates the *Sinn* expressed by “ ξ is a planet” in Gottlob's *Gedanke* picks out an object. In the case of (1) this assumption holds and the inference goes through; in the case of (2) it fails, and the inference is blocked, exactly as required.

But all is not well with the system. In fact, we knew ahead of time that the system would be inconsistent, for it has been built as an extension, not merely of Frege's functional calculus, but also of his calculus for value-ranges, including the infamous Basic Law V (which appears in Klement's system as Axiom FC^{+V7}). Russell's paradox, as well as new paradoxes concerning *Sinne*, can thus be proved in the system (pp. 158–60). This, of course, is entirely to be expected. However, even if we strip away the value range calculus, the resulting system,

Pp. 48–50”, *Analysis*, 38 (1978): 71–2.

¹² Meinongians, like myself, who use quantifiers without existential loading, will treat the matter rather differently.

¹³ Remember that, for Frege, in *oratio obliqua* contexts, expressions refer, not to their usual *Bedeutungen*, but to their usual *Sinne*.

FC^{+SB} , contains an analogue of the Russell–Myhill paradox.¹⁴ Informally, for every concept-expression there is what Klement calls a universal *Gedanke*, namely the *Gedanke* that everything falls under that concept. Some of these universal *Gedanke* fall under the concept they generalize and some do not. Consider now the concept *F* such that an object falls under it just in case it is a universal *Gedanke* which does not fall under the concept it generalizes. A universal *Gedanke* falls under *F* just in case it does not. The formal proof (pp. 164–9) is long and complex. Since Frege’s functional calculus is known to be consistent, the error must lie in the axioms presented for the logic of *Sinn* and *Bedeutung*. As Klement says: “We can only conclude that the expansion of Frege’s [conceptual notation] in line with his own semantic theories—indeed, in ways he himself suggests—reveals internal difficulties and flaws within his overall philosophical position that have hitherto gone unnoticed and unaddressed” (p. 169). Thus, exactly 100 years after Frege asked Russell precisely how his doctrine of *Gedanken* would lead to contradictions, we learn the answer. Progress in philosophy is often slow.

Klement identifies four paradoxes in Frege’s theory of *Sinn* and *Bedeutung*. As his subsequent discussion makes clear, they in no way depend upon idiosyncratic features of his system FC^{+SB} : in Chapter 6 he provides very strong arguments for thinking that similar paradoxes arise in other systems of intensional logic.¹⁵ This is not, I think, unduly surprising. The paradoxes arise, as Russell realized (and as Frege failed to see) from Cantorean diagonalization arguments based on the realm of *Sinne* and *Gedanken*. The same considerations that apply to *Sinne* and *Gedanken* will apply, *mutatis mutandis*, to other types of intensional objects. If intensional objects are to do the job for which they are intended, their identity conditions will have to cut exceedingly fine, and so there will be an awful lot of them. In Frege’s system there will be a *Sinn* for every class, yet by Cantor’s powerset theorem we know that there must be more classes of *Sinne* than *Sinne*.¹⁶ Cutting back on the intensional objects, however, undermines the purpose for which they were introduced, in particular in handling intentional discourse. Cardinality alone, however, does not account for all the paradoxes: diagonalization is the source of the problem. Klement’s modified Epimenides, for example, takes this form:

¹⁴ Indeed, as Klement shows later (pp. 186–8), with the calculus of value-ranges added, the system contains an exact analogue of Russell’s original paradox of propositions from Appendix B of the *Principles*.

¹⁵ In these cases fully formal proofs are not forthcoming because the theories in question lack the formal articulation that Klement has given to Frege’s theory.

¹⁶ Klement has provided further discussion of these cardinality problems for *Sinne* in “The Number of Senses”, *Erkenntnis*, 58 (2003): 302–23 and “Does Frege Have Too Many Thoughts? A Cantorian Problem Revisited”, *Analysis*, 65 (2005): 44–9.

“Church’s favourite *Gedanke*” picks out a *Gedanke* as its *Bedeutung*. Suppose the *Gedanke* it picks out is: the *Gedanke* that Church’s favourite *Gedanke* does not pick out the True as its *Bedeutung*. Then Church’s favourite *Gedanke* picks out the True just in case it does not.

The problem here does not arise from the number of *Gedanken* Church entertains, but from their nature. Yet there seems no reason why Church should be logically precluded from choosing this as his favourite *Gedanke*. If intensional logic is to be adequate to the intentional discourse it serves, then there must be a *Sinn* (or other intensional entity), expressed by “Church’s favourite *Gedanke*”, which (assuming Church has preferences among his *Gedanke*) picks out some *Gedanke* as its *Bedeutung*. Moreover, “Church’s favourite *Gedanke* does not pick out the True as its *Bedeutung*” must express some *Gedanke*. What reason could we give for thinking that, *as a matter of logic*, the *Gedanke* expressed by the sentence could not be identical to the *Gedanke* picked out by the definite description?

In the final chapter, Klement considers what revisions could be made to Frege’s system to block the paradoxes. The prospects are not good: modest revisions fail to eliminate all the paradoxes; more radical ones result in systems that are alien to the spirit of Frege’s philosophy. Of all the options considered, the most plausible is to follow Russell and ramify (the policy Church adopted in the end for his own intensional logic). Various ramification policies would be possible and Klement suggests some of them, though he does not develop any of them formally. The problems are not technical—the paradoxes would all be eliminated—but philosophical: how would we motivate ramification within a Fregean framework? Russell motivates his own ramified theory by appeal to the vicious circle principle, but how could the vicious circle principle get a purchase in Frege’s system, where *Sinne* are all independent objects? Klement suggests that it might be possible if one abandons the independence of *Sinne*, adopting instead an account on which “a *Sinn* is metaphysically dependent in some way upon its *Bedeutung*” (p. 226). This, as he notes (p. 227), is a “substantial deviation from Frege’s own views”. Worse, it would entail that there were no *Sinne* without *Bedeutungen*. This would seem to undermine one of the main (non-Fregean) reasons which make *Sinne* desirable, namely to account for the intelligibility of referential expressions in cases where *Bedeutung* is lacking. It might be possible to mitigate this ill effect, by identifying a fundamental level of (simple?) *Sinne* which do depend upon *Bedeutungen*, and then to construct, in accordance with the strictures of the ramified theory, other levels (complex *Sinne*?) which do not.¹⁷ Quite apart from the difficulties of constructing the

¹⁷ More elegantly, one might suppose a base-level language in which *Sinne* depend upon *Bedeu-*

hierarchy (and establishing its adequacy), the primary philosophical problem here would lie in identifying those *Sinne* which belong to the base level and justifying their inclusion there.

Klement's formalization of Frege's logic of *Sinn* and *Bedeutung* is an enormous step forward in Frege scholarship. It allows him to conduct his discussion at a level of rigour that has hitherto been impossible. Quite apart from its technical sophistication, Klement's book is informed by an extensive and detailed knowledge of Frege's philosophy and, despite the very great intricacies of the issues involved, Klement writes with a consistent clarity which has all too often eluded many writers on Frege. Altogether this book is an achievement of the highest order.

tungen, and then construct a hierarchy of higher-level languages which were not subject to this condition. The model here would not be the Tarskian hierarchy of languages, but something more like the hierarchy Russell proposes in *An Inquiry into Meaning and Truth*.
