NEW MANUSCRIPT LEAVES AND THE PRINTING OF THE FIRST EDITION OF
PRINCIPIA MATHEMATICA

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Three half-leaves of the final manuscript of Principia Mathematica have come to light in the Bertrand Russell Archives. They were originally tucked in Russell’s own copy but avoided archival notice because their versos had been employed for an index of propositions used in theorem *350·62. The leaves form the whole of a folio 152 and the top half of 153 and include *336·51 through part of *336·52, on pages 400–1 of Volume 111. Markings by the Cambridge University Press add to our knowledge of the typesetting and proofreading of PM and give some indication of the fate of the remainder of the approximately 5–6,000 manuscript leaves, of which only one had been known to have survived.

INTRODUCTION

Although the original manuscript of Principia Mathematica (3 vols., 1910, 1912, 1913), on which Bertrand Russell collaborated with A. N. Whitehead … was destroyed, much of the preliminary work and some rejected sections of it are in this collection, as well as the introduction and appendices to the second edition (1925 and 1927).

(Feinberg, p. 61')

1 This page of A Detailed Catalogue of the Archives of Bertrand Russell cites a letter to Continuum 1 Limited from Lord Russell’s secretary (Christopher Fairley), 25 April 1967. KB saw the letter then, but it is not to be found in the Felton or Feinberg papers in BRA.
Three half-leaves of the final manuscript of *Principia Mathematica* have come to light in the Bertrand Russell Archives. Although slight in bulk, they provide valuable clues on how *Principia* was typeset, proofread and printed.

These leaves for Volume 111, pages 400 and 401, include the statements of theorems *A3651*, *A36511* and *A3652*. The first two fragments are the top and bottom of a leaf of manuscript foliated as 152, the third the top half of folio 153 (hereafter referred to as “152t”, “152b” and “153t”). The leaves were found in Volume 1 of Russell’s first-edition copy of *Principia* when it arrived at the Archives. They were catalogued in file R210.147501 and then stored in box 8.41 with the rest of that material ever since. The versos of the three half-leaves are connected (and follow other tracing notes), tracing back the “Props used in proof of *A35062*” (a proposition that is printed at PM, 3: 416). The tracing begins on 152b (see figure 1) and continues on 153t with *A2613521226*, which is mentioned in the middle of 152b, followed by seven more numbers, and 152t, which begins with *A3327166...* which follows another thread begun on 152b. This pattern suggests that a printer’s sheet containing *A3651* was sent to Russell with the corresponding manuscript, corrected and the proofs returned. (The authors’ proofreading took place on octavo sheets that were folded, or perhaps cut, into sixteen-page gatherings.) Russell would have soon discarded the manuscript, using some of the leaves as scrap when he turned to tracing, in his concatenated way, the propositions used in the proof of *A35062* at the end of the next sheet of proofs, which covered pages 401–16.

As Russell indicated to Lady Ottoline Morrell in a 1911 letter, checking that prior theorems were cited properly was carried on during the

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2 On 26 or 27 May 2005, while we were examining the versos and other papers that had been removed from Russell’s copy of Volume 1 of PM (see B&S, p. 7). While we don’t remember who actually turned the first sheet over, or which it was, it took only seconds to identify the find. KB recognized the era of the handwriting, while BL had to reason that there was no manuscript of the main text of the second edition of PM. Volumes 1 and 11 were reset from the first edition, while Volume 111 was photographically reproduced. When a check revealed that *A336* is in Volume 111, that cinched the case. A quick search of the rest of the file revealed the other two sheets. Not enough attention has been paid to Russell’s recycling of manuscript leaves, which is what he did in this case and with many of his *Principia* papers during the writing.

3 Cited in note 10.
proofreading process. Indeed, as an aid to accuracy, Russell had main-
tained a concordance of cross-references in the burgeoning *Principia*.

The three half-leaves are the only known leaves in addition to the
long-known manuscript of *x*208·4 in Volume 11, held in the Ottoline
Morrell papers in the Ransom Library at the University of Texas at
Austin. The Bertrand Russell Archives manuscripts thus bring the total
of known manuscripts to two and a half leaves of the approximately five
to six thousand for the three volumes. Russell describes the first 4,000

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4 [RAI 230.0312701–5, headed “Prop. Where Used”. The 260 leaves take the index
only as far as *x*276·43. Every proposition that is referred to is listed, followed by a list
of propositions that cite it. This manuscript represents a large effort. Russell extracted
from it a list of “Unused Props”, also found in his copy of *PM*. If his tracings, as they are
known to us, were routine, they would have been another large effort.

5 A photocopy is in bra, attached to Russell’s letter to Ottoline cited in note 10.
Figure 2 The top half of folio 152 (reduced by 30%).
Figure 3 The bottom half of folio 152 (reduced by 30%).
leaves as being packed into “two large crates” when they were ready to be sent to Cambridge University Press on 18 October 1909, and he indicated that he held back a quantity that could easily be finished later. While colourful rumours have circulated as to the fate of the bulk of the manuscript, it is likely that Russell destroyed the manuscript “copy” after each printer’s sheet of proofs was proofread and returned to the printer for correction. This was evidently Russell’s practice with published articles from the period. The manuscripts for the material added for the second edition of *Principia* are all in the Archives, however, and are helpful with the understanding of this material. There was no intervening typescript. As Russell recalled, “It was not, of course, the sort of manuscript that could be typed, or even copied” (*Auto.*, 1: 152).

**THE NEW HALF-LEAVES**

Folio 152t, earlier numbered “12”, contains *336·51* and the first half of its proof:

\[
*336·51 \vdash: \kappa \in FM \text{ sr} \cdot R, S \in \kappa \cdot v \in Nc \text{ ind} - v^0 \cdot \square:

(R^a)(i^v\kappa_0)(S^v a) \equiv (R^{v^v} a)(i^v\kappa_0)(S^{v^v} a)
\]

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7 These include the legends that they were in Whitehead’s possession and then burnt with other papers at his request following his death, and that Dora Russell took them with her to Soviet Russia in 1929. (Lloyd Brereton, who had later been married to Doreen Joad, Russell’s secretary in the late 1920s, told KB this about 1970.) Russell told KB in an interview in 1967 that he had destroyed the manuscript out of revulsion for it, but did not say whether this was in routine batches as the proofs were corrected, or in a single violent act.

8 This includes the manuscripts of the “Introduction to the Second Edition” as well as the three Appendices, a, b and c, as well as two collections of manuscript pages. “The Hierarchy of Propositions and Functions” (RA1 230.031400) is a large manuscript from which leaves of the Introduction and Appendix a were drawn. The “Amended List of Propositions” (RA1 230.03190) consists of 52 leaves of working notes for Appendix b.
See figure 2. The typeset version is reproduced as well. Note the more economical layout of formula lines and the more pleasing spacing of symbols.

Folio 152b (figure 3) completes that proof and concludes with the statement of the corollary printed at the top of page 401:

\[ \exists 336 \cdot 511 \vdash \kappa \in FM \land R, S \in \kappa \land \nu \in NC \land \imath \in 0 \cdot \exists : RU_\nu S \equiv \exists R^\nu U_\nu S^\nu \]

Folio 153t, formerly numbered “13” (see figure 5), contains the statement of \#336 \cdot 52 and the beginning of its proof:

\[ \exists 336 \cdot 52 \vdash \kappa \in FM \land Q, R, S, T \in \kappa \land \chi \in \mathbb{G}((\bar{Q} \mid R) \cap \mathbb{G}((\bar{S} \mid T)). \exists : (\bar{Q} \mid R) V_\chi(\bar{S} \mid T) \equiv \exists (S \cdot R \cdot \chi) \cdot \exists (Q \cdot T \cdot \chi) \]

The title of \#336 is “The Series of Vectors” and occurs in Part vi “Quantity”, Section b “Vector-Families”. The initial “Summary of Section b”, pages 339–49 in Volume 111, states the concern of the section as “... the theory of magnitude, so far as this may be developed without measurement...” Measurement, in turn, is identified as “the application
**Figure 5** The top half of folio 153 (reduced by 30%).
of ratios and real numbers to magnitudes” (3: 339). The theory of magnitude in *Principia* is based on the theory of vectors, which are relations that form a “vector family”, i.e. a class of one-one relations all having the same converse domain, and all having their domain contained in their converse domain” (3: 233).

*FM* is the class of vector families. Temporal quantities, i.e. intervals of time, are a straightforward example. Moments of time will bear relations to each other, and those relations will form a vector family. A relation (being earlier by a certain interval) may hold between moments *x* and *y* and between *y* and *z*. In that case *xRz*, and similarly for any finite number *ν* greater than 0, i.e. member of NC ind – ε0. The number of steps of the relation *R* between moments will be the source of the number which measures the lapse of time between them. Times that are measured by rational and real numbers are represented as ratios (proportions) between families of vectors *κ*. If each point in the field (union of the domain and range) of a vector family can be reached from some single “connected point”, the family is connected, i.e. in *FM* conx. Families which are conx and transitive are serial, in *FM* sr. The non-zero vectors, i.e., those that do not relate a point to itself, are denominated by *κφ*, and *κφ* is the sum of those vectors, much as the “earlier than” relation is derived from the relation of “immediately preceding”, both of which hold between moments. Putting these notions together, *336 §51* is seen to assert that if *κ* is a serial vector family including *R* and *S*, and *ν* is a finite integer greater than 0, then the point to which *a* is related by *R* and the point to which *a* is related by *S*, are related by a non-zero vector in *κ* if and only if the point to which *a* is related by *ν* steps of *R* is related by a non-zero vector in *κ* to the point to which *a* is related by *ν* steps of *S*. This is a step in developing the notion of a ratio of steps of *R* to steps of *S* to derive a measurement. *Uκ* and *Vκ* are “the general relations from which greater and less [among magnitudes] are derived” (3: 395), the features that they need to do this being demonstrated beginning with *336 §511* and *336 §52.*

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* The theory of real numbers in *PM* is described by John Bigelow and Robert Pargetter in *Science and Necessity* (Cambridge: Cambridge U. P., 1990), pp. 352–63, where it is attributed to Whitehead and Frege. This section of *PM* receives a brief description in Grattan-Guinness, *Roots*, pp. 408–10.
MARK-UP, COPY-EDITING AND RUSSELL’S PROOFREADING

These theorems were not of special significance, and their manuscripts have survived by chance. That they were removed from the main manuscript, however, suggests that the whole manuscript was not kept intact until some dramatic incident resulted in the loss or disappearance of the whole manuscript. It seems rather to have been destroyed or the leaves reused as they were returned to Russell with the proofs. Indeed, in regard to the only other leaf known to survive, Russell told Lady Ottoline, after receiving “a new lot of proofs from the Press”: “I enclose a page of the ms (which please burn) to amuse you. Every one of the numbers on the left is a reference, which has to be verified.” If Russell had been reaccumulating the manuscript as it was returned by the printer, he is unlikely to have told her to burn the specimen leaf. Our hypothesis is that Russell burnt the manuscript partially and serially, that is, as he was done with each portion of it. Its vast bulk was surely a consideration against reaccumulating it.

Yet to verify or trace the references, the authors needed an up-to-date text of *Principia*. A copy of the manuscript was not possible, and we have just seen that Russell had no interest in retrieving the specimen leaf from Ottoline. In the long periods before Volumes 1 and 11 were published, the authors, or at least Russell, probably maintained a duplicate set of revised proof sheets. Victor Lowe, who extensively interviewed Russell in his 90s on Whitehead, has written that “Reading the proofs of Volumes 11 and 111 was a long business. Most of it was done by Russell.”

Whitehead seems to have sent his revisions directly to

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10 No. 242, 3 Nov. 1911, RA3 Rec. Acq. 69; original ms. at Texas. There are nine bracketed references on the leaf and another four embedded in proof lines—all of them to the left on this leaf, as chance would have it.

11 Such sheets may survive. Russell gave the Polish editor of Wycliff’s *Logica*, Michael H. Dziewicki, an incomplete set in 1914 (see Dziewicki to Russell, June 1914, RA1 710.049648); the whereabouts (if any) of his papers are still unknown. This sort of gift was not unique for Russell. In 1923 he offered Hans Reichenbach his proof sheets of the second edition of Volume 1 down to page 304 (mentioned by Grattan-Guinness, *Roots*, p. 500n.43; see Russell–Reichenbach, 20 Sept. 1923, Reichenbach papers, Archives of Scientific Philosophy, U. of Pittsburgh Library; photocopy in RA3 Rec. Acq. 897).

Russell, with the latter having the job of making a master copy to take to the Press. Whitehead wrote him during the proofreading of Volume 1:

Nothing on 2nd proofs of sheets 13 and 14, except that on p 213 in note to \( *21 \cdot 01 \), alter 'and also relate' to 'and thus relate'.... By the bye—I have been meaning to ask you to return my marked proofs in time for me to compare them with the revises—it will save me a lot of bother.

(14 July 1910, RA1 710.057445)

“Revises” were the revised proofs. The need for frequent communication was manifest, and at one point Whitehead felt obliged to apologize to his partner: “I am awfully sorry to have kept the proofs in this unconscionable way.... As usual the notation (eg. *182) struck me as beautiful. You have surpassed yourself” (20 Sept. [1911]).

Alterations and other markings on the newly found leaves demonstrate that they were the final version sent to the compositors. The new folio numbers, written in pencil by Russell, made those leaves the 152nd and 153rd leaves in Section B of Part VI; about 400 were expected in the entire Part. The original foliation of 12 and 13 may have started at article \(*336\). For clarity Russell inserted the proposition number (\(*336\) again) in pencil on folio 152b (see fig. 3); he had done similarly on the leaf for \(*208 \cdot 4\). Someone, perhaps Russell himself, overwrote in pencil all thirteen subscripted \( d \)'s to make clear that it was the partial derivative symbol \( \partial \) that was to be printed. Pencil notations that are definitely not in Russell’s hand will be discussed later. The ink alterations on the new leaves include the insertion of the reference to \(*331 \cdot 42\) as an afterthought, as was the shift in position for “\( T \in K \)”. As was his custom in logic manuscripts, Russell overlined “\( V \)” (see fig. 5; he also overlined “\( W \)”, even on envelopes). Presumably this was to distinguish Roman letters from similar Greek ones. The new leaves are no more neatly written than a great many of his rejected leaves.

Collation reveals some significant variants between the manuscript leaves and the printed book. The amount of variation for the three
half-leaves provides an indication of how content the authors must have been with their final manuscript when they passed proofs. The manuscript was accurate, but not 100 percent so. By this time in his life Russell seldom changed much and yet had an sharp eye for errors. By this time in his life Russell seldom changed much and yet had an sharp eye for errors. In the absence of extant proofs we can only conjecture who made the changes.

In the premiss for step (i) in the proof of *336·51, the notation “. Induct” was removed following the reference to *334·13 (see fig. 2), and that number itself was changed to *334·131. (Induction was used with the premiss *334·13 in the proof for *334·131 on page 386.) Russell and Whitehead likely made the changes so far, and perhaps the staff of the Press the rest. A bold square dot was supplied where it had been overlooked at the end of the second line of the demonstration of *336·51. Other compositorial changes concerned layout matters: the lines of formulae were laid out, sometimes realigned under major operators, and broken according to professional standards, although the positioning of references was not changed; a period was inserted after theorem numbers and the heading “Dem” to accord with Cambridge house-style; and—important for readability—the relative spacing of the symbols now reflected syntactic elements. No instructions for the manuscript’s passage into print were marked on the extant leaves, yet the whole attained an austere typographical beauty.

With the possible exception of the overwritten d’s, there is no mark-up or copy-editing of Russell’s text for the Press’s compositors (or compositor). As for other markings, folio 152b has the name “Rackham” pencilled in and circled in the left margin above the instruction for the of the square-bracketed double reference, *71·362· *330·5.

See KB, “Russell’s Mathematical Proofreading”, Russell, n.s. 3 (1983): 157–8. The only known surviving page of proof—with Russell’s corrections—is reproduced in this article. The proof page was found in his copy of Volume 1 of the first edition of PM.

Induction does not appear in the “Alphabetical List of Propositions Referred to by Names” in Volume 1, although upon its first use, in *260·25 of Volume 11, the authors say: “In the above proposition, ‘Induct’ refers to *120·13” of Volume 11.

In April 1910 Russell had informed Ralph Barton Perry that “there is only one compositor who can read our queer symbols” (Russell–Perry, in H. M. Sheffer papers, correspondence box, Houghton Library, Harvard, as cited in Grattan-Guinness, Roots, p. 385). Lowe noted, and was presumably told by Russell: “The printing was slow work; the Cambridge University Press had only one compositor who could set up the queer symbols” (Whitehead, the Man and His Work, 1: 289). Thus Mr. Rackham may have been Principia’s sole compositor.
signature for the next printer's sheet (see fig. 3). Surnames such as Blanshard, Gunn and Moore (and ten others) occur in the manuscript of The Principles of Mathematics; for the second edition of Principia, we find the names Symonds and Dawson. All these names are surely those of compositors to whom a swatch of manuscript was assigned to hand-set into type. The printing process at Cambridge seems to have been uniform over the period.

Below Rackham’s name we find the underlined text “R. & W. III. 26—401”. As was the practice at Cambridge University Press—see the manuscript of Parts I, II and III of the Principia—a title identifier, “r. & w. 111.”, was indicated thus for the beginning of the signature line on the first page of each printer’s sheet in Volume 111. The underlined text (without the reference to 401) was inserted right at the stage in the manuscript where page 401 of the printed text was to begin, which it did at the beginning of sheet 26. It is the last page (416) of this sheet that has the proposition whose references were traced on the versos of folios 152t, 152b and 153t. The presence of the signature line suggests that the typesetting of Principia went almost directly into pages and that the authors did not see galley proofs. Thus the mathematico-aesthetic decisions about where to start a new page of formulae had been made by the time the authors started correcting proofs. As for the printing of the volumes in their 750, 500 and 500 copies (B&R, 1: 20–2), respectively, we know

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18 Cambridge did not acquire its first hot-metal (Monotype) composing machines until 1913 (David McKitterick, A History of Cambridge University Press, Vol. 3: New Worlds for Learning, 1873–1973 [Cambridge: Cambridge U. P., 2004], pp. 13, 171), and there is no sign that the typesetting of the first two volumes of the second edition of PM was not also done by hand. (The third volume was photo-reproduced.) McKitterick’s citations of the Press archives suggest that there may be documents of interest for the printing history of PM. For the archives, see Elisabeth Leedham-Green, “University Press Records in the University Archives: an Account and a Checklist”, Transactions of the Cambridge Bibliographical Society, 8 (1981–85): 389–418.

19 All of Russell’s Cambridge books were printed with what R. B. McKerrow calls a “title-signature” on the recto of the first leaf of each gathering: “r. g.” (Foundations of Geometry, 1897); “r. l.” (PL, 1900); and “r.” (PoM, 1903). The first volumes of PM bore the title-signatures “r. & w.” and “r. & w. ll.”, respectively. Similar title-signatures are to be found in Whitehead’s Cambridge books and others of the period. The periods were dropped in the reset volumes of the second edition of PM, and the title portion sometimes altogether in Volume 111, where the gatherings are different.

20 The Press employed a substantial staff of “readers” of proofs. Possibly they worked on galleys for PM. Maybe it’s their fingerprints that are smudged on the new leaves.
from a letter from Whitehead to Russell that the printer’s sheets, once the process of correction had abated, were printed off in sufficient numbers without waiting for the volume to be completed in type. Perhaps then the type was distributed—otherwise a stock of many thousands of unusual types would have been necessary—and we know that by the time of the second edition of Principia the type was not standing.

The mammoth task of verifying references and checking first page-proofs and then revises for Principia Mathematica was exacting work. Yet, after the “continuous dead heave” of Russell’s part in writing the book, which he considered to be “practically finished” by early 1910 (Papers 11: 15), it was mainly the time the proofs took that he regretted. “They take up a great deal of time when they are in full swing”, he told Ottoline. “It is a nuisance how proofs go on after one’s mind has travelled to other things…..” “There are only one or two more sheets of proofs to come, and then I shall have all that time free.” How much time was that? We soon get an indication: “My proofs are finished, and I haven’t had so many letters to write, so I get 2 or 3 hours a day to work at matter….” Yet Russell returned to the task a decade later, in far busier circumstances, with the second edition of Principia. After that he never again wrote or saw through the press such complex volumes.

22 Whitehead to Russell, 25 August 1911: “If the sheet [for *174·12] is printed off, keep this as an erratum” (RA 710.077462). In January 1911 Russell had told Lucy Donnelly that Volume 11 was “half printed” and that he expected publication in June. This remark and the long printing period support our claim that the sheets were printed soon after Whitehead and Russell signed off on the text of a given sheet. There were 50 such sheets, making 808 pages, in Volume 11 (B&R 1: 21). For the letter to Donnelly, see Grattan-Guinness, “The Royal Society’s Financial Support of Principia Mathematica”, p. 100; original at RA 710.049478.

23 It is not known whether stereo plates were made for the first edition of PM. The existence of plates would indicate that the Press expected the volumes to be reprinted (see McKitterick, 3: 118), but such plates might have been melted down during the First World War. Nor is it known precisely what types had to be designed and cut, as very many of the symbols can be found in contemporary Cambridge books. “The special types cut for Principia Mathematica”, Russell was informed as late as 1956, were ”still stocked at Cambridge” (R. W. David of CUP to Russell, 10 May 1956, RGAC 25).

24 The last four quotations are in letters to Morrell, respectively: no. 84, pmk. 26 May 1911; no. 167, pmk. 6 Aug. 1911; no. 673, pmk. 16 Jan. 1913; no. 706, pmk. 22 Feb. 1913.

25 We wish to thank Nick Griffin, Ivor Grattan-Guinness and Carl Spadoni for discussing with us some of the problems dealt with in this paper.