RUSSELL'S CONCEPTS "NAME", "EXISTENCE" AND "UNIQUE OBJECT OF REFERENCE" IN LIGHT OF MODERN PHYSICS

Paul Weingartner
Philosophy / Universität Salzburg
A–5020 Salzburg, Austria
PAUL.WEINGARTNER@SBG.AC.AT

With his theory of descriptions Russell wanted to solve two problems concerning denotation and reference, which are formulated here as Problem I and Problem II. After presenting each problem, we describe the main points of Russell's solution. We deal with Russell's concepts of existence and then elaborate his presuppositions concerning the relation of denoting and referring. Next we discuss the presuppositions or principles which underlie Russell's understanding of the *objects* of reference. These principles are such that if the objects of reference are material objects, they are objects of classical mechanics, or very close to such an interpretation. Finally we show how these principles have to be relaxed if the objects of reference are objects of quantum mechanics or special or general relativity.

With his theory of descriptions Russell wanted to solve two problems concerning denotation and reference, which will be formulated subsequently as Problem I and Problem II. After presenting each problem, we describe the main points of Russell's solution (secs. 1–2). In section 3 we deal with Russell's concepts of existence. Section 4 elaborates Russell's presuppositions concerning the relation of denoting and referring. Section 5 discusses the presuppositions or principles which underlie Russell's understanding of the *objects* of reference. These principles, as will be shown, are such that if the objects of reference are material objects, they are objects of classical mechanics or at least very close to such an interpretation. Sections 6 and 7 show how these principles have to be relaxed if the objects of reference are objects of quantum mechanics or objects of special or general relativity.

russell: the Journal of Bertrand Russell Studies n.s. 27 (summer 2007): 125–43
The Bertrand Russell Research Centre, McMaster U. ISSN 0036-01631

1. PROBLEM 1

Problem 1:

(1) If names are defined as linguistic expressions "directly designating an individual", and
(2) if this individual is the meaning of the name,[1] and since
(3) linguistic expressions like "the round square" do not designate an individual, then it follows:
(4) that such linguistic expressions are not names and do not have meaning.

However, linguistic expressions (sentences) like "the round square does not exist" are surely meaningful and true, which can hardly be understood if the expression "the round square" is neither a name nor has any meaning.

According to Russell, a theory which gives an adequate solution to Problem 1 should satisfy the following conditions:

(a) it has to accept as true all three premisses and the conclusion in Problem 1
(b) Russell's definition of name (as indicated in note 1)
(c) his identification of reference and meaning in the case of names[2]
(d) his requirement that although expressions like "the round square" or "Hamlet" or "the even prime greater than 2" … etc. are neither names nor have meaning (in isolation), they may occur in sentences which can be meaningful and true.[3]

[1] "A name is a simple symbol whose meaning is something that can only occur as subject, i.e. something of the kind that, in Chapter XIII, we defined as an 'individual' or a 'particular'" (Russell, *IMP*, p. 173). "We shall define 'proper names' as those terms which can only occur as *subjects* in propositions.… We shall further define 'individuals' or 'particulars' as the objects that can be named by proper names" (*ibid.*, p. 142).

[2] "A *name* … is a simple symbol, directly designating an individual which is its meaning, and having this meaning in its own right, independently of the meanings of all other words …" (*IMP*, p. 174). "[I]f '*a*' is a name, it *must* name something: what does not name anything is not a name, and therefore, if intended to be a name, is a symbol devoid of meaning …" (*ibid.*, p. 179).

[3] "Suppose we say: 'The round square does not exist.' It seems plain that this is a true proposition ..." (*PM* 1: 66).

Russell's theory of descriptions—as it is contained in *Principia Mathematica* and in *Introduction to Mathematical Philosophy*—satisfies conditions (a) to (d). Because of conclusion (4) above, descriptive phrases like $(\iota x)\phi x$ do not have meaning and are not names, they are "incomplete symbols";[4] thus they are not substitution instances of real variables. However, although $(\iota x)\phi x$ does not have meaning and thus cannot be defined—i.e. it cannot stand alone as the *definiendum* of a definition—$(\iota x)\phi x$ can stand in a larger context such that this context has meaning.[5] Russell gives two important sentences which have meaning and in which $(\iota x)\phi x$ occurs. Each of these sentences can figure as the *definiendum* of a definition. These two sentences are: "the so-and-so is a such-and-such", or "$(\iota x)\phi x$ has the property ψ". And "the so-and-so exists", or "$(\iota x)\phi x$ exists":

$$*14.01 \qquad [(\iota x)\phi x]\,\psi(\iota x)\phi x = (\exists b)(x)(\phi x \equiv (x = b)) \wedge \psi b \qquad \text{Df}$$

$$*14.02 \qquad E!(\iota x)\phi x = (\exists b)(x)(\phi x \equiv (x = b)) \qquad \text{Df}$$

In both cases the uniqueness (that at least and at most one object satisfies ϕx) follows from the right part (the *definiens*) of the definition.

Observe, however, that Russell is not entirely consistent with his use of $(\iota x)\phi x$. Definition *30.01 is not in accordance with *14 (the chapter on definite descriptions) of *Principia Mathematica* in this respect:

$$*30.01 \qquad R`y = (\iota x)xRy \qquad \text{Df}$$

Here both parts, *definiendum* and *definiens*, can hardly be incomplete symbols without meaning, since it holds in general for definitions that the *definiendum* has the same meaning as the *definiens*.[6] Russell is aware of the difficulty here and gives a "more formally correct" formulation of this definition with functions and scope indication (p. 232f.). However,

[4] "Thus all phrases (other than propositions) containing the word *the* (in the singular) are incomplete symbols: they have a meaning in use, but not in isolation. For 'the author of *Waverley*' cannot mean the same as 'Scott', or 'Scott is the author of *Waverley*' would mean the same as 'Scott is Scott', which it plainly does not; nor can 'the author of *Waverley*' mean anything other than 'Scott', or 'Scott is the author of *Waverley*' would be false. Hence 'the author of *Waverley*' means nothing" (*PM* 1: 67; see also 180).

[5] "[W]e do not define 'the x which satisfies $\phi\hat{x}$,' but we define any proposition in which this phrase occurs" (*PM* 1: 173).

[6] *Cf. PM* 1: 11. This was also noted by Lambert 1990, p. 141f.

this is not a real way out of the problem since ∗30.01 is still a definition of *Principia Mathematica*. A related difficulty appears with definition ∗62.01, where "ϵ" (membership) is defined.

2. PROBLEM II

Problem II:

(1) If linguistic expressions of the form "the author of *Waverley*" are permitted as substitution instances of names, and
(2) if the sentence "the author of *Waverley* is Scott" is true if and only if Scott is identical with the author of *Waverley*, and since
(3) expressions connected by the identity sign may be substituted for each other, then it follows:
(4) that the sentence "the author of *Waverley* is Scott" is logically equivalent to the sentence "Scott is Scott".

However, the sentence "Scott is Scott" is an instance of the law of identity—which is a logical truth, or theorem, of *Principia Mathematica*—whereas the sentence "Scott is the author of *Waverley*" needed empirical investigation for establishing its truth.[7]

According to Russell, the conclusion of Problem II is false. Hence one of the premisses has to be false. The second premiss is true and so is the third (substitution with the help of identity). Therefore the first premiss must be false. And this means that descriptions (descriptive phrases) are not substitution instances of real variables: $a = (\imath x)\phi x$ is not a substitution instance of $a = y$, from which it follows that $(\imath x)\phi x$ is not a value of y. Therefore, although $(x) x = x$ is a theorem (∗13.15), $(\imath x)\phi x = (\imath x)\phi x$

[7] In Russell's words: " 'Scott is the author of *Waverley*.' … This proposition expresses an identity; thus if 'the author of *Waverley*' could be taken as a proper name, and supposed to stand for some object c, the proposition would be 'Scott is c'. But if c is any one except Scott, this proposition is false; while if c *is* Scott, the proposition is 'Scott is Scott', which is trivial, and plainly different from 'Scott is the author of *Waverley*' " (*PM* 1: 67). "If a is identical with b, whatever is true of the one is true of the other, and either may be substituted for the other in any proposition without altering the truth or falsehood of that proposition. Now George IV wished to know whether Scott was the author of *Waverley*; and in fact Scott *was* the author of *Waverley*. Hence we may substitute *Scott* for *the author of 'Waverley'*, and thereby prove that George IV wished to know whether Scott was Scott. Yet an interest in the law of identity can hardly be attributed to the first gentleman of Europe" (*OD*, *Mind* n.s. 14: 485; *Papers* 4: 420).

is not a theorem. Also: $(\exists y) y = x$ is a theorem (∗13.19), but $(\exists y) y = (\imath x) \phi x$ is not a theorem.

However, existence in the sense of uniqueness is a sufficient and necessary condition for $(\imath x) \phi x = (\imath x) \phi x$:

∗14.28 $$\mathrm{E}!(\imath x) \phi x \equiv (\imath x) \phi x = (\imath x) \phi x$$

And since it is the case that

∗14.21 $$\psi (\imath x) \phi x \rightarrow \mathrm{E}!(\imath x) \phi x$$

it is also the case that:

$$\psi (\imath x) \phi x \rightarrow (\imath x) \phi x = (\imath x) \phi x.$$

A consequence of this is that the law of identity does not hold for objects which do not satisfy uniqueness. Thus "Hamlet = Hamlet" and "the round square = the round square" are both false. Whereas there is widespread agreement that inconsistent objects are not identical, i.e. do not satisfy the law of identity, it is not so clear why objects which are not logically inconsistent, but which factually do not exist, cannot be self-identical. Such objects might be roughly of two sorts: those which violate some law of nature (like Pegasus or huge giants), and those which are in accordance with laws of nature but do not exist because of some accidental reason, i.e. some initial conditions (like red swans or blue ravens). Concerning the second case, we can also easily imagine that humans with the heart on the right side or left-hand screwed snail shells, which are both very rare, would (factually) not exist at all.

In this connection there is another claim of Russell which is not completely correct: "Generalizing, we see that the proposition $a = (\imath x) \phi x$ is one which may be true or may be false, but is never merely trivial, like $a = a$ ..." (*PM* 1: 67). What Russell has in mind here is that $(\imath x) \phi x$ must not be understood as a name (or as a real variable), otherwise it would be trivial (i.e. $a = a$) if true. However, it is permitted to substitute "$x = a$" for "ϕx", which gives $(\imath x)(x = a) = a$, and this is a theorem (∗14.2) of *Principia Mathematica*. Thus the proposition $a = (\imath x) \phi x$ is in fact "trivial" in the sense of being a theorem—i.e. not a factual empirical truth—if "ϕx" is instantiated by "$x = a$". (This point has also been discussed by Lambert 1990, p. 143.)

3. RUSSELL'S CONCEPTS OF EXISTENCE

There are mainly three concepts of existence formalized in the system of *Principia Mathematica*: Existence represented by the existential quantifier $(\exists x)\phi x$, existence expressed through uniqueness $\mathrm{E}!(\iota x)\phi x$, and existence expressed by non-emptiness of a class $\exists!\alpha$.[8]

3.1 Existence represented by the existential quantifier

Existence in the sense of the existential quantifier (applied to a propositional function) is presented in *Principia Mathematica* $*9$ ($*9.1.11$) from which the universal quantifier is derived ($*10.01$). Semantically the notion is based on the non-emptiness of the universe of discourse and the fact that objects belong to it by virtue of their being self-identical ($*24.01$).

(1) First, there is a universal class or in other words the universal class is not empty:

$*24.52$ $$\exists!\mathrm{V}$$

This proposition rests on $*9.1$ $\phi x \rightarrow (\exists z)\phi z$ or on $*10.1$ $(x)\phi x \rightarrow \phi y$. The underlying idea is that all the individual variables or variables of lowest type refer to one and the same universe of discourse and this universe is not empty, i.e. there is something.

(2) Secondly, the universal class is defined as the class of objects which are self-identical:

$*24.01$ $$\mathrm{V} = \hat{y}(y = y) \quad \mathrm{Df}$$

(3) Based on $*24.01$, on the theorem of identity $x = x$ and on the principle $*20.15$, which converts equivalent predicative statements into identical classes and vice versa, the universal proposition $(x)\phi x$ can be defined in such a way that it means that a certain related set is identical with the universal set:

$*24.102$ $$(\mathrm{V} = \hat{y}(\phi y)) \equiv (x)\phi x$$

[8] For a detailed elaboration see Weingartner 1966.

(4) From this it follows that the universal negative proposition $(x) \neg \phi x$ means that a certain related set is identical with the null-set:

$$*24.103 \qquad (x) \neg \phi x \equiv (\hat{y}(\phi y) = \Lambda)$$

(5) From this theorem we can derive a kind of definition of the existential statement:

$$(\exists x) \phi x \equiv \neg (\hat{y}(\phi y) = \Lambda)$$

This says: there is an x with the property ϕ if and only if it is not the case that the class of all y satisfying ϕy is identical with the null-class. Since being identical with the null-class was interpreted as being inconsistent, not being identical with the null-class was interpreted as being consistent.

On these lines a view developed which is expressed by the rather problematic slogan, "Existence in Logic and Mathematics means consistency." Russell did not make such a claim, it seems. But Albert Menne and Hao Wang, for example, did,[9] although already in 1950 Bernays pointed out correctly that the above slogan is only a half-truth and neglects important differences.[10] What is shown in proofs is how one can proceed from establishing an example or from constructing a model to consistency, or from a discovered contradiction (inconsistency) to non-existence. But what is not shown is the other direction of the claimed equivalence: how one could proceed from an established consistency to existence. Although the above slogan is supported by the fact that in several domains of logic and mathematics a proposition is satisfied by at least one object (or model), or satisfied by no object (or model), the warning of Bernays is important: in logic and higher-order mathematics, consistency is a necessary and sufficient condition only for non-standard models. Therefore "existence" has to be relativized to the system or domain.

3.2 Existence represented by uniqueness

$$*14.02 \qquad \mathrm{E}!(\imath x)\phi x = (\exists b)(x)[\phi x \equiv (x = b)] \qquad \mathrm{Df}$$

[9] *Cf.* Menne 1959, p. 103, and Wang 1962, p. 347.

[10] Bernays 1950. *Cf.* also Beth 1956, §§9, 11, 31, 32, and 1959, Part v.

"The kind of existence just defined covers a great many cases."[11]

For this kind of existence, uniqueness—i.e. that there is at least one and at most one object—is a necessary and sufficient condition. It has the following properties:

(a) It applies only to objects which already have properties.
(b) The existence defined is a kind of "individual existence" in the sense of uniqueness. Observe, however, that uniqueness is also satisfied by natural numbers. This shows that this kind of individual existence is not specific for individuals in space and time, like for example the Aristotelian substance.
(c) It is suitable for negative existential statements, e.g. "there is no Pegasus" or "there is no number which is even and prime and greater than 2."
Observe however that it does not fit very well with negative existential statements of the sort "there is no perpetuum mobile", since in this case no individual existence is denied.
(d) Because this kind of existence can be denied with regard to certain objects, these objects cannot belong to the universe of discourse and therefore descriptions are not substitution instances of real variables (*cf.* §2 above).
(e) Primary and secondary occurrences (and a scope indication) of the descriptive phrase allow us to distinguish two kinds of negations with regard to existence. Using Russell's example, "the King of France exists and is not bald" can be precisely distinguished from "it is not the case that the King of France exists and is bald."
(f) The question "$\mathrm{E}!(\imath x)\phi x$?" is widely applicable: in all investigations in which we do not want to presuppose the existence (uniqueness) of $(\imath x)\phi x$ (i.e. of a new object of investigation) before a proof or argument or empirical evidence has been given. Examples: discoveries of new elementary particles, of new stars, of black holes … proofs of the existence of God.

3.3 Existence expressed by non-emptiness of a class

Class-existence is defined by Whitehead and Russell thus:

[11] *PM* 1: 31. *Cf.* what has been said in §1.

∗24.03	$\exists!\alpha = (\exists x) x \in \alpha$	Df
∗20.42	$\alpha = \hat{y}(y \in \alpha)$	Df ($\hat{y}$: the class of y such that)
∗24.51	$\neg \exists!\alpha \equiv \alpha = \Lambda$	(Λ: the null set)
∗20.56	$E!(\imath\alpha)(x \in \alpha \equiv_x \phi x)$	

∗20.56 says that every propositional function (ϕx) defines a class. However, these classes are type-theoretically restricted.

In the subsequent sections we will concentrate on the type of existence that is defined with the help of uniqueness.

3.4 Other proposals for definite descriptions and existence as uniqueness

This is only a very rough sketch of some important peculiarities of other proposals compared to Russell's:

(1) Hilbert-Bernays 1934: "the so-and-so", $(\imath x)\phi x$, is permitted as a term only if uniqueness for it has been established beforehand by a proof. The theory is iota- (i.e. $(\imath x)\phi x$) eliminating.
(2) Rosser 1953: Rosser's theory of description is not iota-eliminating.
(3) Frege-Carnap 1956: The assignment to non-designating terms is arbitrary. The theory presupposes individual constants.
(4) Montague-Kalish 1957: Like Rosser's, this theory is not iota-eliminating. It is complete with respect to a special semantics defined by Montague-Kalish with the help of an extended Henkin-version. Complete in this sense are also the theories of description by Hilbert-Bernays and Frege-Carnap.
(5) Scott 1967: Bound variables range only over the given domain of individuals. The values of terms and free variables need not belong to the domain, but to an "outside domain". Non-referential singular terms need not to be co-designative.
(6) Lambert 1962 and van Fraassen-Lambert 1967: This theory is a "free-description-theory". "The so-and-so" is constructed as a singular term. If uniqueness is not satisfied, the term does not assign anything in the domain of discourse.

Compared to these theories, Russell's theory of description is iota-eliminating, relatively (see below) universally applicable, but it is not complete in the sense of Montague-Kalish. Concerning applicability, it is more universal than the one of Frege-Carnap, since the latter requires individual constants.

However, it will become clear especially from sections 6 and 7 that neither Russell's theory, nor one of the other proposals, seems to be suitable to handle the "objects" of modern physics, i.e. objects in quantum theory or in the theory of relativity: they are incomplete concerning their properties, they do not satisfy uniqueness or temporal identity, they are not rigid, i.e. they change their essential properties when they are arbitrarily moved in space. But they still belong to the universe of discourse (not to the null set or to the outer domain, nor are they non-referring), since they are described by physical laws. On the other hand, the objects presupposed by these theories of description come close to the objects of classical mechanics: they are not supposed to change properties like mass or geometrical shape just by moving through space, and they are not supposed to lose uniqueness during an interval between a first and second observation (measurement).

4. PRESUPPOSITIONS OF RUSSELL'S PROPOSAL CONCERNING THE RELATION OF DENOTING OR REFERRING

4.1 A name directly designates an individual (object) which is its meaning.[12] That is, the relation of denoting, designating or referring is a two-place relation, and reference is identified with meaning. It seems that Russell's view concerning descriptions presupposes the same; and if uniqueness is not satisfied, there is no reference and no meaning.

In Russell's theory of denoting or referring concerning both real variables and descriptive phrases, there seems to be no place for more elements with regard to the relation of denoting or referring. According to the medieval theory, this relation involves three components:

name	concept	reference
description	$\left\{\begin{array}{c}\text{conceptual construction}\\ \text{meaning, content}\end{array}\right\}$	object

A similar three-place relation was adopted by Meinong, but Russell was proud to be able to skip Meinong's content.

4.2 In Russell's understanding, the relation of denotation (designation) or reference is the same if the objects of reference are mathematical (or

[12] *Cf.* the quotation from Russell (*IMP*, p. 174) in note 2.

conceptual) entities or physical objects; that is, this relation is independent of whether the relata are conceptual objects (which are neither spatial nor temporal) or physical objects in space and time.

4.3 Mathematical entities are always rigid in the sense that they either (sharply) satisfy uniqueness or do not. Physical entities, on the other hand, are not always rigid. But in Russell's understanding all objects of reference are rigid.

To substantiate §§4.2 and 4.3 one has to know first that according to Russell "all the objects of common-sense and developed science are logical constructions out of events...."[13]

Secondly, that these logical constructions which are built from physical objects are like conceptual entities and thus rigid and impenetrable: "[T]he events out of which we have been constructing the physical world are very different from matter as traditionally conceived.... The matter that we construct is impenetrable as a result of definition...."[14] Under "matter as traditionally conceived" Russell understands matter as a permanent indestructible substance.

5. PRINCIPLES OF RUSSELL'S PROPOSAL CONCERNING THE OBJECTS OF REFERENCE

The presuppositions or principles listed here cannot be substantiated directly by giving quotations in the literal sense from Russell's works. But they seem to be hidden by Russell's treatment of objects of reference and by consequences of such treatment (see the quotations in §4.3 above). In this respect it is of particular importance that objects in space and time—physical objects—are understood as logical constructions of objects in the sense of classical mechanics (even if sometimes entities at the microlevel like electrons are mentioned). Russell's *ABC of Relativity*, well written for its time of publication, did not influence—so it seems—in any specific way his theory of denoting or his theory of descriptions. In the *ABC of Relativity* there is no connection made to his theory of descriptions.

In general, Russell's view concerning physical objects is, as is to be expected, guided by §4.2 and §4.3: objects of reference (of names or de-

[13] Nagel 1944, p. 331.
[14] Russell, *AMa*, p. 385; *cf. The ABC of Relativity* (1925), p. 185.

scriptions which satisfy uniqueness) are interpreted as rigid in a similar sense to mathematical entities or—applied to physics—in a similar sense to objects of classical mechanics.

5.1 Value-completeness

If $(\imath x)\phi x$ satisfies uniqueness, then the object of reference is a bearer of value-definite (or value-complete) properties. This presupposition was accepted and defined by Kant: of all possible predicates of an object as a bearer of predicates, one of each pair of opposite (or contradictory) predicates must belong to it. In Kant's words: "… every thing as regards its possibility, is likewise subject to the principle of complete determination, according to which if all the possible predicates of things are taken together with their contradictory opposites, then one of each pair of contradictory opposites must belong to it."[15]

A physical consequence of §5.1 is that every individual (or physical) object possesses always a well-defined position in space. This holds also for Russell, according to whom the most elementary physical objects are his "events": "[T]he matter in a place is all the events that are there, and consequently no other event or piece of matter can be there. This is a tautology, not a physical fact.…"[16] The above consequence is, however, typical for the domain of classical mechanics and does not hold generally (*cf.* §6 below).

5.2 Mechanical object

If $(\imath x)\phi x$ satisfies uniqueness, then the object of reference is a bearer of such (essential) properties as mass, charge and geometrical shape, which transform covariantly under the transformations of the Galilean group. That means that the object remains rigid under translation in space, under orientation in space, under translation in time and under inertial movement with arbitrary velocity. In this sense "mechanical object" or "mechanical system" can be characterized by the Galilean symmetry group.[17]

From this it will be clear that the opposite implication does not hold:

[15] Kant 1787, B600. *Cf.* the discussion in Mittelstaedt and Weingartner 2005, pp. 268, 271f. and 276f.

[16] *AMa*, p. 385.

[17] *Cf.* Mittelstaedt 1986, p. 219f. A more detailed and precise definition for "classical physical object" or "object of classical mechanics" is given in Mittelstaedt and Weingartner 2005, p. 271f.

that an object which satisfies the Galilean group satisfies $(\imath x)\phi x$. It is a whole class of objects (the objects of classical mechanics) which satisfies the Galilean group and not a single object only.

5.3 Uniqueness

If $(\imath x)\phi x$ satisfies uniqueness, then the object of reference is *unique*, according to classical mechanics, by its definite (or accidental) properties: by position (p), momentum (q) and point in time (t). This holds under the additional assumption of the impenetrability of the object in a space–time point (which does not follow from the dynamical laws). But this assumption seems to be hidden in Russell's view of event and place (see the quotation from him in §5.1 above).

Whether Newton has already proved uniqueness is difficult. The question is whether he has shown that, besides the one, there does not exist a different, second trajectory satisfying the same initial conditions along which a body can move in obeying his laws, including his law of gravitation. According to Arnold, Newton showed by checking many solutions of the laws that they depend smoothly (continuously) on the initial data. But the theoretical proof seems to have been given first by Johann Bernoulli (*cf.* Arnold 1990, p. 31f.).

5.4 Reidentifiability

If $(\imath x)\phi x$ satisfies uniqueness, then the object of reference is *reidentifiable* through time; i.e., that it has temporal identity. This reidentifiability in turn requires two conditions to be fulfilled:

(a) There has to be a dynamical law which connects the object in state $S_1(p, q, t_1)$ with the reidentifiable object in state $S_2(p, q, t_2)$.
(b) The objects have to be impenetrable such that there can be only one object at a space–time point (see the quotation from Russell in §5.1 above).

5.5 Observer-invariance

If $(\imath x)\phi x$ satisfies uniqueness, then all observers of the object of reference—in other words, all laboratories with rods and clocks in which the object is investigated—are equal; there is no designated observer or laboratory. All observers will arrive at the same result concerning the unique object of reference.

5.6 Transworld identity

According to our understanding of "law of nature", the laws of nature are valid in all physically possible worlds which differ from our world only with respect to individual states or initial conditions.[18] Thus individual states or initial conditions are not designated by any law either in this world or in another physically possible world. Therefore no law determines whether some individual initial state of our world can be found in any of the other physically possible worlds. This is particularly true of the dynamical laws of classical mechanics. Consequently, although a dynamical law connects two individual states of our world, and although it will connect two individual states in another world, it does not connect two individual states of two different worlds. From this it follows immediately that transworld identity of individual states is not guaranteed in classical mechanics.

The same holds for the objects of reference of classical mechanics. For any such object $(\imath x)\,\phi x$ satisfying uniqueness, its identity cannot be guaranteed in any other possible world, independently of how the accessibility relation is defined. Although such an object is reidentifiable in *one* world, it is not reidentifiable from one world to another one.[19] It follows that an application of Kripke's semantics to classical mechanics will lead only to a redundant extension, since its interpretation of "possible" and "necessary" reduces to factual (i.e. to true or false) in our world.

6. ARE THE PRINCIPLES ABOUT THE OBJECTS OF REFERENCE VALID WHEN APPLIED TO THE MICROLEVEL?

6.1 Value-completeness

Applied to stable elementary particles like electrons, protons and neutrons (or to stable composed systems), there is no general value-completeness or value-definiteness; the object of reference is, in general, not a bearer of value-definite (or value-complete) properties (*cf.* §5.1 above). At time t the object, as it is known through measurement results, can be the bearer only of a selected or limited number of properties, i.e. those which are mutually commensurable. The conceptual construction of the object as it is known by measurement results — Russell's logical construc-

[18] For a detailed justification *cf.* Weingartner 1996, Chap. 7, and Mittelstaedt and Weingartner 2005, pp. 181ff.

[19] For a detailed justification *cf.* Mittelstaedt 1986, pp. 241ff.

tion—is necessarily incomplete. Therefore the description $(\iota x)\phi x$ of such an object, since it is not value-definite (*cf.* §5.1), will not satisfy uniqueness. As a consequence of that, the conceptual (or logical) construction which is incomplete cannot, in general, be identified with the reference. Thus if we interpret the conceptual construction as the meaning, it should not be identified with the reference (in contradistinction to Russell).

6.2 Permutation invariance

The Schrödinger equation holds for *kinds* of objects, not for single, individual objects. In general, the laws of quantum mechanics (*QM*) are permutationally invariant, i.e. they are invariant with respect to an exchange of particles of the same kind. This means that numerically different individual particles of the same kind are treated identically by the laws. The laws do not distinguish between two electrons, two protons ..., etc.; they remain the same laws when we exchange two electrons, two protons, two neutrons or also two photons.[20] From this it follows that one of the conditions for $(\iota x)\phi x$—the condition that at most one x satisfies ϕx—is violated since more than one object (a whole class of objects of the same kind) satisfies the law. Thus, uniqueness of the *QM*-object is not satisfied.

6.3 Uniqueness

A *QM*-object can also not be uniquely described as an individual object by accidental properties. Recall from §5.3 above that an object of classical mechanics can be so described, namely by the three magnitudes of position (p), momentum (q) and time (t). The reason that this is not possible for the *QM*-object is because the totality of accidental properties, which were needed for individualization or uniqueness, is not available at the same time. That is, the description by accidental properties is never complete, and thus we cannot get uniqueness for the respective objects if they are understood to be permanent in some reasonable way (*cf.* §6.4 below).

6.4 Reidentification

The *QM*-object is not identifiable through time; there is no temporal

[20] For more on permutational symmetry *cf.* Mittelstaedt and Weingartner 2005, pp. 74, 77, 82.

identity. In fact there are the following two possibilities:

(a) There is a position-measurement at t_1; that is, we can have uniqueness of the object (or state of the system)—impenetrability being presupposed—only at the time t_1; in this case the object or state dissolves later at t_2, so that we do not have uniqueness anymore, i.e. no permanent object.

(b) The two states $\psi(t_1)$ and $\psi(t_2)$ are connected by a law of *QM* through time (t_1, t_2). But in this case only $\psi(t_1)$ is unique with regard to one object or state (or system), since $\psi(t_2)$ can then be satisfied by more than one object and therefore does not guarantee a designation of the original unique object. Although there is permanence given by the connection of the law, there is no guarantee that what is connected is the original object at a later time.[21]

6.5 Transworld identity

For quantum mechanical objects or systems, uniqueness—the condition for using $(\iota x)\phi x$—is not satisfied. The reason is this: the characterization by their essential and permanent properties fails because with them only classes of objects or systems (like electrons, protons, photons) can be determined. But also a characterization by accidental properties like position and momentum at a certain time is impossible, since only a part of such properties is simultaneously available. Still another possibility for a unique characterization would be a description of a sufficiently complete historical development of the object (instead of giving only the actual properties at a certain point of time).[22] However, it is an unsolved problem how such a description could be obtained and used for the individuation of quantum mechanical objects. Since transworld identity of objects implies reidentifiability of *one unique* object in different worlds, it follows that transworld identity of quantum mechanical objects is not possible. There may be, however, a kind of weak analogy of transworld identity if the following restrictions and deviations with respect to a Kripke-style semantics are made:

[21] For more details see Mittelstaedt 1986, pp. 227ff.

[22] The idea of using the history of human actions and decisions as a principle of individuation of human souls (after separation from the human body—which could not serve anymore as individuating) was proposed by Thomas Aquinas (*De Veritate*, 19, 1) as one, though not the only, possibility, since it is not sufficient in all cases (e.g. children who die immediately after birth).

(1) Possible worlds are replaced by measuring processes.
(2) "There is a world W' (different from the actual world W) in which the proposition A' about system S' is true" is replaced by: "There is a measuring process M' (different from the actual [or earlier] measuring process M) with the result that proposition A' about system S' is true."
(3) The accessibility relation $R(S, S')$ is satisfied, when $A(S, M) = A'(S', M')$.
(4) There is a non-zero probability for reaching result A' about S' by measuring process M' from the earlier result A about S by measuring process M.
(5) If (4) is satisfied, there is a non-zero probability for a weak temporal identity of the system (object) S.

7. ARE THE PRINCIPLES ABOUT THE OBJECTS OF REFERENCE VALID WHEN APPLIED TO *SR* AND *GR*?

7.1 Value-completeness

Special relativity (*SR*): Value-completeness, or value-definiteness, of properties of an object (or reference system, *cf.* §5.1) is not satisfied for an observer at every point of time. But in this case—we have just Minkowski space-time and inertial movement and no acceleration or gravitation—the observer may always wait until the object appears in his past light cone.

General relativity (*GR*): As soon as acceleration or gravitation is taken into account, there are always some domains in space-time with objects that will never appear in the past light cone of the observer. This is so even if the observer moves on a geodesic, i.e. free from forces. It is plain, then, that the description of such objects cannot be value-complete.

7.2 Permanence

SR and *GR*: The essential properties of the object of reference (as the bearer) are not permanent, with one exception, namely charge. The other properties, like mass, length and geometrical shape, change in case of fast inertial motion in accordance with the Lorentz-transformation; this holds also in local inertial reference systems of Riemannian space-time (*GR*).

7.3 Uniqueness

According to §5.3, uniqueness of objects in classical mechanics can be satisfied by special values of the accidental properties of position (p), momentum (q) and point of time (t). But in §6.3 it was shown that uniqueness is not satisfied on the microlevel in *QM*. Concerning *SR*, uniqueness with regard to p, q, t holds only partially; namely it holds only for objects appearing in the past and future light cone of the observer (dynamical laws being presupposed). With respect to *GR*, uniqueness is dependent on the space–time curvature.

7.4 *Reidentifiability*

With respect to both *SR* and *GR*, the object of reference is not in general reidentifiable through time; this is so because essential properties of the object like geometrical shape and mass may change depending on movement. Therefore reidentifiability holds only approximately in local reference frames of space-time.

7.5 *Time and simultaneity*

With respect to both *SR* and *GR*, there is neither a universal time, nor universal simultaneity. Each different observer (i.e. each different laboratory or reference system) has its own time and simultaneity. Therefore the object of reference is not the same for all observers.

SUMMARY

As a consequence of what has been elaborated in sections 5 to 7 we may say that Russell's idea of characterizing an individual object with the help of a definite description expressing uniqueness is applicable to individual objects of everyday life and to physical objects of classical mechanics. But it is only approximately applicable with restrictions to objects in the domain of quantum mechanics and in the domain of the special and general theories of relativity.

REFERENCES

Arnold, V. I. 1990. *Huygens and Barrow, Newton and Hooke.* Basel: Birkhäuser.

Bernays, P. 1950. "Mathematische Existenz und Widerspruchsfreiheit". In *Études de philosophie des sciences, en hommage à Ferdinand Gonseth à l'occasion de son soixantième anniversaire.* Neuchatel: Griffon. Pp. 11–25.

Beth, E. 1956. *L'existence en mathématique.* Paris: Louvain.

—. 1959. *The Foundations of Mathematics.* Amsterdam: North Holland.

Carnap, R. 1956. *Meaning and Necessity.* Chicago U. P.

Hilbert, D., and Bernays, P. 1934. *Grundlagen der Mathematik.* Vol. 1. Berlin: Springer.

Kant, I. 1787. *Kritik der reinen Vernunft.* 2nd edn. Trans. N. K. Smith as *Immanuel Kant's "Critique of Pure Reason".* London: Macmillan, 1929.

Lambert, K. 1962. "Notes on E! III: A Theory of Descriptions". *Philosophical Studies* 13: 51–9.

—. 1990. "Russell's Theory of Definite Descriptions". *Dialectica* 44: 137–52.

Menne, A. 1959. "Zur logischen Analyse der Existenz". In I. M. Bochenski, *Logisch-Philosophische Studien.* Freiburg: Alber. Pp. 97–106.

Mittelstaedt, P. 1986. *Sprache und Realität in der modernen Physik.* Mannheim: Bibliographisches Institut.

—, and Weingartner, P. 2005. *Laws of Nature.* Heidelberg-Berlin: Springer.

Montague, R., and Kalish, D. 1957. "Remarks on Descriptions and Natural Deduction". *Archiv für Mathematische Logik und Grundlagenforschung* 3: 50–73.

Nagel, E. 1944. "Russell's Philosophy of Science". In P. A. Schilpp, ed., *The Philosophy of Bertrand Russell.* Evanston, Ill.: Northwestern U. Pp. 319–49.

Rosser, B. 1953. *Logic for Mathematicians.* New York.

Russell, B. 1905. "On Denoting". *Mind* 14: 479–93. *Papers* 4: 415–27.

—. 1919. *Introduction to Mathematical Philosophy.* London: Allen and Unwin.

—. 1925. *The ABC of Relativity.* London: Kegan Paul.

—. 1927. *The Analysis of Matter.* London: Kegan Paul.

Scott, D. 1967. "Existence and Description in Formal Logic". In R. Schoenman, ed., *Bertrand Russell, Philosopher of the Century.* Boston: Atlantic–Little, Brown. Pp. 181–201.

Thomas Aquinas. *De Veritate.* Trans. as *The Disputed Questions on Truth,* 3 vols. Chicago: Regnery, 1953.

Van Fraassen, B., and Lambert, K. 1967. "On Free Description Theory". *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 13: 225–40.

Wang, H. 1962. "Process and Existence in Mathematics". In V. Bar-Hillel, ed., *Essays on the Foundations of Mathematics.* Amsterdam: North Holland. Pp. 328–51.

Weingartner, P. 1966. "Der Begriff der Existenz in Russell's Theorie der Deskription". In Weingartner, ed., *Deskription, Analytizität und Existenz.* Salzburg: Pustet. Pp. 69–86.

—. 1996. "Under What Transformations are Laws Invariant?". In Weingartner and G. Schurz, eds., *Law and Prediction in the Light of Chaos Research.* Heidelberg-Berlin: Springer. Pp. 47–88.

Whitehead, A. N., and Russell, B. 1925–27. *Principia Mathematica.* 2nd edn.