PROPOSITIONAL ANALYSIS

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Graham Stevens has written a short book on a di
cult subject: the unity of
the proposition. While the title of the volume is The Russellian Origins of
Analytical Philosophy, the underlying theme is the comparatively little discussed
problem of how the elements of a proposition come together to form a unity,
as indicated in the book’s subtitle: “Bertrand Russell and the Unity of the Pro­
position”. Stevens traces this concept, as a unifying thread or “central concern”,
throughout Russell’s many philosophies which constitute for Stevens a “consis­
tent and constantly evolving philosophy” (p. 2) stretching from the earliest log­i­cal writings to the later works of the 1930s and 1940s. This is an important and
well written volume, which Russellians should have in their personal and uni­
versity libraries.

That the “unity of the proposition” is a problem at all requires the reader to
return to the neo-Hegelian idealism which Russell encountered as a student at
Cambridge.1 Russell, influenced by Peano and Frege, broke with the doctrine of
internal relations, defended by Bradley and others, according to which external
relations are unreal and false appearance. However, an important element of
idealism remains even during the Principia period, due to Russell’s continued
belief that a proposition is a non-linguis
tic, non-mental, abstract entity; these
elements are all of the same ontological kind, which can be identified through
analysis and recombined so that their synthesis reconstitutes the original “unity
of the proposition”. This turns out to be a very tall order which will occupy
Russell for nearly half a century.

Consider the standard example Russell gives: “Desdemona loves Cassio”. According to Russell, this proposition (independently of its truth or falsity)
actually contains its “simple constituents”—with “Desdemona” and “Cassio”
being things and “loves” a concept. Russell, as a founder of analytic philosophy,
has to defend the legitimacy of this analysis from the claim that it has reduced

1 Chapter 1: “Russell, Frege and the Analysis of Unities”. 
the structured unity of the proposition into an unstructured set of parts (or heap) where the original order of combination has been lost.

The problem of is that merely listing “Desdemona”, “Cassio” and “loves” does not indicate the way in which the terms were originally combined or related. For reasons to be made clear later, this is referred to in the literature as the “narrow order problem”. In particular, the question arises as to how analysis enables us to assert that it is Desdemona who loves Cassio (supposing this to be the case), rather than the other way around (since love may not be requited). Stevens deals with logical form in a later chapter devoted to the problem of belief statements, but the notion applies here as well: analysis seems to have skipped the logical form, the way in which the constituents are combined. But if the logical form is included, then there seems to be an extra constituent in the proposition not given by direct inspection, and analysis would appear to have added something above and beyond what was there before the analysis. Adding linking relations to indicate how the components were originally combined does not help either. If there is some relation $L_z$ which relates “Desdemona” and “loves” and another relation $R_z$ which relates “loves” and “Cassio”, then there are five constituents, not three. Moreover, there is now room for infinite regress, with a relation $L_z$ linking “Desdemona” and $L_z$, $R_z$ linking “loves” and “Cassio”, and so on.

But without adding these additional items, there is an apparent failure of analysis to fully provide the order of the elements present in the original synthesis. Stevens quotes Russell to the effect that “these constituents taken together do not reconstitute it [the proposition]” (quoted, p. 23). Again, Russell: “though analysis gives us the truth, and nothing but the truth, yet it can never give us the whole truth” (ibid.). Stevens notes that this amounts to “a tension bordering on paradox at the very heart of logical analysis, that the original synthesis involved in the proposition contains more than the analysis” (p. 31).

A further problem follows from Russell’s view that all terms in a proposition have a common ontological status. This is linked to his doctrine of the unrestricted variable, according to which a variable ranges over all objects: “The range of a real variable, Russell’s philosophy demanded, must include everything; there are no types of entities. Everything that is, is a logical subject (or term, in Russell’s terminology)” (p. 23). Although Russell has broken with the absolutist aspect of idealism—the view that reality is “one” (an idealist view based on the rejection of external relations)—he has not broken with another aspect of idealism: a type of monism which holds that reality is all of one kind or substance. Facts and propositions are of the same “type” (to abuse the term): true propositions are facts, and facts are true propositions, and both are non-mental, of a single kind. Ultimately, Stevens argues, this will come into conflict with Russell’s own theory of types, as developed in his formal logic in Principia.
Chapters 2 and 3 concern Russell’s theory of types, in part a detour from the problem of the unity of the proposition, as Russell struggles to deal with the antinomies of set theory, including his own contribution of the set of all sets not members of themselves. These two chapters provide a concise and cogent introduction to the issues, as well as a sophisticated treatment of the philosophical and logical problems involved, including the substitutional theory of classes and relations and the ramified theory of types, as well as discussion of the theory of descriptions in “On Denoting”. These chapters, more than any others in the book, exemplify its main title, “The Russellian Origins of Analytic Philosophy”, as Stevens explains the intricate and intimate relation Russell developed between ontological problems and logical technique which is a hallmark of analytic philosophy.

Stevens’ analysis of the theory of types is based on the distinction made clear by Frank Ramsey between the “theory of types” and the “theory of orders”, which together form the ramified theory of types. The theory of simple types, or types for short, was developed to exclude the logical paradoxes, since on this view — now widely accepted — the “set of all sets not members of themselves” \( \{x \mid x \notin x\} \) cannot be formulated, as the member of a set (the “\( x \)” on the left of the membership sign) must be of lower type (in general, one type lower) than the set of which it is a member (the “\( x \)” on the right of the membership sign). As the \( x \)’s in question clearly violate this requirement (they are both of the same type), the problem is eliminated.

But Russell also added a theory of orders to avoid semantic paradoxes of the Cretan types: the statement, as uttered by a Cretan, that “All propositions asserted by Cretans are false”, if true is false, and if false is true — another paradox. Russell analyzed the problem as involving reference to a totality (all propositions uttered by Cretans) to which a further proposition is added (that these assertions are all false). The illegal move is to presume that the totality is complete and then add a further element. On Stevens’ view of orders, sets involving all propositions “must be broken into smaller sets of different order, each of which may legitimately have a total” (p. 37). This poses problems for actual mathematical practice, as there is now a stratification of statements about all objects of a certain kind — for example, that every bounded set of reals has a least upper bound. On the ramified theory of types, the least upper bound would now be of a higher order than the set of which it is the least upper bound. To restore legitimacy,

2 Chapter 2: “Russell’s Paradox and the Theory of Types”, and Chapter 3: “Ramification and *Principia Mathematica*”.


4 Ramsey helped Russell prepare the second edition of *Principia Mathematica* for publication, several years before his premature death at age twenty-seven in 1930.
Russell invokes his famous (or infamous) axiom of reducibility, which never acquired widespread acceptance.5

Stevens’ discussion of the theory of types (simple and ramified) is especially clear and appropriate for a philosophy student being introduced to this notoriously difficult subject; but as Stevens also analyzes subsequent interpretations (notably that of Quine), the discussion is useful for professionals in the field as well, nicely combining two distinct but important audiences. Moreover, Stevens uses the type/order distinction to provide his own answer to the question of what Wittgenstein said in 1913 that reduced Russell to abandoning his manuscript, Theory of Knowledge, of the same year.6 We return to the problem of the unity of the proposition, the attempted solution of which now leads Russell to abandon propositions altogether (though temporarily as it turns out) in favour of judgments.

Ultimately, Stevens argues that Russell is able to solve the problem of propositions only through a return to a psychological perspective. Russell’s introduction of the theory of judgment, while soon to be abandoned as well, is a transitional step towards that re-psychologizing of the issue of propositions, as judgments are analyzed as requiring a judging mind. For the moment, Russell sidelines the problem of propositions, and tries an approach to unity based on consideration of judgments. As Stevens notes: “According to Principia, propositions are incomplete symbols that require the context of a judging mind in order to achieve meaning; propositions, in other words, are to be replaced by judgments” (p. 79). A significant difference between the old style proposition and the newly introduced judgment is that a judgment has two verbs: the primary or judging verb, which links the judging mind to the terms whose relation is being judged, and the subordinate verb, linking the objects in what was formerly a complete proposition but now no longer is.

In Chapter 4,7 Stevens considers Russell’s example, “Othello believes that Desdemona loves Cassio”, which involves five terms: “Othello”, “believes”, “Desdemona”, “loves”, and “Cassio”. Stevens symbolizes the relation between the judging and the series of objects in the judgment, including the subordinate verb, as follows: $B [o, d, l, c]$, where “$B$” stands for belief and “$o$” for Othello, the judging subject. The general form of a judgment now is $J [S, x_1, x_2, \ldots, x_n]$, where at least one of the $x_i$ is a subordinate verb. Which one it is, and how to distinguish it from the rest, will ultimately be an insurmountable problem.

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5 A post-Tarskian analysis situates the problem differently: no language can formulate its own truth predicate, which belongs to the next higher meta-language. As a result, Cretan-type statements can be shown to be improperly formulated, without recourse to a theory of orders as in the ramified theory of types.

6 Subsequently published as Papers 7.

7 Chapter 4: “The Rise and Fall of the Multiple-Relation Theory of Judgment”. 
But, immediately, the “narrow direction problem” returns, as the ordering of the elements of the subordinate verb is no more immediately given (by direct inspection) than was the ordering of the elements in the earlier problem of the proposition. Russell attempts to deal with this by adding the notion of logical form as part of the judgment: e.g., $B \{ o, x R y, d, L, c \}$ in Stevens’ notation, where $x R y$ is the required logical form. However, it still isn’t clear that “$d$” takes the place of “$x$” and “$c$” that of “$y$” in this instance of the logical form, so that this analysis does not distinguish between $d L c$ and $c L d$.

There is a further problem, “the wide direction problem” (p. 95), and it is Stevens’ claim that this is what Wittgenstein addresses in his written 1913 critique: that the multiple-relation theory of judgment cannot properly distinguish between an element of the judgment that is an object and one which is a relation between the objects—e.g., that “Love desdemona’s Cassio” is not ruled out (p. 96), as we don’t know which of the three terms is the relation, and which are the objects being related. But there is more, as this element of critique was already in Wittgenstein’s *Notes on Logic* (1913). The “unwritten” criticism that finally reduced Russell uncharacteristically to silence has been much discussed, and Stevens mentions the Sommerville–Griffin interpretation that the multiple-relation theory was incompatible with the ramified theory of types. Stevens recognizes that this view is partially correct, but adds that it is important to distinguish which part of the ramified theory is involved and why this is such a problem. Stevens sees Wittgenstein’s critique as involving the theory of orders, following the distinction due to Ramsey (where order is construed in terms of semantic levels, not sequencing of elements as in the distinct narrow and wide direction problems).

Stevens argues that Wittgenstein in his “unwritten” criticism objected that Russell’s theory of judgment breaks down not merely in terms of the narrow and wide direction problems, but in terms of an issue related to type—though this issue is in a sense an extension of the wide direction problem. The subordinate relation, as a term in the judgment complex, must be of higher type than the terms that it relates, in order to be distinguished from them, and in order for them to be correctly related by it. But in the proposed multiple-relation theory of the judgment, the subordinate relation and its objects appear as $x_i$’s without distinction of type (as Stevens suggests in his formalism for it mentioned earlier, $J \{ S, x_1, x_2, \ldots x_n \}$). Introducing type distinctions in order to make sense of judgments contradicts Russell’s antecedent commitment to the doctrine of the unrestricted variable, according to which the elements of the judgment are all on the same logical footing, and therefore not distinguished as to type. Stevens claims this contradiction lies at the heart of Russell’s decision not to publish:

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… Russell’s last-ditch effort to preserve a version of the ontological purity of the doctrine of the unrestricted variable had come to grief. The purpose of the multiple-relation theory is to generate orders of judgments without positing any corresponding ontological divisions. Every logical subject is to be treated as an ontological equal. Wittgenstein’s objection to the multiple-relation theory, however, exposes this enterprise as a failure.

(P. 103)

The multiple-relation theory of judgment was supposed to require at most distinctions of order (at the semantical level), but not of type (at the ontological level). Distinctions of type, Stevens says “are supposed to emerge from the multiple-relation theory, not provide the foundations for it” (p. 101). Similar to Russell’s analysis of the contradiction among the basic axioms in Leibniz’s philosophy, Stevens argues that a different, but equally fatal, contradiction lies at the core of Russell’s theory of the unity of the judgment—in this case, between the doctrine of the unrestricted variable that presupposes that the objects over which the variable ranges are all of the same kind, and the need to introduce type distinctions in the analysis of the judgment, in particular, in its subordinate part. Whether this explanation fully solves the mystery in a significantly different way than previous attempts is a matter for further debate.

In any case, Russell abandons the multiple-relation theory of judgment, and moves to a new framework for solving the problem of the unity of the proposition, as described by Stevens in Chapter 5.9 The title of the main section of this chapter, “Russell’s Naturalism and His Re-Psychologizing of the Proposition” (pp. 115ff.) clearly indicates the anti-Fregean move which is afoot. By the end of the 1910s Russell had assimilated the “dual aspect” neutral monism of William James, according to which there is a “neutral stuff” which viewed one way is physical, and viewed another way is mental. Mental and physical, rather than being distinct substances, are dual aspects of an underlying neutral stuff. Russell tended in his writings on science to identify this neutral stuff with events in space-time as described in relativity theory, though it is also characterized as “sense-data” (whether perceived or not). Whether the neutral stuff is viewed as physical or mental depends on the type of law—physical or mental—that applies. But for the purposes of the problem at hand (the unity of the proposition), Russell, now armed with neutral monism, finds justification for using a psychological approach which had been earlier barred by Frege’s anti-psychologism.

Stevens analyzes “On Propositions” (1919) as the beginning of this new approach. Russell distinguishes “word-propositions”, whose meaning is given by “image propositions”, which in turn have an “objective reference dependent upon the meanings of the constituent images”. The fact which is the objective reference is the “truth maker” of the proposition, which in turn is the “truth

9 Chapter 5: “Propositions Naturalized”.
bearers”. There is now a tripartite relation between the statement (“word proposition”)—which is linguistic—the proposition itself (“image proposition”)—which is psychological—and the external fact whose presence (or absence) confers truth on the proposition.

Stevens concludes that Russell had, in 1919, “at last secured an answer” to the problem of the unity of the proposition. Propositions, construed as mental entities, are of the same “kind” as the facts to which they refer, as both mental and physical are flip sides of the coin of neutral events.10 Psychologizing of propositions now becomes a hallmark of Russell’s analysis. Stevens quotes Russell from *An Inquiry into Meaning and Truth* (1940): propositions are “to be defined as psychological occurrences of certain sorts—complex images, expectations, etc.” (cited p. 117) and concludes: “This conception of propositions as entities, in some sense, inhabiting the minds of thinkers was given an increasingly naturalist slant as Russell’s views developed” (pp. 117–18).

The use of the plural “minds of thinkers” could be made more explicit. Presumably there are individual differences in how the reader of this review and the reviewer form images based on the very same words. The same difficulty exists at the level of the statements, as slightly variant words can formulate the same (or similar) statement. The notion of equivalence classes of mental (or cerebral) states as suggested by Mario Bunge11 would be useful: a proposition would then be an equivalence class of mental images, such that though no two are exactly identical, the images are sufficiently similar as to constitute the same proposition. Similarly, we would have word-statements as equivalent classes of word-instances.

Having discussed Russell’s solution to the problem of the unity of the proposition, Stevens might well have ended his book at this point, but he goes on to a final chapter 6,12 where he reviews Russell’s later work—*An Inquiry into Meaning and Truth* (1940), and *Human Knowledge: Its Scope and Limits* (1948). In these works Russell removes the ontological excesses he had admitted in his earlier logical atomism as concerns negative facts and general facts. Molecular facts, the purported correlates of propositions combined through the logical connectives, were rejected from the start: the logical connectives do not correspond to anything ontological, an insight due to Wittgenstein. Russell now applies the newly found psychological status of propositions to negative propositions, which are taken to be disbeliefs in what is claimed, not beliefs in negative facts (pp.

10 To continue the coin metaphor, it’s as if, in the case of a true proposition, there is an isomorphism between the design on the “head” side and that on the “tail” side, even if the designs are distinct in terms of detail.


12 Chapter 6: “Facts, Propositions, and Truth”.
General beliefs (involving quantifiers) are taken as a disjunction of cases (each of which has an ontological counterpart if true) along with the statement, which is epistemological in nature, that the disjunction of cases is complete (pp. 144–6).

This short chapter is interesting for its summary of Russell’s later views, usually neglected by commentators, but considered as an integral part of Russell’s overall philosophy by Stevens. Curiously, however, Stevens sees the solution of the negative and general fact issues as extraneous to the psychologism to which Russell has now moved: “Notwithstanding the general complaints we might have about psychologism in any form, however, there are other difficulties that remain for Russell’s theory of propositions” (p. 128); namely, negative and general facts. But it seems that Russell’s psychologism is integral to the solution as traced by Stevens; for, without the “retreat” to psychologism and its thinking subject, there would be no way to reformulate negative propositions as disbeliefs of the subject, or general facts as involving, in addition to the ontological enumeration of cases, the epistemological claim that the subject’s knowledge is complete. This is implicit in what Stevens says, but should have been made explicit.

Nonetheless, Stevens is committed to a continuity from early (Principles and Principia) Russell, through the Russell of neutral monism, to the later Russell (up to the later 1940s). Moreover, Stevens argues that this later philosophy of Russell has been unfairly neglected by those who see him as past his prime after Principia. While Stevens accepts the judgment that these later works do not rise to the “towering standards set by the Principles of Mathematics, ‘On Denoting’, Principia etc.” (p. 146), he does not see them as “hopelessly out of date”, as Monk, for example, does. Rather they complete a project of analyzing the “unity of the proposition” begun in those earlier works and essential to the programme of analytic philosophy. Stevens concludes with the words: “there is still plenty to be learnt from the philosopher who continually wrestled with just these kinds of problems for approximately half a century” (p. 147).

Graham Stevens’ book is a welcome contribution to Russell scholarship, accessible to students as well as professionals. The book—at just under 150 pages of text, with a further twenty-one pages of end-notes and fairly extensive, though not quite complete bibliography of twelve pages—provides not only a concise overview of Russell’s metaphysics and epistemology, but a detailed analysis of Russell’s views on issues such as the simple and ramified theory of types, the theory of substitution, the multiple-relation theory of judgment, the theory of descriptions, and of course the unity of the proposition, which is its *leitmotif*. Stevens writes clearly and cogently. He bases himself on a reading of primary texts, with asides on notable interpretations with which he disagrees in whole or in part, including Wittgenstein, Quine, Griffin, Landini and others. The book is an outgrowth of a PhD thesis supervised by Ray Monk, with whom the author
has cordially differed, especially as concerns Russell’s later work. Stevens’ book is highly recommended for anyone interested in the background to Russell’s early logical concerns, through the development of his work in *Principia* and later developments up to *Human Knowledge* in the late 1940s. It does not deal at any length with Russell’s criticism of ordinary-language philosophy, or the debates around *My Philosophical Development* and its appendices in 1959, but no one book can encompass everything. Stevens has done quite a bit as it is, producing what is surely now part of the standard for any discussion of unity of the proposition and the continuity of Russell’s analytic views on logic, metaphysics and epistemology.