

RUSSELL AND THE ONTOLOGICAL ARGUMENT

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It is well known that in *Principia Mathematica* Russell offers a theory of definite descriptions and holds that ‘existence’ is not a property. It is less well known that in “On Denoting” he discusses the version of Anselm’s ontological argument for God formulated by Descartes, accepting the premiss “Existence is a perfection” and assessing the argument as valid but question-begging. This is different from his later comments in *A History of Western Philosophy* which find the argument invalid. Indeed, given the sanctions of *Principia*, one might have thought he would find the argument logically ungrammatical. This paper shows how Russell might formulate and evaluate Anselm’s ontological argument and the version offered by Descartes in a way that avoids the conflict.¹

I. INTRODUCTION

In “The Metaphysician’s Nightmare”² Russell tells the story of a “poor friend” Andrei Bumbrowski who, in investigating sophisms concerning the non-existent, was led to the great truth that the word “not” is superfluous and the categorical imperative: *Thou shall not use the word “not”*. Undaunted by the apparent self-refutation, Bumbrowski re-frames it by striking such words from the dictionary and prescribing that proper speech shall be composed entirely of the words that remain. In

¹ Special thanks to Richard Fumerton, Francesco Orilia, Russell Wahl and the Editor for helpful comments on this paper.

² In Russell, *Nightmares of Eminent Persons* (London: Bodley Head, 1954), pp. 30–5; CSBR, pp. 229–32.

Bumblowski's nightmare, he confronts philosophers worshipping Satan as "He [Who] is pure Nothing, total non-existence, and yet continually changing." Every negation emanates from Him, every moralist whose morality consists in "don'ts", every tyrant compelling fear, is eventually subsumed into the black hole of nothingness that is Satan. "What you say is absurd", protests Bumblowski: "You are trying to persuade me that the non-existent exists. But this is a contradiction.... I denounce Him as a bad linguistic habit." The philosophers insist: "If the non-existent is nothing, any statement about it is nonsense. And so is your statement that it does not exist. I am afraid you have paid too little attention to the logical analysis of sentences, which ought to have been taught you when you were a boy. Do you not know that every sentence has a subject, and that, if the subject were nothing, the sentence would be nonsense? So, when you proclaim, with virtuous heat, that Satan—Who is the non-existent—does not exist, you are plainly contradicting yourself."

St. Anselm's *Proslogion* (1078) is most famous for an ontological argument for the existence of God which, in one version or another, has come to occupy the attention of many important philosophers.³ Though an easy target of parody, the argument is captivating because it is a watershed of deeply perplexing issues concerning logic, 'non-existence' and the intentionality (aboutness) of thought. In his *Autobiography*, Russell tells the amusing story of his early attraction to the ontological argument. He writes: "For two or three years ... I was a Hegelian. I remember the exact moment during my fourth year [in 1894] when I became one. I had gone out to buy a tin of tobacco, and was going back with it along Trinity Lane, when I suddenly threw it up in the air and exclaimed: 'Great God in Boots!—the ontological argument is sound!'" (*Auto.* 1: 63). It wasn't very long before Russell came to change his mind.⁴ Armed with his theory of definite descriptions in "On Denoting" (1905), Russell had the full apparatus for a treatment of the argument. Russell writes:

"The most perfect Being has all perfections; existence is a perfection; therefore, the most perfect Being exists" becomes:

³ Russell discussed the argument many times in his long life. It appears in an 1894 "Paper on Descartes II" in *Papers* 1: 178–84. It appears in the 1946 monograph *My Own Philosophy* in *Papers* 11: 67–82. See N. Griffin, *Russell's Idealist Apprenticeship* (Oxford: Clarendon P., 1991), pp. 71–8, for a discussion of the paper on Descartes.

⁴ See Carl Spadoni, "'Great God in Boots!—the Ontological Argument Is Sound'", *Russell* o.s. nos. 23–4 (1976): 37–41.

There is one and only one entity x which is most perfect; that one has all perfections; existence is a perfection; therefore that one exists. As a proof, this fails for want of a proof of the premiss “there is one and only one entity x which is most perfect.”⁵

In a footnote Russell adds:

The argument can be made to prove validly that all members of the class of most perfect beings exist; it can also be proved formally that this class cannot have more than one member; but, taking the definition of perfection as possession of all positive predicates, it can be proved almost equally formally that the class does not even have one member. (*Ibid.*)

Russell says that “as a proof” the argument fails for want of a proof of the first premiss. He surely has in mind that the theory of definite descriptions shows that the argument begs the question.

Russell takes “Existence is a perfection” to be the second premiss of the argument. This, however, poses a serious problem. How can the second premiss be transcribed into the formal language for logic as set out in Whitehead and Russell’s *Principia Mathematica*? The problem has not passed unnoticed. Cocchiarella avoids the problem by pointing out that nothing in Russell’s theory of definite descriptions demands that ‘existence’ not be taken as a genuine property.⁶ But others have gone so far as to maintain that “... Russell’s theory of descriptions is unsuitable for Anselm’s purposes ...” and “... fails in fairly reproducing Anselm’s argument....”⁷ This paper explores what Russell might have said in reply, while remaining faithful to *Principia*’s thesis that ordinary-language statements of existence are not to be transcribed into the language of logic with an existence predicate.

⁵ In D. Lackey, ed., *Essays in Analysis* (London: Allen & Unwin, 1973), p. 117; *Papers* 4: 426.

⁶ “Meinong Reconstructed versus Early Russell Reconstructed”, in Nino Cocchiarella, ed., *Logical Studies in Early Analytic Philosophy* (Columbus: Ohio State U.P., 1987), p. 123.

⁷ William E. Mann, “Definite Descriptions and the Ontological Argument”, in Karel Lambert, ed., *Philosophical Applications of Free Logic* (New York: Oxford U.P., 1991), p. 266.

2. THE PROBLEM OF TRANSCRIPTION

To understand the nature of the problem of transcription, let us remind ourselves of some features of Russell's approach to definite descriptions. In Russell's view, the proper language of a calculus of logic should have no terms besides variables. Thus, to transcribe ordinary-language statements involving names and descriptions we must employ predicates. For example, to transcribe "Vulcan is a planet" into the language of logic requires that we replace "Vulcan" with an ordinary description such as "the astronomical body whose gravitation accounts for the perihelion of the smallest planet of our solar system". Using " Px " for " x is a planet" and " Vx " for " x is a body whose gravitation accounts for the perihelion of the smallest planet of our solar system", we can transcribe the statement as $(\exists x)(\forall y \equiv_y y = x \cdot \& \cdot Px)$.⁸ In *Principia*, we find these abbreviations:

$$\begin{aligned} E!(\iota xAx) &=_{\text{df}} (\exists x)(Ay \equiv_y y = x) \\ [\iota xAx][B(\iota xAx)] &=_{\text{df}} (\exists x)(Ay \equiv_y y = x \cdot \& \cdot Bx).^9 \end{aligned}$$

Proceeding in this way we can abbreviate our transcription of "Vulcan is a planet" with the convenient $[\iota xVx][P(\iota xVx)]$. Moreover, we can even make it look close to ordinary language by adopting the convention of dropping scope markers when narrowest scope is intended. This yields $P(\iota xVx)$. For our present discussion, however, it is convenient to employ scope markers using the abbreviation

$$[\iota xAx][Bx] =_{\text{df}} (\exists x)(Ay \equiv_y y = x \cdot \& \cdot Bx).$$

Thus "Vulcan is a planet" is transcribed as $[\iota xVx][Px]$.

In *Principia*, no terms α besides individual variables are allowed. Thus, the axiom schema $(\forall x)Cx \supset C[\alpha/x]$, where α is free for x in C , requires that α be a variable. In *Principia*, we do *not* have

$$(\forall x)(x = x) \cdot \supset \cdot \iota xAx = \iota xAx$$

⁸ The expression $Ay \equiv_y By$ abbreviates $(\forall y)(Ay \equiv By)$. Similarly, the expression $Ay \supset_y By$ abbreviates $(\forall y)(Ay \supset By)$.

⁹ "On Denoting" was published in October of 1905 and does not set out the theory in formal symbols. Russell clearly had such notations in his work notes "On Fundamentals" dated June 1905 (*Papers* 4: Paper 15).

as an instance of universal instantiation. Blocking the universal instantiation in *Principia* is the simple fact that there is no term “ $\exists xAx$ ” in its language. The only terms are variables. *Principia* does have the theorem:

$$(\forall x)Cx \ \& \ E!(\exists xAx) \ . \ \supset \ . \ [\exists xAx][Cx].$$

But this is certainly not a universal instantiation to a term “ $\exists xAx$ ”. *Principia* goes on to render the following theorem schema telling us the conditions under which primary and secondary occurrences of definite descriptions are logically equivalent. We find:

$$E!(\exists xAx) \ . \ \supset \ . \ [\exists xAx][B(Cx)] \equiv B([\exists xAx][Cx]),$$

where B is a truth-functional context (such as those built up from the logical connectives). If existence is assured, and B is truth-functional, primary and secondary scopes are equivalent.

It is natural to ask when Russell came to believe that ‘existence’ is not a property. In a manuscript of 22 December 1905, Russell clearly has the outlines of the theory of definite descriptions later set forth in *Principia*. The following is an excerpt from *12 of the manuscript:

- .. // $\phi! \{(\exists x)(\psi!x)\} . = : (\exists b) : \psi!x . \exists_x . x = b : \phi!b$ Df
- This defines propositions containing describing phrases. $(\exists x)(\psi!x)$ is to be read “the x which satisfies $\psi!x$ ”. Thus if $\psi!x$ is “ x wrote Waverley”, $(\exists x)(\psi!x)$ is “the author of Waverley”. If $\phi!y$ is “Scott = y ”, we find by the definition Scott is the author of Waverley. = : There is a b such that “ x wrote Waverley” is equivalent to “ $x = b$ ”, & Scott is b .
- .. III $\exists x!(\exists x)(\psi!x) . = : (\exists b) : \psi!x . \exists_x . x = b$ Df
- $\exists x$ is short for exists. $\exists x!(\exists x)(\psi!x)$ asserts that the describing phrase $(\exists x)(\psi!x)$ does denote an individual.

The notation is only slightly unusual.¹⁰ At definition *.II, Russell offers his contextual definition of the definite description—though it is without its scope marker. Then at *.III, Russell introduces what, in *Principia*, is the definition of E! ($\exists xAx$).

It may be possible to find even earlier evidence. In the July 1905 issue of *Mind*, prior to the appearance of “On Denoting” in October, Russell published a paper called “On the Existential Import of Propositions”. In this paper he writes:

The meaning of *existence* which occurs in philosophy and in daily life is the meaning which can be predicated of an individual, the meaning in which we inquire whether God exists, in which we affirm that Socrates existed, and deny that Hamlet existed. The entities dealt with in mathematics do not exist in this sense: the number 2, or the principle of the syllogism, or multiplication, are objects which mathematics considers, but which certainly form no part of the world of existent things. This sense of existence lies wholly outside Symbolic Logic, which does not care a pin whether its entities exist in this sense or not.¹¹

Russell says that symbolic logic doesn’t care a “pin” about the ordinary sense of “exists” as applied to individuals. This strongly suggests that he thinks that ordinary existence statements are not transcribed into symbolic logic by means of an existence predicate.¹²

Russell was concerned to reply to Hugh MacColl’s thesis that since classes are collections of entities, the null-class must only be empty of existents. It contains non-existents. Russell explains:

¹⁰ “On Substitution”, RAI 220.0194ob, fol. 4. Transcribed, the text is:

- .II $\phi! \{(\exists x)(\psi!x)\} . = : (\exists b) : \psi!x . \equiv_x . x = b : \phi!b$ Df
 This defines propositions containing denoting phrases. $(\exists x)(\psi!x)$ is to be read “the x which satisfies $\psi!x$ ”. Thus if $\psi!x$ is “ x wrote Waverley”, $(\exists x)(\psi!x)$ is “the author of Waverley”. If $\phi!y$ is “Scott = y ”, we find by the definition Scott is the author of Waverley . = : There is a b such that ‘ x wrote Waverley’ is equivalent to ‘ $x = b$ ’, and Scott is b .
- .III $Ex!(\exists x)(\psi!x) . = : (\exists b) : \psi!x . \equiv_x . x = b$ Df
 Ex is short for *exists*. $Ex!(\exists x)(\psi!x)$ asserts that the denoting phrase $(\exists x)(\psi!x)$ does denote an individual.

¹¹ In *Essays in Analysis*, p. 98; *Papers* 4: 486.

¹² For a discussion of Russell’s views on sentences such as “This exists”, which involve indexicals outside of logic, see Charles Ripley, “Russell and Moore on Existence as a Predicate”, *Russell* o.s. nos. 37–40 (1980): 17–30.

But it is natural to inquire what we are going to say about Mr. MacColl's classes of unrealities, centaurs, round squares, etc. Concerning all these we shall say simply that they are classes which have no members, so that each of them is identical with the null-class. There are no Centaurs.... Similarly, there are no round squares. The case of nectar and ambrosia is more difficult, since these seem to be individuals, not classes. But here we must presuppose definitions of nectar and ambrosia: they are substances having such and such properties, which, as a matter of fact, no substances do have. We have thus merely a defining concept for each, without any entity to which the concept applies. In this case, the concept is an entity, but it does not denote anything. To take a simpler case: "The present King of England" is a complex concept denoting an individual; "the present King of France" is a similar complex concept denoting nothing. The phrase intends to point out an individual, but fails to do so: it does not point out an unreal individual, but no individual at all. The same explanation applies to mythical personages, Apollo, Priam, etc.¹³

At this time, Russell seems to still hold some form of the theory of denoting concepts of *The Principles of Mathematics* (1903). Nonetheless, his plan for applying symbolic logic to existence claims made with ordinary proper names clearly foreshadows what would become central to the theory of *Principia*. We are to replace ordinary proper names such as "God" with definite descriptions so that symbolic logic represents an ordinary assertion of existence as an assertion that there is a unique entity satisfying the condition (or in the class) characterized by the description. Given Russell's view, it seems that he should say that ontological arguments which rely on the premiss that 'existence' is a perfection are simply ungrammatical mumbo jumbo. But this was not his position. How then do we transcribe ordinary language arguments (say about the existence of God) into symbolic logic to check for their validity?

In "On Denoting" Russell does not investigate Anselm's original version of the ontological argument. But it is useful to begin with it in understanding how the theory of definite descriptions offered him a logical analysis of ontological arguments. Anselm states his argument as follows in the *Proslogion*:

God is that, than which nothing greater can be conceived.... And [God] assuredly exists so truly, that it cannot be conceived not to exist. For, it is possible to conceive of a being which cannot be conceived not to exist; and this is greater

¹³ *Essays in Analysis*, p. 100; *Papers* 4: 487. This is a missed opportunity for Russell, since he might have included God with mythological creatures like Apollo and Priam.

than one which can be conceived not to exist. Hence, if that, than which nothing greater can be conceived, can be conceived not to exist, it is not that, than which nothing greater can be conceived. But this is an irreconcilable contradiction. There is, then, so truly a being than which nothing greater can be conceived to exist, that it cannot even be conceived not to exist; and this being thou art, O Lord, our God.¹⁴

Anselm takes it to be logically true that for any property ϕ , if one conceives of God as ϕ then, attending to the concept of God, one thereby conceives of God as both ϕ and existing. Hence, Anselm thinks that if it is conceivable (logically possible¹⁵) that God is ϕ , then it is conceivable (logically possible) that God is both ϕ and existing. With this premiss Anselm's argument may be represented as follows:

$$(\forall \phi)(\diamond \phi(\exists zGz) \supset \diamond(\phi(\exists zGz) \ \& \ E!(\exists zGz)))$$

Therefore $\square E!(\exists zGz)$.

We have replaced the proper name "God" with the definite description " $\exists zGz$ " for "the being a greater than which cannot be conceived". We let $\diamond p$ mean that p is conceivable (logically possible), and we let $\square p$ mean $\sim \diamond \sim p$, i.e., $\sim p$ is inconceivable (p is necessary).

Anselm's derivation seems deceptively simple.¹⁶ We are to universally instantiate to a property of *non-existence*. Thus, imagine universally instantiating to arrive at:

$$\diamond \sim E!(\exists zGz) \supset \diamond(\sim E!(\exists zGz) \ \& \ E!(\exists zGz)).$$

The consequent is clearly a contradiction and hence, by *modus tollens*, we have $\sim \diamond \sim E!(\exists zGz)$. Thus, $\square E!(\exists zGz)$. It is inconceivable that God not exist (necessarily God exists). Russell would find many flaws in this derivation. In *Principia*, neither 'existence' nor 'non-existence' is a property and so cannot be involved in a universal instantiation. Observe that

¹⁴ St. Anselm, *Basic Writings*, trans. N. S. Deane, 2nd edn. (La Salle: Open Court, 1962), Chap. 3.

¹⁵ Few today would agree with Anselm that conceivability and logical possibility coincide.

¹⁶ This formulation avoids the problem Hochberg sees in the argument. See Herbert Hochberg, "St. Anselm's Ontological Argument and Russell's Theory of Descriptions", *New Scholasticism* 33 (1959): 319–30.

$\sim E!(\iota zGz)$ abbreviates $\sim (\exists x)(Gx \equiv_y y = x)$, which is certainly not of the form $\phi(\iota zGz)$.

There are other flaws. Since ϕ is a predicate variable, the only possible secondary scope for the definite description in $\diamond \phi(\iota zGz)$ yields the following: $\diamond([\iota zGz][\phi z])$. Thus, making the scope clear, the first premiss is this:

$$(\forall \phi)(\diamond[\iota zGz][\phi z] \supset \diamond([\iota zGz][\phi z] \ \& \ E!(\iota zGz))).$$

From this perspective, Anselm's flaw is in having failed to notice the scope ambiguity in the use of the definite description. Consider what happens if we construe 'non-existence' as a property $\sim E\hat{z}$. Applying universal instantiation, we arrive at:

$$\diamond[\iota zGz][\sim Ez] \supset \diamond([\iota zGz][\sim Ez] \ \& \ E!(\iota zGz)).$$

The consequent is false, so by *modus tollens* we arrive at

$$\sim \diamond[\iota zGz][\sim Ez].$$

But this does not assert that it is inconceivable that God not exist (necessarily one and only one entity is G). It asserts

$$\sim \diamond(\exists z)(Gy \equiv_y y = z. \ \& \ . \ \sim Ez).$$

This says that it is inconceivable that there be one and only one G who does not exist. This does not assure the existence of God. One might hold, contrary to Anselm, that $\square (\forall z)(\phi y \equiv_y y = z. \ \supset \ . \ Ez)$ is true for all properties ϕ . Of course, on Russell's official view of *Principia*, there is no property E of *existence* and no property of *non-existence*. But the point is that the derivation fails (due to scope) even if *existence* and *non-existence* are properties. In this respect, one might say that it is not the rejection of *existence* as a property but attention to scope that forms the essence of Russell's diagnosis of the flaw in the ontological argument.¹⁷

We might try to do better in understanding Anselm's conception of scope by writing the premiss schematically. This yields the following:

¹⁷ Thanks to Richard Fumerton for this point.

$$\diamond A(\exists zGz) \supset \diamond(A(\exists zGz) \ \& \ E!(\exists zGz)).$$

In some instances of this schema, the definite description has a secondary scope in A and in other instances it has a primary scope. If we accept the schema, then we would be committed, given that $\sim E!(\exists zGz)$ is counted as an instance of $A(\exists zGz)$, to a derivation of the necessary existence of God. For we have $\diamond \sim E!(\exists zGz) \supset \diamond(\sim E!(\exists zGz) \ \& \ E!(\exists zGz))$. The consequent is a contradiction; thus by *modus tollens* we get $\sim \diamond \sim E!(\exists zGz)$. Russell now has two lines of response. He holds that $\sim E!(\exists zGz)$ is not an instance of $A(\exists zGz)$. He also rejects the schematic premiss. Some secondary occurrences of definite descriptions do not entail that $E!(\exists zGz)$, unless of course $E!(\exists zGz)$. For instance, let p be false and let $A(\exists zGz)$ be $p \supset [\exists zGz][Bz]$. Thus the schematic form of Anselm's premiss is not acceptable to Russell.

As we noted, Russell does not discuss Anselm's ontological argument in "On Denoting". He discusses something closer to the version Descartes gave, modifying it in light of his book *The Philosophy of Leibniz* (1900). At *Meditations V*, Descartes wrote:

... it is in truth necessary to admit that God exists, after having supposed him to possess all perfections, since existence is one of them.

The version Russell offers takes it as a premiss that the most perfect being (God) has all perfections. To put this in symbols, we need to represent the notion of a perfection and the notion of a perfect being. Leibniz thought of the argument in a modal way, as did Anselm. He was concerned that Descartes had not established that a most perfect being is possible. To rectify this, Leibniz offered the idea that every perfection is a positive property (a property that is simple so that any two positive properties can be exemplified by the same entity). Leibniz writes:

I call every simple quality which is positive and absolute, or expresses whatever it expresses without any limits, a perfection. But a quality of this sort, because it is simple, is therefore irresolvable or indefinable, for otherwise either it will not be a simple quality, but an aggregate of many, or, if it is one, it will be circumscribed by limits and so be known though negations of further progress contrary to the hypothesis, for a purely positive quality was assumed. From these

considerations it is not difficult to show that all perfections are compatible with each other or can exist in the same subject.¹⁸

Where $P(\phi)$ says that ϕ is a perfection, Leibniz means to have the following in mind: $(\forall \phi)(P(\phi) \supset \text{Positive}(\phi))$. Each perfection is the possession of a given positive property to the maximum or highest degree possible. This is corroborated by Leibniz's remark that "... God is absolutely perfect—*perfection* being nothing but the magnitude of positive reality considered as such, setting aside the limits or bounds in the things which have it. And here, where there are no limits, that is, in God, perfection is absolutely infinite."¹⁹ Thus, each perfection is a positive property. Since all positive properties are compatible, no two perfections are incompatible. One might then define the God property as

$$Gx =_{\text{df}} (\forall \phi)(P(\phi) \equiv \phi x).$$

An entity x is G if and only if x has all and only perfections. From this definition it follows that

$$E!(\exists x Gx) \cdot \supset \cdot P(\phi) \equiv_{\phi} [\exists x Gx] [\phi x].$$

If God exists, then all and only perfections are properties of God. It follows as well that there is at most one G , that is:

$$(\forall x)(\forall y)(Gx \ \& \ Gy \cdot \supset \cdot x = y).$$

The result follows from df¹ of G by Leibniz's principle of the Identity of Indiscernibles: $(\forall x)(\forall y)((\forall \phi)(\phi x \equiv \phi y) \supset x = y)$. This is a logical truth, given that every well-formed formula comprehends an attribute.²⁰ From this we can easily prove that there is at most one G . Assume that Gx and Gy and $x \neq y$. Hence, for some property θ we have θx and $\sim \theta y$. By Gx we have $P(\theta)$ and yet by Gy , we have $\sim P(\theta)$.

Russell was well aware of Leibniz's view that perfections are positive

¹⁸ Gottfried Leibniz, "Two Notations for Discussion with Spinoza", in *The Ontological Argument*, ed. A. Plantinga (Garden City, NJ: Anchor Books, 1965), pp. 55–6.

¹⁹ Leibniz, *The Monadology and Other Philosophical Writings*, trans. Robert Latta (London: Oxford U.P., 1951), p. 240.

²⁰ The property $x = \hat{x}$ is a property of x and hence also of y .

properties. In his comments in *The Philosophy of Leibniz*, he writes that on Leibniz’s view “every quality which is simple or absolute, positive and indefinable, and expresses its object without limits, is a perfection. All such qualities can be predicates of one and the same subject” (PL, p. 174). Russell alludes to Leibniz’s idea of a positive property in a footnote in “On Denoting”. He says he is “... taking the definition of perfection as possession of all positive predicates.” Russell’s footnote suggests that he has: $Gx =_{df} (\forall \phi)(\text{Positive}(\phi) \supset \phi x)$. But the footnote also says that it can be proved formally that the class of most perfect beings cannot have more than one member. That is, it can be proved that there can be at most one G . It seems that Russell just means by $\text{Positive}(\phi)$ that ϕ is the maximum of the given positive property. In short Russell is just replacing $P(\phi)$ with $\text{Positive}(\phi)$. But even so, this does not fully explain Russell’s adoption of df^2 since it does not capture the notion of God as having all and *only* perfections. This may have been a slip on Russell’s part. On the other hand, perhaps Russell imagined deriving that there is at most one G from some perfection itself. For example, perhaps the property of *being omnipotent* assures it, or perhaps the property $(\forall \phi)(\phi \hat{z} \supset P(\phi))$ of having only perfections is itself a perfection. Let us find a compromise by adopting the definition of G as follows: $Gx =_{df} (\forall \phi)(P(\phi) \supset \phi x)$. At the same time let us assume uniqueness is logically provable. This offers some progress. Russell also says in his “On Denoting” footnote that it can be proved almost equally formally that the class of most perfect beings cannot have even one member. Unfortunately, I can offer no progress there.

Having sorted out Russell’s definition of G , we are led to the following straightforward formalization of the ontological argument:

$$\begin{aligned} & [\exists x Gx][(\forall \phi)(P(\phi) \supset \phi x)] \\ & P(E) \\ & \text{Therefore } (\exists x) Gx. \end{aligned}$$

On this formulation, the ontological argument does *not* have the form:

$$p, q; \text{ therefore } p.$$

Moreover, it does not have this form even with the conclusion $E!(\exists x Gx)$. Nonetheless, it clearly begs the question.

Two serious problems plague this formulation. First, it trivializes the

argument, obviating its second premiss by putting the definite description $\exists xGx$ in a primary scope in the first premiss. The second premiss plays no role whatsoever in the argument. This undermines the work Russell alludes to in his footnote. Russell says that with “perfection” defined as possession of all positive predicates, it can be shown that there is at most one G . The unique existence of a G is already asserted in the first premiss by the primary scope of the definite description $\exists xGx$. The second serious flaw in the formulation is that it allows a premiss that is ungrammatical according to *Principia*. The second premiss uses the expression E as a predicate for a property ‘existence’. The formalization does not present a well-formed argument.

In *A History of Western Philosophy* Russell offers something which might help as an explanation of his comments on the ontological argument in “On Denoting”. He wrote:

St Anselm was ... an Italian, a monk at Bec, and archbishop of Canterbury (1093–1109)... He is chiefly known to fame as the inventor of the “ontological argument” for the existence of God. As he put it, the argument is as follows: We define “God” as the greatest possible object of thought. Now if an object of thought does not exist, another, exactly like it, which does exist, is greater. Therefore the greatest of all objects of thought must exist, since, otherwise, another, still greater, would be possible. Therefore God exists.

This argument has never been accepted by theologians. It was adversely criticized at the time; then it was forgotten till the latter half of the thirteenth century. Thomas Aquinas rejected it, and among theologians his authority has prevailed ever since. But among philosophers it has had a better fate. Descartes revived it in a somewhat amended form; Leibniz thought that it could be made valid by the addition of a supplement to prove that God is *possible*. Kant considered that he had demolished it once and for all. (HWP, p. 417)

In discussing Kant’s rejection of the argument, he wrote:

The ontological proof, as he sets it forth, defines God as the *ens realissimum*, the most real being; i.e., the subject of all predicates that belong to being absolutely. It is contended, by those who believe that proof valid, that, since “existence” is such a predicate, this subject must have the predicate “existence”, i.e. must exist. (HWP, p. 709)

This passage seems to suggest that the validity of the ontological argument requires that ‘existence’ be accepted as a property. But in his remarks on the Leibniz/Descartes form of the argument, Russell wrote the

following:

Leibniz easily proves that no two perfections, as above defined, can be incompatible. He concludes: "There is, therefore, or there can be conceived, a subject of all perfections, or most perfect Being. Whence it follows also that He exists, for existence is among the number of the perfections."

Kant countered this argument by maintaining that "existence" is not a predicate. Another kind of refutation results from my theory of descriptions. The argument does not, to a modern mind, seem very convincing, but it is easier to feel convinced that it must be fallacious than it is to find out precisely where the fallacy lies. (HWP, p. 586)

The idea in this passage seems to be that there are two lines of objection: Kant's, which is that 'existence' is not a property; and Russell's which, in applying the theory of definite descriptions, finds the argument valid but question-begging. This suggests that perhaps in "On Denoting" Russell intended to say that even if 'existence' is a property (so that the argument is logically formulable), it begs the question. But later in *A History of Western Philosophy* Russell goes on to say that "... as a result of analysis of the concept 'existence', modern logic has proved this argument invalid" (p. 787). These comments seem better suited to a discussion of Anselm's ontological argument. As we saw, Russell's theory of definite descriptions reveals that Anselm's argument is invalid. In fact, we found it to be invalid even if 'existence' is taken as a property. Thus, Russell's comments do not help us to understand his intent in "On Denoting", where the argument is found to be valid.

Surely Russell had in mind a more faithful formulation of the ontological argument. The formulation must not treat 'existence' as a property. A faithful formulation must neither obviate the second premiss nor make it ungrammatical. But if the second premiss is to play an important role in the argument, then how can Russell formulate the argument and find it valid but question-begging? Borrowing a phrase from Aquinas, who rejected Anselm's ontological argument, let us investigate five ways.

3. THE FIRST WAY

One might imagine transcribing the second premiss of the ontological argument into the language of *Principia* by means of $Gx \supset_x (\exists y)(x = y)$. This yields the following:

$$\begin{aligned} &[\exists x Gx][(\forall \phi)(P(\phi) \supset \phi x)] \\ &Gx \supset_x (\exists y)(x = y) \\ &\text{Therefore } (\exists x) Gx. \end{aligned}$$

This approach to formulating the argument fails. To be sure, *Principia* accepts the existential commitments of classical logic. Thus, the second premiss is grammatical and a logical truth. Both $(\forall x)(\exists z)(z = x)$ and $(\exists z)(z = z)$ are regarded as logical truths of classical logic. They follow from the schema

$$*10.01 \quad (\forall x) Cx \supset C[y/x],$$

where y is free for x in C . The presence of the free (“real”) variable “ y ” is the culprit. Whitehead and Russell write:

The assumption that there is something, which is equivalent to this proposition, is implicit in proposition *10.01 that what is always true is true in any instance. This would not hold if there were no instances of anything; hence it implies the existence of something. ... The assumption that there is something is involved in the use of the real variable, which would otherwise be meaningless.

(*PM* I: 226)

In *Introduction to Mathematical Philosophy* (1919), however, Russell came to hold that it is a “defect in logical purity” that *Principia* has a commitment to at least one individual (*IMP*, p. 203n.). In Appendix A of *Principia*’s second edition (1925–27), Russell offers a deductive system without free variables. Given his earlier comments, it is likely that he hoped that this is the first step toward a new formal logic that is free of the commitment to at least one individual. In any event, it is clear that in *Principia* Russell did not think that the property $(\exists z)(z = \hat{x})$ is an ‘existence’ property. The expression “ $(\exists z)(z = x)$ ” abbreviates “ $(\exists z)(\forall \phi)(\phi!z \equiv \phi!x)$ ” and says that x has all its predicative attributes in common with something. This certainly does not say that x exists. There is also the theorem “ $x = x$ ” and the theorem “ $(\exists \phi)\phi x$ ”. None of these say “ x exists”. Indeed, there are infinitely many theorems in *Principia* containing “ x ” free. Every instance of “ $p \supset p \cdot v \cdot \phi x$ ” is a theorem. None of these theorems say that x exists. The presence of such theorems certainly does not capture the metaphysician’s thesis that ‘existence’ is a property, even if she holds that it is metaphysically necessary that everything exists. To

have an existence predicate one would have to introduce singular terms besides variables into the language of logic together with a primitive sign “E”.

Our task in formulating the ontological argument is therefore complicated. We want a formalization of the argument on behalf of Russell that presents the first premiss in such a way that the definite description ιxGx has a *secondary* occurrence in it. To formulate the first premiss in such a way that the definite description ιxGx has a primary occurrence utterly trivializes the argument, obviating the second premiss and making Russell’s work to show it is valid (though question-begging) entirely vapid. In the formulation of the argument that we are seeking, the second premiss must play an essential role in the proof. If, however, the second premiss plays an essential role, then in what sense can the argument be said to be question-begging?

The intuitive idea is that a deductive argument begs the question if and only if at least one of its premisses *asserts* the conclusion. Hence, the argument

p, q ; therefore p

begs the question since p is asserted in the first premiss. But the argument

If p then q , if p then r ; therefore if p then q and r

is not question-begging since the conclusion is not asserted in any one of the premisses. It would take our topic too far afield to attempt an analysis of what it is for a deductive argument to beg the question.²¹ Happily, such an analysis is not needed for our task. In the context of the present discussion, we have strong conditions of adequacy. The criteria of adequacy set before us apply *only* to the Russellian formulation of the Descartes/Leibniz ontological argument. We shall say that it begs the question only if the second premiss is used essentially in the proof and the first premiss is logically equivalent to the conclusion. This, in turn, requires that the second premiss be logically true. If the second premiss

²¹ It is sometimes claimed that every deductive argument begs the question. I disagree, but we can avoid the issues here.

were contingent, then the first premiss would not logically entail the conclusion since it essentially uses a contingent premiss.²²

4. THE SECOND WAY

Our central problem is to find a viable transcription of “existence is a perfection” that fits with a secondary scope of the description λxGx in the first premiss. We can do this if we represent the ontological argument on Russell’s behalf as follows:

$$\begin{aligned} &(\forall \phi)(P(\phi) \supset [\lambda xGx][\phi x]) \\ &G(\lambda x\lambda/x) \supset_{\psi} E!(\lambda x\lambda/x) \\ &\text{Therefore } (\exists x) Gx. \end{aligned}$$

Observe that on this rendition, the first premiss has the form $G(\lambda xGx)$, with its definite description in a secondary scope. This is nice because it is this form that seemed to Meinongians to be the foundation of Intentionality. Its truth seems given by the very descriptive concept itself. In “On Denoting”, Russell explicitly discusses and criticizes the principle, $A(\lambda xAx)$, as “apt to infringe the law of contradiction” so the second way comports with the tenor of Russell’s discussion.

One might worry, however, that our first premiss assumes a distinct god for each property that is a perfection. That is certainly out of sorts with what would be intended by the first premiss of the Descartes/Leibniz ontological argument. We need, therefore, to show that there can be at most one G . We saw that this is to be immediate from the definition of G .

In the present formulation, the ontological argument is question-begging (in our strong sense). The secondary scope of the definite description λxGx in the first premiss is logically equivalent to a primary occurrence. We have:

$$(\forall \phi)(P(\phi) \supset [\lambda xGx][\phi x]) \equiv [\lambda xGx][(\forall \phi)(P(\phi) \supset \phi x)].$$

Given that the second premiss is logically true, the conclusion is logically

²² Our criteria of adequacy do not require that the first premiss be logically necessary. That would undermine interest in the question of whether the conclusion logically implies the first premiss.

equivalent to the first premiss. The left-to-right direction is the ontological argument itself. The right-to-left direction is immediate since we have $E!(\exists xGx) \supset [\exists xGx][Gx]$. Hence we have our result.

Trouble comes, however, in assuring that our second premiss is a logical truth. Recall that the second premiss is this: $G(\exists x\psi x) \supset_{\psi} E!(\exists x\psi x)$. This is readily assured if logically there must be at least one property that is a perfection, for then $G(\exists x\psi x)$ logically entails $E!(\exists x\psi x)$. To see this, assume $G(\exists x\psi x)$. By definition this is $(\forall \phi)(P(\phi) \supset [\exists x\psi x][\phi x])$. By universal instantiation we arrive at $P(S) \supset [\exists x\psi x][Sx]$. If it is logically true that some property S is a perfection, then the antecedent is true and is so by *modus ponens* $[\exists x\psi x][Sx]$. This primary occurrence of the definite description readily yields $E!(\exists x\psi x)$.

But how can we assure that it is logically necessary that some property is a perfection? One way is to adopt the following as a logical axiom:

$$(\forall \phi)(P(\phi) \equiv \sim P(\sim \phi \hat{x})).$$

This axiom assures that every property is such that either it or its complement is a perfection. Since comprehension principles in standard second-order logic assure the existence of properties and their complements, we have our result. It is far from clear, however, that this axiom is plausible—let alone a logical truth. A better candidate is this, $(\forall \phi)(P(\phi) \supset \sim P(\sim \phi \hat{x}))$. This allows that some property is such that neither it nor its complement is a perfection. Unfortunately, this weakened axiom does not assure the existence of at least one perfection. We therefore need a definition of a property being a perfection that yields the logical existence of at least one perfection. It is not at all clear what might be the definition.

5. THE THIRD WAY

The weakness of the second way is that its second premiss is not doing much work in the argument. The second premiss encapsulates the crux of the argument's derivation of the conclusion from the first premiss. So there is a sense in which it is being used. But there is also a sense in which the logical necessity of there being a property that is a perfection does all the heavy lifting. The first premiss yields the conclusion without appeal to the second premiss.

A better approach that suggests itself is to move up a type in Russell's simple type theory. This yields the third way to formulate the argument.

We have:

Ontological Argument (OA)

$(\forall \psi)(\text{Perfection}(\psi) \supset \psi(G))$
 Perfection $\{(\exists x)\hat{\phi}x\}$
 Therefore $(\exists x)Gx$.

Our task is next to define “Perfection(ψ)”. Thus we put:

$\text{Perfection}(\psi) =_{\text{df}} [\exists xGx][\hat{\phi}x] \supset_{\phi} \psi(\phi)$.

A property ψ is a perfection property just when every property ϕ of God (the perfect being) exemplifies it, if God exists. Note that we can very easily arrive at $E!(\exists xGx)$ from the first premiss. We have only to notice that Perfection $\{[\exists xGx][\hat{\phi}x]\}$. That is, the property of being a property exemplified by God is itself a perfection property. Hence, from our first premiss we have $[\exists xGx][Gx]$. This entails $E!(\exists xGx)$.

Advocates of the ontological argument who make ‘existence’ a property have always struggled with the problem that ‘existence’ does not seem to be a perfection of an individual. After all, there is a sense in which a Satan is more evil if he exists than otherwise. But on the present formulation, $(\exists x)\hat{\phi}x$ is certainly a perfection property in the sense that $[\exists xGx][\hat{\phi}x] \supset_{\phi} (\exists x)\phi x$. This is logically true.²³ With our definition of “Perfection(ψ)” the problem vanishes.

Parodies of the present form of the ontological argument vanish, too. In a letter of 3 January 1912 to Lady Ottoline Morrell, Russell shows he was well aware of the problem of parody. He wrote:

What is plain is this: Man can imagine things that don't exist, and sometimes he can see that they are better than things that do exist: this is involved in all rational action.... Some people say that the mere fact that one can *imagine* ideas shows that they exist somewhere—but if this were true it would apply to bad things too—the Devil would be proved by the same argument as proves God.... Is there not the same reason to regard bad thoughts as promptings of the Evil One that there is to regard good ones as inspirations of God? (SLBR I: 413)

²³ It is important not to confuse Perfection(ψ) with $P(\phi)$. We might have written Perfection $\{\psi(\hat{\theta})\}$ and $P(\phi\hat{z})$ to indicate the type difference.

The parody objection attempts to show that similar reasoning yields the existence of Satan (who is maximally evil and ungodly). In addition to the ontological argument for God, consider the following parody for Satan:

$$\begin{aligned} & (\forall \psi)(\text{Evilness}(\psi) \supset \psi(\text{U}G)) \\ & \text{Evilness}\{(\exists x)\hat{\phi}x\} \\ & \text{Therefore } (\exists x)\text{U}Gx. \end{aligned}$$

The parody would offer a definition of an evilness property as follows:

$$\text{Evilness}(\psi) =_{\text{df}} [\exists x \text{U}Gx][\phi x] \supset_{\phi} \psi(\phi).$$

But the definition of $\text{Evilness}(\psi)$ fails to be motivated. The definition says that a property ψ is an Evilness property just when it is exemplified by every property ϕ of the most ungodly being, if there is such. On this definition, if there is no maximally evil ungodly being, every property of a property is an evilness property—even the property $[\exists xGx][\phi x]$. This seems untenable. Surely there are evilness properties without Satan. Let Hy mean that y has all the properties of Hitler. Now if there is Satan, $\text{Evilness}\{\hat{\phi}y \equiv_y Hy\}$ fails to be true, since obviously not every property ϕ of Satan is such that $\phi y \equiv_y Hy$. The above definition of “ $\text{Evilness}(\psi)$ ” makes it the case that the existence of Satan prevents $\hat{\phi}y \equiv_y Hy$ from being an evilness property. Theologians traditionally held that without God, perfection is trivialized and any properties would become as much a perfection property as any other. As Leibniz put it, “... creatures derive their perfections from God’s influence, but ... they derive their imperfections from their own nature, which is incapable of being without limits. For it is in this that they are distinguished from God.”²⁴ This is not to say that there is no good without God; it is to say there is no perfection without God. There is no such analog for Satan. There are certainly evilness properties without Satan. The point of all this slapdash theology is that it assures that there is an asymmetry which, in its present form, prevents parody of the ontological argument.²⁵ Indeed, if God has only per

²⁴ *Monadology*, p. 240.

²⁵ In 1944, Russell offered a nice parody of Leibniz’s idea that God surveyed all the worlds that are logically possible and, being beneficent, decided to create the one of the possible worlds that, although it contained a good deal of evil, contained the greatest

fections and Satan has no perfections, we have:

Perfection $\{P(\hat{\phi})\}$
 Evilness $\{\sim P(\hat{\phi})\}$.

Hence it follows that $[\exists xGx][\phi x] \supset_{\phi} \sim [\exists x^UGx][\phi x]$.²⁶ Since the ontological argument for God yields $E!(\exists xGx)$, we have $[\exists xGx][x = x]$. But then from the above we arrive at $\sim [\exists x^UGx][x = x]$ and so $\sim E!(\exists x^UGx)$. Manicheism is impossible. Of course, this doesn't show whether it is to be God who exists or Satan (or neither). But the parody fails if it was supposed to show that the technique of the ontological argument can generate Satan as well as God.

The third way also offers a nice response to Gaunilo's famous objection to the argument. Gaunilo, a Benedictine monk, put his objection by saying that a similar argument form could work for the concept of a perfect lost island. Put in terms of the third way, the objection is that the first premiss could be asserted for any property Ω whatsoever—for example, let Ω be the property of 'being a perfect lost island'. Consider the following argument:

$(\forall \psi)(\text{Perfection}(\psi) \supset \psi(\Omega))$
 Perfection $\{(\exists x)\hat{\phi}x\}$
 Therefore $(\exists x)\Omega x$.

Anselm replied to Gaunilo, prefacing his remarks with this:

It was a fool against whom the argument of my *Proslogium* was directed. Seeing, however, that the author of these objections is by no means a fool, and is a Catholic, speaking in behalf of the fool, I think it sufficient that I answer the Catholic.²⁷

excess of good over evil. "This is a pretty fable", Russell writes, "... [but] it is exactly equally possible that the world was created by a wholly malicious devil, who allowed a certain amount of good in order to increase the sum of evil." See Russell, "The Value of Free Thought", in Al Seckel, ed., *Russell on God and Religion* (Buffalo: Prometheus Books, 1986), p. 257.

²⁶ We have $[\exists xGx][\phi x] \supset_{\phi} P(\phi)$ and also $[\exists x^UGx][\phi x] \supset_{\phi} \sim P(\phi)$. Universally instantiate and transpose the latter to get $P(\phi) \supset \sim [\exists x^UGx][\phi x]$. Then apply hypothetical syllogism and universal generalization.

²⁷ "Anselm's Apologetic", in St. Anselm, *Basic Writings*, p. 153.

Anselm explains that he had been misunderstood and that his reasoning may be applied only to the God concept. In the present form, this reply has some merit. We are not at liberty to alter the definition of “Perfection(ψ)” to suit Ω . Recall that the definition is this: $[\exists xGx][\phi x] \supset_{\phi} \psi(\phi)$. We cannot plausibly change it in a way that matches the change in the first premiss, replacing G by Ω . The concept of a *perfection property*, as we have defined it, is inseparable from the concept of *perfection* that is given in the definition of the God property G . One can prove

$$(\forall \phi)(\forall \psi)(\text{Perfection}(\psi) \ \& \ P(\phi) \ . \supset \ . \text{E}! [\exists xGx] \supset \psi(\phi)).$$

Moreover, one can prove:

$$(\forall \theta)((\forall \psi)(\text{Perfection}(\psi) \supset \psi(\theta)) \ . \supset \ . G_x \supset_x \theta x).$$

This follows because $\text{Perfection}(G_x \supset_x \hat{\phi}x)$. Therefore, the assertion of a first premiss as $(\forall \psi)(\text{Perfection}(\psi) \supset \psi(\Omega))$ is implausible in the extreme. It would entail that everything that is G is a lost island.

Having extolled some of the virtues of the third way, it is high time we show that the argument begs the question (in our strong sense). Our task is to show that the first premiss is logically equivalent to the conclusion. That is, we have to prove

$$(\exists x)Gx \equiv (\forall \psi)(\text{Perfection}(\psi) \supset \psi(G)).$$

For the right-to-left direction, we have the following:

1. $(\forall \psi)(\text{Perfection}(\psi) \supset \psi(G))$ premiss.
2. $(\forall \psi)([\exists xGx][\phi x] \supset_{\phi} \psi(\phi) \ . \supset \ . \psi(G))$ 1, df “Perfection(ψ)”.
3. $[\exists xGx][\phi x] \supset_{\phi} (\exists x)\phi x \ . \supset \ . (\exists x)Gx$ 2, universal instantiation.²⁸
4. $[\exists xGx][\phi x] \supset_{\phi} (\exists x)\phi x$ 3, theory of definite descriptions.
5. $(\exists x)Gx$ 3, 4, *modus ponens*.

For the left-to-right direction, assume $(\exists x)Gx$. Given that there is at

²⁸ Strictly speaking, we must employ the comprehension principle for attributes to arrive at an attribute θ such that $(\forall \phi)(\theta(\phi) \equiv (\exists x)\phi x)$, and then we universally instantiate to θ . This yields $[\exists xGx][\phi x] \supset_{\phi} \theta(\phi) \ . \supset \ . \theta(G)$. From here line 3 follows from basic logic.

most one G , we have $E!(\neg xGx)$. Assume $\text{Perfection}(y)$. This yields $[\neg xGx][\phi x] \supset_{\phi} \psi(\phi)$. Next universally instantiate to G and we have $[\neg xGx][Gx] \supset \psi(G)$. Since $E!(\neg xGx)$ readily yields $[\neg xGx][Gx]$, we have our result that $\psi(G)$. Observe that the stronger conclusion $E!(\neg xGx)$ is even more obviously logically equivalent to the first premiss.

On the present definition of “ $\text{Perfection}(\psi)$ ” the definite description “the perfect being” has a secondary scope in the first premiss. We can see this by simply replacing “ $\text{Perfection}(\psi)$ ” in the first premiss by its *definiens*. This yields:

$$(\forall \psi)([\neg xGx][\phi x] \supset_{\phi} \psi(\phi) \cdot \supset \cdot \psi(G)).$$

This is logically equivalent to the primary occurrence in

$$[\neg xGx][(\forall \psi)(\phi x \supset_{\phi} \psi(\phi) \cdot \supset \cdot \psi(G))].$$

The third way, therefore, seems to be the best way to form a Russellian transcription of the Descartes/Leibniz ontological argument.

6. THE FOURTH AND FIFTH WAYS

It remains, however, to consider the question as to whether every ontological argument for God in the form OA begs the question (in the strong sense that the first premiss is logically equivalent to the conclusion) on *any* definition of “ $\text{Perfection}(\psi)$ ” which makes the first premiss non-contradictory²⁹ and makes $\text{Perfection} \{(\exists x)\hat{\phi}x\}$ logically true.

Let us examine the case where we make the second premiss logically true by means of the following definition:

$$\text{Perfection}(\psi) =_{\text{df}} E!(\neg xGx) \supset \psi(G).$$

This says that a perfection property is a property that God property G has, if God exists. This forms a fourth way to formulate the argument. Quite obviously, $\text{Perfection} \{(\exists x)\hat{\phi}x\}$ is logically true since by our definition, this says: $E!(\neg xGx) \supset (\exists x)Gx$. The trouble with this fourth way

²⁹ Of course, *plausibility* is a weaker standard but not formally precise. Note that a definition such as “ $\text{Perfection}(\psi) =_{\text{df}} \psi(G) \cdot \mathbf{v} \cdot p \supset p$ ” makes the second premiss logically true, but makes the first premiss logically false.

to formulating the ontological argument is that it does not explain the way the argument begs the question as a logical equivalence of a primary scope of the definite description ιxGx and the secondary scope of the definite description ιxGx in the first premiss. On the present approach, the definite description ιxGx does not occur in the first premiss at all.³⁰ Nonetheless, our task of revealing that the first premiss is logically equivalent to the conclusion is now very easy. We have to show $(\exists x)Gx \equiv (\forall \psi)(\text{Perfection}(\psi) \supset \psi(G))$. The right-to-left direction is the proof of the ontological argument itself. For the left-to-right direction, assume $(\exists x)Gx$. Given there can be at most one G , we arrive at $E!(\iota xGx)$. Now assume $\text{Perfection}(\psi)$. This yields, $E!(\iota xGx) \supset \psi(G)$. Hence, we arrive at $\psi(G)$. It is worth noting as well that the stronger conclusion $E!(\iota xGx)$ is logically equivalent to the first premiss. This follows because $\text{Perfection}(E!(\iota x\hat{\phi}x))$.

The fact that the definition of “Perfection(ψ)” of the fourth way also makes the conclusion logically equivalent to the first premiss gives some credence to the intuition that every relevant instance of *OA* will beg the question in our strong sense. If we restrict definitions of “Perfection(ψ)” to pure expressions of our formal language of logic, then I suspect it is true. But the general result is doubtful. Consider the following, fifth way, to formulate an *OA* argument:

$$\text{Perfection}(\psi) =_{\text{df}} \text{Good } E!(\iota xGx) \supset \text{Good } \psi(G).$$

A property ψ is a perfection property on this new definition just in case it is good for the God property G to exemplify it if it is good for God (the unique thing which exemplifies G) to exist. We are reading $\text{Good}(p)$ as saying that it is intrinsically good that p is the case, or perhaps, all things considered it is good that p is the case. The notion is not intended to mean that it *would* be good *were* p to be the case. Hence, we have: $\text{Good}(p) \supset p$. Now given our new definition of “Perfection(ψ)”, are we assured that $\text{Perfection} \{(\exists x)\hat{\phi}x\}$ is logically true? This says

$$\text{Good } E!(\iota xGx) \supset \text{Good}(\exists x)Gx.$$

One might object that even when p logically entails q , it doesn't follow

³⁰ Recall that $E!(\iota xGx)$ does not involve an occurrence of a definite description.

that if Good(p) then Good(q). It may well be good that there is exactly one G , and not good that there is *a* G since many G 's (many gods) would be a horror. We must be careful not to violate Moore's principle of organic unity.³¹ But this objection is not telling in the context of a definition of G which assures that it is logically necessary that there is at most one G .

To show that the argument begs the question in our strong sense, we must prove that the conclusion is logically equivalent to the first premiss. That is, we must show $(\exists x)Gx \equiv (\forall \psi)(\text{Perfection}(\psi) \supset \psi(G))$ is logically true. The right-to-left direction is the proof of the ontological argument itself. For the left-to-right direction, however, we have a problem. To see this, assume $(\exists x)Gx$. We have to demonstrate that $(\forall \psi)(\text{Perfection}(\psi) \supset \psi(G))$. Notice that on the new definition of $\text{Perfection}(\psi)$, the first premiss logically entails Good E! ($\neg \exists xGx$). This reveals that if the argument is question-begging in our strong sense, then $(\exists x)Gx \supset \text{Good E!}(\neg \exists xGx)$ is logically true.

Observe that nothing we have said turned on G being the property of possessing all perfections. Insofar as the argument goes, G could be a property picking out a unique Satan in the suburbs.³² But in such a case, we should be inclined merely to say that the first premiss is false, not that the argument begs the question. I imagine that many a theologian thought that " $(\exists x)Gx \supset \text{Good E!}(\neg \exists xGx)$ " is *necessarily* true since God is the paradigm of goodness. After all, this says that if God exists then it is good that a unique God exists. But within the parameters of the present discussion, to make it a *logical* truth one would have to rely on the definition of G as the possession of all perfections. So it relies on a definition of $P(\phi)$ and the Herculean task of finding purely logical connections between what would seem to be irrevocably informal concepts of *perfection* and *good*. Moreover, the definition would have to concur with the following formulation: $(\forall \phi)(P(\phi) \supset \sim P(\sim \phi\hat{x}))$. It is not easy to find such a definition. Friends of the ontological argument may take comfort in the result that not all instances of *OA* are question-begging.

³¹ See G. E. Moore, *Principia Ethica* (Cambridge: Cambridge U.P., 1903).

³² I allude here to Russell's entertaining collection of stories, *Satan in the Suburbs and Other Stories* (New York: Simon & Schuster, 1953).

7. A MODAL VERSION OF THE RUSSELLIAN FORMULATION

In conclusion, one may wonder (though Russell would have cringed at the thought) what would happen if we tried our hand at a modal version of the third way.³³ Recall that the reason Leibniz introduced the notion of a property being positive is that he was concerned to address the weakness he saw in the Cartesian version of the ontological argument. The weakness is that one must first establish that it is possible that something is G . Maydole³⁴ has a modal version of the ontological argument which addresses this. The version is something of an improvement over the notes Gödel made toward a modal ontological argument—notes which may have been jottings for his amusement. Maydole’s argument is couched in an S_5 logic which accepts the following:

$$\begin{aligned} & \Box p \supset p \\ & \Box(p \supset q) \cdot \supset \cdot \Box p \supset \Box q \\ & \Diamond \Box p \supset \Box p \\ & \text{Carnap–Barcan wff: } \Diamond(\exists x)Ax \supset (\exists x)\Diamond Ax. \end{aligned}$$

For ease of exposition,³⁵ let us give Maydole’s argument as follows:

$$\begin{aligned} & (\forall \phi)(\forall \psi)(P(\phi) \ \& \ \Box(\phi z \supset_z \psi z) \cdot \supset \cdot P(\psi)). \\ & (\forall \phi)(P(\phi) \supset \sim P(\sim \phi \hat{x})). \\ & P(\Box G \hat{x}) \\ & \text{Therefore } (\exists x)\Box Gx. \end{aligned}$$

The proof of the conclusion runs as follows. First we establish that $\Diamond(\exists x)\Box Gx$. Assume for *reductio* that $\sim \Diamond(\exists x)\Box Gx$. Then $\Box(\forall x)\sim \Box Gx$.

³³ Russell would have cringed because he held that the only necessity is logical necessity. See the Russell–Copleston debate (*WINC*, pp. 147–8; *Papers* II: 526–7). Cocchiarella has shown that a “primary” semantics assures that for any first-order formula, logical necessity coincides with Tarski’s semantics for logical truth. In this primary semantics, $(\exists x)\Box \phi x$ is logically false. See Cocchiarella, “On the Primary and Secondary Semantics of Logical Necessity”, *Journal of Philosophical Logic* 4 (1975): 13–27.

³⁴ Robert Maydole, “The Modal Perfection Argument for the Existence of a Supreme Being”, *Philo* 6 (2003): 299–313.

³⁵ Using $x > y$ to mean x is greater than y , we can express Maydole’s definition of Gx as follows: $Gx =_{\text{df}} \Box(\forall y)(x \neq y \supset x > y) \ \& \ \Box(\forall y)(y \not> x)$. But Maydole’s definition of G is not of interest here.

From this it follows that $\Box(\Box Gz \supset_z \sim \Box Gz)$. Hence, by the first premiss, we arrive at $P(\sim \Box G\hat{x})$. But, by the second and third premisses, we have $\sim P(\sim \Box G\hat{x})$ and a contradiction. Hence, $\Diamond(\exists x)\Box Gx$. Now from the Carnap–Barcan formula we arrive at $(\exists x)\Diamond\Box Gx$. In the modal logic of S_5 this yields our desired conclusion $(\exists x)\Box Gx$.

Oppy has challenged Maydole’s first premiss on the following grounds.³⁶ For every property ϕ , we have $\Box(\phi z \cdot \supset_z \cdot \phi z \vee p)$. But let p be the statement of the occurrence of a most horrific event, say, a nuclear holocaust. By the second premiss, it would follow that if $P(\phi)$ then $P(\phi\hat{x} \vee p)$. But at a world in which p is true, the property $\phi\hat{x} \vee p$ is certainly not goodness-making for a being exemplifying it. Maydole’s first premiss is false.

A modal version of our third way offers a technique to prove $\Diamond(\exists x)\Box Gx$ that avoids the problem Oppy raised against Maydole. We have:

$$\begin{aligned} &(\forall \psi)(\text{Perfection}(\psi) \supset \psi(\Box G\hat{z})) \\ &\text{Perfection} \{ \Diamond(\exists x)\hat{\phi}x \} \\ &\text{Therefore } (\exists x)\Box Gx. \end{aligned}$$

The first premiss says that for every perfection property, the property $\Box G\hat{z}$ has it. The second premiss, reminds us that $\Diamond(\exists x)\hat{\phi}x$ is a perfection property. This holds since we have $[\exists xGx][\phi x] \supset_\phi \Diamond(\exists x)\phi x$. Hence, the first two premisses readily yield the theorem $\Diamond(\exists x)\Box Gx$. Thus we have avoided Maydole’s problems entirely. Next, by the Carnap–Barcan formula, we arrive at $(\exists x)\Diamond\Box Gx$. Applying the S_5 principle $\Diamond\Box p \supset \Box p$, we arrive at the conclusion $(\exists x)\Box Gx$ and so $(\exists x)Gx$.

The flaw in this modal ontological argument based on our third way can be found without indicting S_5 modal principles or the Carnap–Barcan formula. The conclusion is logically equivalent to the first premiss. We have

$$(\exists x)\Box Gx \equiv (\forall \psi)(\text{Perfection}(\psi) \supset \psi(\Box G\hat{z})).$$

The right-to-left direction is just the modal ontological argument itself. For the right-to-left direction, assume $(\exists x)\Box Gx$. Given we can prove there can be at most one G , we have $[\exists xGx][\Box Gx]$. Now assume Per-

³⁶ Graham Oppy, “Maydole’s 2QS5 Argument”, *Philo* 7 (2004): 203–11.

fection(ψ). By definition, this yields: $[\neg xGx][\phi x] \supset_{\phi} \psi(\phi)$. Instantiating, we arrive at $[\neg xGx][\Box Gx] \supset \psi(\Box G\hat{z})$. By *modus ponens* we get $\psi(\Box G\hat{z})$. The argument begs the question in our strong sense. Interestingly, we may further weaken the first premiss. Consider this:

$$(\forall \psi)(\text{Perfection}(\psi) \supset \diamond \psi(\Box G\hat{z})).$$

We have Perfection $\{(\exists x)\hat{\phi}x\}$ and hence we arrive at $(\exists x)\Box Gx$. But from the above we can see that, by similar reasoning, we arrive at $\psi(\Box G\hat{z})$. Hence, from $\Box p \supset p$, which yields $p \supset \diamond p$, we have $\diamond \psi(\Box G\hat{z})$. These modal versions of the argument beg the question in our strong sense just as surely as does the third way.