

BERTRAND RUSSELL IN RECENT BOOKS ON LOGIC HISTORY

IRVING ANELLIS

Peirce Edition / Indiana U.–Purdue U.

Indianapolis, IN 46202–5157, USA

IANELLIS@IUPUI.EDU

Dov M. Gabbay and John Woods, eds. *Logic from Russell to Church*. Vol. 5 of *The Handbook of the History of Logic*. Amsterdam, etc.: North-Holland, 2009. Pp. xii + 1056. US\$260; £157.99; €185.95.

Leila Haaparanta, ed. *The Development of Modern Logic*. Oxford and New York: Oxford U. P., 2009. Pp. x + 904. US\$150.00.

The essay “Bertrand Russell’s Logic” by Andrew D. Irvine in Volume 5 of the *Handbook of the History of Logic* is one of the shortest in the volume (pp. 1–28). But something of Russell pervades many of the other essays as well, as we might expect given the centrality that *Principia Mathematica* has occupied in the historiography of logic through much of the twentieth century. This is especially evident in the lengthiest of the essays, Andrea Cantini’s “Paradoxes, Self-reference and Truth in the 20th Century” (pp. 875–1013), and more than tangentially in Dale Jacquette’s “Logic for Meinongian Object Theory Semantics” (pp. 29–76) and Michael Potter’s “The Logic of the *Tractatus*” (pp. 255–304).

In their preface, the editors of the *Handbook* explain (p. vii) the rationale for treating Frege and Russell in separate volumes.¹ One reason is that it was a matter of simple convenience—to keep the volume covering logic from Leibniz to Frege of manageable size. The other is historical: while admitting that Frege and Russell shared important goals and the philosophical position of logicism, it is noted that Frege’s most important work in logic and foundations was carried out prior to his learning of Russell’s paradox, whereas the entire corpus of Russell’s work, *Principia* included, was designed to a considerable extent in efforts to deal with the paradox, as was much of the work in logic, set theory, and philosophy of logic that came as a consequence of the discovery of that paradox. Gabbay

¹ Gabbay and Woods, eds., *The Rise of Modern Logic from Leibniz to Frege*, Vol. 3 of the *Handbook of the History of Logic* (Amsterdam, etc.: North-Holland, 2004).

and Woods also make explicit the philosophical differences between Frege and Russell regarding the epistemological status of axioms as one of the significant differences between these two. More particularly, the editors assert (p. viii) that, in the very brief period between publication of the second volume of Frege's *Grundgesetze* (1903) and Russell's *Principles of Mathematics* (1903), there was a pronounced shift initiated in the goals of mathematical logic, away from naïve apriority of axiom systems, towards study of the limits and properties of formal systems. The contributors to these metalogical concerns, Herbrand, Hilbert, and Gödel, among others, are the subject of the bulk of the essays included in Volume 5 of the *Handbook*.

The task that Irvine undertakes is initiated by the effort to understand the unity of Russell's technical body of work in mathematics, logic, metaphysics, and epistemology. Thus the discrepancy between the neo-Hegelian concept of relations, inherited from Bradley, with certain types of relations—logical, spatial, temporal, for example—with the mathematical understanding of relations, is given as the ground for Russell's abandonment of idealism in favour of realism (pp. 4–5). For example, there is nothing inherent in two relata, say Abelard and Héloïse, which contributes to the relation of love between the pair; rather, its root is found in the mental states of one or the other of the lovers, so that the relation itself is not an intrinsic property of one of the pair. Likewise, Russell reaches this position in mathematics by his recognition that spatial points are individuated by the relation between them, not by some intrinsic relational property of one of the points. Many of the other results which Russell obtained in technical philosophy, such as the theory of descriptions, arose, directly or indirectly, out of efforts to deal with logical problems associated with the Russell paradox. The distinction between a proposition and a propositional function, for example, allows and underwrites the difference between a (proper) name and a definite description. The theory of types, although essentially devised as a technical apparatus to escape—via establishment of an hierarchy of levels that distinguish between individuals and functions, functions of functions, etc.—has its metaphysical correlate that enables the distinction between individuals, properties of individuals, properties of properties, etc.

Irvine's essay is concerned with Russell's work in logic only to the extent that it illuminates and provides motivation for the issues in philosophy of logic and philosophy of language that emerge from Russell's attempts to construct a logical system or that gave rise to Russell's work on the logical system first attempted in *The Principles of Mathematics* and then in the *Principia*. Following the philosophical account of the theory of types, Irvine explains Russell's logicism (pp. 12–15), his concern, indeed desire, for mathematical certainty (pp. 15–21), and the role of his logic in analytic philosophy (pp. 21–4). These three sections alone occupy fully half of the text of Irvine's essay.

The remainder of the core text of the essay does nothing to inform the reader

of the development of Russell's presentation from the *Principles* to the *Principia*. Even more disconcerting, perhaps than the lack of an account of the history of Russell's work in logic, is that there is no exposition whatever of the details of the theory set forth either in the *Principles* or in the *Principia*. There is no account, either, of the various technical devices which Russell set forth in his proposed devices for dealing with the paradoxes, either the simple or ramified theory of types. Nor does one find an account of Russell's earlier work, for example on series or the logic of relations. Rather, one must turn to Potter's essay on Wittgenstein to have more than a cursory account of the philosophical significance and motivation for the theory of types. And, even there, what one finds are Wittgenstein and Ramsey's evaluations of the theory. It is perhaps assumed that readers of the *Handbook* are so thoroughly familiar with Russell's work, with the structure and contents of the *Principles* and of *Principia Mathematica*, that even a brief description of these works is superfluous. This might be acceptable. But in contrast with the essays covering the work of Herbrand, Hilbert, Gödel, and others, it is a deficiency not easily overlooked. There is no effort even to evaluate or analyze Russell's principal technical works in logic, still less to do the same for his lesser-known papers in technical logic and set-theory.

Dale Jacquette, in "Logic for Meinongian Object Theory Semantics", is concerned primarily to develop a logical semantic for Meinong's *Gegenstandstheorie*, as a logic of intention which is based upon the distinction between the content of thought and the corresponding object, which may or may not have the ontological status of existence. In a lengthy footnote (p. 31 n.3), Jacquette expounds on Russell's analysis and criticism of Meinong's *Gegenstandstheorie*. Despite N. Griffin's attestations to the contrary,² Jacquette attributes to Russell the mischaracterization of Meinong's position, referring not only to Russell's reviews of Meinong's work, but to "On Denoting" and the theory of descriptions as Russell's logically-grounded theory of reference as the means for dispelling the alleged absurdities of Meinong's theory. Jacquette readily admits that Meinong was not a logician. And since Meinong himself did not develop a formal logical system, Jacquette uses this essay to construct one on his behalf. It is an intentional logic with what Jacquette describes (p. 34) as non-standard propositional and predicate machinery, providing a theory of inference and operating with a semantic that accounts for existent and fictive entities.³ Jacquette readily admits (p. 34) that any such attempt to construct, on Meinong's behalf, a formal logic for his *Gegenstandstheorie* must necessarily be speculative, given that Meinong himself made no attempt to devise a formal logic for his metaphysics.

² Nicholas Griffin, "Russell's 'Horrible Travesty' of Meinong", *Russell* o.s. nos. 25–8 (1977), 39–51.

³ For an earlier and fuller account, see Dale Jacquette, *Meinongian Logic: the Semantics of Existence and Inexistence* (Berlin and New York: Walter De Gruyter, 1996).

Potter's treatment of the logic of the *Tractatus* focuses upon the articulation of the metaphysics behind Wittgenstein's philosophy of logic as mediated by his efforts to balance Frege and Russell's divergent philosophies of language. Wittgenstein's aim is understood to be the search for a means of dealing with singular terms as referring to objects and propositions that are either true or false as expressions regarding those objects, and to do so without falling into the philosophical difficulties that arise either in Frege's or in Russell's theory of sense and reference. Wittgenstein's treatment of singular terms, then, endeavours to find an harmonious way to combine Frege's distinction between sense and reference on the one hand, and Russell's theory of reference, for which "Morning Star" and "Evening Star" are not two logically proper names, but disguised definite descriptions for Venus, on the other. Citing Russell's discussion in "On Denoting" of Gray's *Elegy* (pp. 256–7), Potter argues that Russell was aiming not so much against Frege's *Sinn* and *Bedeutung* distinction, but his own position in the *Principles* that there are some singular terms that have both a reference and a denotation (or Fregean *Bedeutung* and *Sinn*). As read by Potter (p. 257), Wittgenstein sided with Russell on the ground that Frege's distinction amounted to a dualism that has propositions concerned with a *tertium quid* rather than expressions of facts about the world.

Wittgenstein does not, Potter tells us (p. 257), clearly define the two most basic terms that he employs in the *Tractatus* to elaborate the basis of the theory of reference that he seeks to formulate when he introduces them. Rather, the entire *Tractatus* is an elaboration of those terms. *The sense* of a proposition is what the proposition expresses about the facts or possible facts about the world. *Substance* is what the facts of the world are that the proposition seeks to express. It is by virtue of this substance that propositions have their meaning; otherwise, the truth of one proposition would depend entirely upon the truth of other propositions. We might add that, from the standpoint of philosophy of logic, this coincides with, and provides an extralogical basis for, the distinction between the *truth* and the *validity* (or more strictly, *provability*) of propositions, a distinction which was extensively mined by Gödel as he wove back and forth between the one and the other in his proof of his first incompleteness theorem. Much of the remainder of Potter's essay is an elucidation and analysis of Wittgenstein's definitions of sense and substance and an explanation of the means of breaking out of an infinite regress where the sense of a proposition is the tool for determining the truth-value of a proposition. The rescue measure depends upon Russell's separation of internal and external negation and rejection of the Fregean conception according to which propositions divide the universe of discourse into two mutually exclusive classes, "The True" and "The False". Most of the remainder of Potter's essay, although of vital interest to historians of twentieth-century philosophy, philosophers of logic, and Wittgenstein scholars, holds little for those who might wonder what contributions Wittgenstein made,

if any, to technical developments in logic. There is virtually nothing about truth tables in the *Tractatus*, and very little discussion, except in passing, about what Wittgenstein (and Ramsey) contributed to the theory of types, except as background to his discussion of Wittgenstein's theory of the relation of language to metaphysics.

Of all the essays in this volume of the *Handbook* that touch on Russell that I have so far considered, only Cantini's deals with the technicalities of logic and set theory that were undertaken either to counteract, eliminate, or even utilize the paradoxes of self-reference. Cantini is concerned to elaborate the impact of the paradoxes on technical developments in logic and set theory, including the development of axiomatic set theories in place of the naïve set theory of Cantor that led to the Cantor, Burali-Forti, and Russell paradoxes, and the development of non-classical logics to provide alternatives that are putatively paradox-free or which, through excision of the Law of Excluded Middle, offer a plausible account of contradictory conclusions or even, like non-well-founded set theories, celebrate self-referential collections. Recognizing the distinction between the logical or set-theoretical paradoxes on the one hand and the semantic paradoxes on the other, Cantini focuses primarily, but not exclusively, upon the semantic paradoxes. His primary thesis, however, is that there is a major shift after Russell brought the concerns of the paradoxes to the fore for logicians working in foundations of mathematics. Prior to 1903, the interest in the paradoxes was to find linguistic means of treating them and, as it were, detoxifying them. After 1903, the concern devolved into the elaboration of theorems elucidating the negative properties of formal systems. The historical concern, in addition to considering the impact which the discovery of the paradoxes had on those who considered them into the early years of the twentieth century and their technical mathematical efforts to deal with them, is with the technical details of the shift that led to the development of proof theory and to the study of the properties of formal systems and the theorems expressing those properties. Beyond this, Cantini deals directly with Russell's historical contributions only to the extent of noting his influence in publicizing the paradoxes, and technically by dealing with his elaboration of the theory of types, as well as the foundational-philosophical problems to which that theory gave rise.

For those articles in the *Handbook* which relate to Russell—Irvine's, Jacqueline's, Potter's, and, albeit to a much lesser extent, Cantini's, it is a serious misnomer to label them as devoted to the history of logic. They belong to philosophy of logic, or, more strictly, to the history of philosophy of logic. Even when Potter raises the question of Wittgenstein's efforts to understand arithmetic, the question is raised only for the sake of elaborating how Wittgenstein could arrive at a definition of number that would be applicable for the purpose not merely of enumeration, but for counting various concrete objects rather than one specific set of concrete objects. When those essays which I have considered

are compared with those which examine the technical logical contributions of Herbrand or Hilbert, Gödel or Post or Tarski, for example, one has to wonder what justification the editors might set forth to account for their inclusion.

Turning to “The Development of Mathematical Logic from Russell to Tarski, 1900–1935” by Paolo Mancosu, Richard Zach, and Calixto Badesa in *The Development of Modern Logic*, edited by Leila Haaparanta (pp. 318–470), we discover that “Itinerary II” (pp. 330–41) is devoted to Russell’s work. In this essay, the authors provide a concise sketch of Russell’s work in logic, tracing the main lines of his work, from his early familiarity with the logic of relations as found in the work of Boole, Peirce, Schröder, and Whitehead, through his acquaintance with the axiomatic development of logic of Peano and his school, to his own work in the *Principles* and the *Principia*, and including a discussion of the philosophical issues to which the logical and set-theoretic paradoxes and associated questions in philosophy of logic and philosophy of language gave rise, as found in such of Russell’s works as “On Denoting” on definite descriptions and in his development of the theory of types; but neither do they neglect the motivational background of the paradoxes in philosophy, discussing the particulars of the Vicious Circle Principle and the paradoxes of self-reference. The authors are interested principally in the technical aspects of Russell’s work as part of the history of early twentieth-century logic and foundations and the genesis of the special sub-fields of mathematical logic such as model theory, recursion theory, and proof theory. The first section (pp. 330–3), covering the early work through the writing of the *Principles*, explains that, in translating the Boole–Peirce–Schröder logic of relations into Peanesque notation, Russell was able to introduce Cantorian set theory into the logic presented in the *Principles*. In the next section (pp. 333–5), Russell’s various proposals to deal with problems of self-predication are mentioned, and then a discussion is undertaken (pp. 335–6) of “On Denoting” to briefly explore the role of singular terms and their connection with propositional functions. This is followed (pp. 336–9) by a return to means for dealing with difficulties arising from impredicative definitions, namely with an account of the ramified theory of types and the concern for properly understanding propositional functions. Finally, a sketch is given (pp. 339–40) of the logic of the *Principia*, with particular attention to the foundational and philosophical aspects of that work. It is noted in particular (p. 339) that the project “consisted in showing that all of mathematics could be developed through appropriate definitions in the system of logic defined in *Principia*.” With that in mind, the bulk of the discussion is concerned with the role of the assertion sign in establishing the truth of propositions. At the same time, it is also noted (p. 340) that the theory of types makes it cumbersome to formalize the quantificational parts of the calculus of the *Principia*.

The final section of Mancosu, Zach, and Badesa’s essay (pp. 340–1) mentions the historical role of the *Principia* as the exemplar of the foundational philo-

sophy of logicism, but focuses attention on developments arising for mathematical logic in the wake of the publication of the first edition of the *Principia*, specifically the reduction of the Boolean connectives to a single connective (the Sheffer stroke), the analysis by Bernays of the propositional part of the *Principia*, C. I. Lewis's challenge to material implication by introducing strict implication, and last, but far from least, the shift, starting in the 1930s, away from second- and higher-order predicate calculus in favour of first-order predicate calculus as the standard paradigm of mathematical logic.

This is the essay that approaches what we should have expected to have found in the *Handbook* in lieu of Irvine's contribution.

The best way to approach a broad survey of Russell's influence in, and contributions, to the history of logic and philosophy of logic for those not already familiar with Russell's work, would be to begin by reading both the Irvine and Mancosu, Zach, and Badesa's essays in tandem. This would provide a broad perspective of the general tenor of Russell's work and influence but, regrettably, hardly a deep and detailed account of his work, and still less a solid technical presentation of his most important work, from "Sur la logique des relations avec des applications à la théorie des séries" (1901) and associated articles, through *The Principles of Mathematics* and "Mathematical Logic as Based on the Theory of Types" (1908) to *Principia Mathematica* and, finally, his introduction to the second edition of the *Principia*. What one would have hoped for is that an in-depth survey such as that, rather than Irvine's largely philosophical sketch, would have been included in—and would have been far more been appropriate for—the *Handbook of the History of Logic*. This is not to say that Irvine's essay is without value; rather, it would be better placed elsewhere, in a survey of the history of philosophy of logic, and consequently it is, ultimately, a disappointment in the present *Handbook*.
