PRINCIPIA MATHEMATICA AT 100

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All the papers in this special issue of *Russell* are connected with the *PM@100* conference held at the Bertrand Russell Research Centre at McMaster University to celebrate the centenary of the publication of the first volume of Whitehead and Russell's *Principia Mathematica*. The conference took place 21–24 May 2010, and over 60 people from ten different countries attended to hear more than 30 papers covering the origins, impact, philosophical motivation and mathematical content of that vast and still relatively unexplored work. All but two of the nine papers in this collection were presented at the *PM@100* conference. Of the two exceptions, Anellis's paper was included in the conference programme, but in the event Anellis was unable to attend and so the paper was never presented. Blackwell's paper, by contrast, was presented as an after-dinner talk, with logicians in attendance, to the 2010 annual meeting of the Bertrand Russell Society which was held at McMaster over the same weekend at the *PM@100* conference.

The nine papers published here exemplify the range of the conference itself. Graham Stevens' paper considers the view, much discussed by contemporary philosophers of language (*cf.* notably Neale 1990, 1993, 2008), that all noun phrases are either semantically structured quantifiers or semantically unstructured singular terms. This view seems like a natural extension of the sharp distinction Russell made in his theory of descriptions between definite descriptions, which are handled quantificationally,

and logically proper names, which are genuine singular terms. And yet there is a difficulty, which Stevens addresses in his paper. One reason Neale reformulates Russell's theory of descriptions in terms of binary or restricted quantifiers is to narrow the gap Russell famously opened between the logical form of a sentence, as revealed after its analysis by the theory of descriptions, and the grammatical form that the unanalyzed sentence has in natural language. The outcome of this seems to be that Russell's claim that definite descriptions are incomplete symbols is lost in the reformulation. Stevens argues that this is not the case, but only if "incomplete symbol" is properly understood, not as a symbol having no meaning, but as one which does not contribute an object to a proposition. Ray Perkins deals with another aspect of the very same issue, but with a quite different approach. Perkins is concerned with a famous argument in *Principia* designed to show that definite descriptions are incomplete symbols. The argument has been frequently criticized, starting immediately with the book's publication (Jones 1910–11), precisely because of its dependence on an alleged ambiguity in the notion of meaning it deploys. Perkins defends the argument against this charge and, addressing the concerns of the historical Russell rather than his modern followers, goes on to explore the implications of the argument for Russell's epistemology, metaphysics and his conception of analysis.

The theory of descriptions is one of Russell's two best-known and most important contributions to *Principia*; the theory of types is the other. Yet the theory of types was never considered, even by its creator, as the triumphant success that the theory of descriptions was. It brought with it too many problems. One of them was the axiom of reducibility, discussed in Russell Wahl's paper. The axiom was essential if the logicist project was to go forward; without it, for example, least upper bounds in real number theory would fall outside the sets they bounded. And yet the axiom's status as an axiom of logic is very much in doubt. It requires the existence of very many predicative propositional functions. If these are understood constructively, the axiom is surely false; yet if they are understood in a realist way as genuine entities, then the axiom looks more like a metaphysical thesis than an axiom of logic. Wahl surveys the available options and, building on work of Linsky (1999) and Mares (2007), fashions a realist interpretation on which the axiom looks more like a logical axiom, akin to the axioms of quantification theory, than a metaphysical principle. Another problem with type theory is Whitehead and Russell's use of typical ambiguity. In order to state the system with the necessary

generality, Russell and Whitehead were obliged to use symbols as if they were ambiguous between many different types. At the very least this looks like an embarrassment for the theory; at worst it looks as if the type-theoretic rules that block the paradoxes may be lifted whenever it is convenient to do so. Brice Halimi's paper, however, suggests that there is no need to give up hope. He presents a very subtle way of formalizing typical ambiguity using the λ-calculus, and even provides a category-theoretic semantics. His paper illustrates the way in which current research in logic can be used to throw light on, and even rehabilitate, parts of the *Principia* system which have long made some of us shudder.

Conor Mayo-Wilson directly addresses the nature of the logicism of *Principia*. Russell always held that the derivation of mathematics from logic, as was begun in *Principia*, would in fact help to establish his life-long goal of demonstrating the "certainty" of mathematics. Yet, as has been pointed out Godwyn and Irvine (2003) and others, this was not because the mathematics thus demonstrated would inherit certainty from the logical axioms of *Principia*. It would be absurd to suppose that with proposition ∗110.643, established 115 pages into Volume II of *Principia*, $1 + 1 = 2$ achieved a degree of indubitability that had hitherto eluded it. In fact, as Russell repeatedly described, the axioms of the system often gained their credibility because they permitted the derivation of mathematical truths that were already certain. Mayo-Wilson acknowledges the usually overlooked coherentist aspects of Russell's epistemology, and argues that the logicist project achieved its epistemological goals using methods familiar from inductive, Bayesian reasoning, by which an axiomatization of a mathematical theory provides explanations of the theorems and achieves certainty though its coherence and simplicity. While a paradigm of deductive mathematical reasoning, the epistemological project of *Principia* is seen to rely on principles of inductive, scientific reasoning.

With Ryan Christensen's paper we move to consider developments after the publication of the first edition of *Principia*. Christensen is concerned with Frank Ramsey's definition of truth, devised at the time the second edition of *Principia* was being published. Strictly speaking the definiens is ill-formed, and this had led to various attempts to construe the propositional quantifiers in ways that would render it admissible. Christensen argues that none of these attempts is entirely satisfactory and proposes instead a new operator taking terms to propositions. Such a device would have been useful to Russell in *The Principles of Mathemat-*

ics, where he struggled unavailingly with similar problems, and again in the second edition of *Principia*, when propositions were once again admitted as genuine entities.

Roman Murawski's paper deals with the work of Leon Chwistek, another logician who contributed to the development of the *Principia* system soon after the publication of the first edition. Chwistek is nowadays best known (indeed, often only known) for his attempt to run the *Principia* project without the axiom of reducibility. Murawski, however, surveys the other work in the philosophy of mathematics of this exceptionally idiosyncratic and under-appreciated thinker. Irving Anellis considers to what extent *Principia* created a "Fregean revolution" in logic. He considers Frege's position among the precursors of *Principia*, and he reminds us, also, that long after the first edition was published, the Aristotelian syllogism still had its defenders.

The volume ends on a lighter note. The derivation of mathematics from basic logical principles is not, perhaps, a natural occasion for mirth, and by Russell's account the process of producing the book was grim indeed. Nonetheless Russell was able to insert a few jokes even into *Principia Mathematica*. In the final paper in the collection Kenneth Blackwell explores some of its remote corners searching for the humour in *Principia*. Russell saved his wit for important issues, and it rewards the effort of examining the importance of the issues about which he joked. Thus even the remark about Cook and Peary "claiming" to have found the North Pole is helpful in timing the composition of the Introduction to *Principia*.

<div align="center">REFERENCES</div>

Godwyn, Martin, and Andrew D. Irvine. 2003. "Bertrand Russell's Logicism". In Nicholas Griffin, ed. *The Cambridge Companion to Bertrand Russell*. Cambridge: Cambridge U. P.

Jones, E. E. C. 1910–11. "A New Law of Thought". *Proceedings of the Aristotelian Society* 11 (1910–11): 166–86.

Linsky, Bernard. 1999. *Russell's Metaphysical Logic*. Stanford: CSLI.

Mares, Edwin. 2007. "The Fact Semantics for Ramified Type Theory and the Axiom of Reducibility". *Notre Dame Journal of Formal Logic* 48: 237–51.

Neale, Stephen. 1990. *Descriptions*. Cambridge, Mass.: MIT Press.

—. 1993. "Term Limits". *Philosophical Perspectives* 7: 89–123.

—. 2008. "Term Limits Revisited". *Philosophical Perspectives* 22: 375–442.