THE WIT AND HUMOUR OF
PRINCIPIA MATHEMATICA

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Except for its belated proof of 1 + 1 = 2, *Principia Mathematica* doesn't feature in studies of mathematical humour. Yet there is restrained and understated humour in that work, despite the inauspicious conditions under which it was written. Russell, to take one of the authors, had an irrepressible talent for enlivening his subject matter. This paper explores even the "obscure corners" of *PM* to uncover its humour and wit, which, for non-logicians, can be an entree to the work.

T omorrow evening[1] you'll be watching portions of an unfinished film biography of the 93-year-old Bertrand Russell. We're lucky to have a transcript of much more of it, and in the transcript Ralph Schoenman prompts Russell: "But you did allow yourself a few jokes in the *Principia*, Bertie?" He responded: "Very few. Very few. I know that in Chapter 10 I proved that one and one are two. I remarked this is occasionally useful."[2] This is exactly the phrase Whitehead and he used at ∗110.643, well into Volume II. Is this it for humour, or at least wit, in *Principia Mathematica*? Russell himself rated the book as dull. He offered this advice to budding authors hoping to write a best-seller like the Koran or *Capital* :

"Do not attempt to be amusing; avoid lucidity at all costs; be profound, voluminous, mysterious and dull. Then you may hope to become one of the

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[1] Delivered at the annual meeting of the Bertrand Russell Society, 21 May 2010.

[2] "Recorded Interview with Bertrand Russell", 22 Nov. 1965, filmed by Emile de Antonio, transcript (with many errors), fol. 28; RA rec. acq. 680. Perhaps Russell said "Chapter 110". He sometimes referred to starred entries as chapters.

world's best sellers." Not that this recipe is a sure way to success. My friend Whitehead and I produced a book having all four requisites that I have just enumerated, but, alas, it never became a best seller. There was one thing that we had forgotten, a thing which both Hitler and Stalin remembered—I mean moral uplift.[3]

He otherwise characterized *Principia* as "appear[ing] to lack that philosophic profundity of which obscurity is the most easily recognizable feature."[4] Nevertheless, I think we would agree that *Principia* has its share of obscurity. Russell even admitted this in some pre-publication humour: "It is amusing", he wrote after packing up the 4,000 pages of the manuscript ready by October 1909, "to think how much time and trouble has been spent on small points in obscure corners of the book, which possibly no human being will ever discover."[5] He had no fear that people who read "bits" wouldn't praise the book, "because otherwise they would have been wasting their time."[6]

Like C. D. Broad, who studied logic under Russell, we may be grateful for "the stimulation of his wit and humour", especially if it opens up *Principia* for us.[7] Russell was very aware of his ability to inject humour into his writings and lectures. G. H. Hardy spread the word at Cambridge that Russell "was full of jokes".[8] Students attended his lectures "for the jokes and for the thrill of an occasional paradox".[9] Still, I am not going to maintain that *Principia Mathematica* teems with hidden humour. Russell had a grim time writing out the book in Whitehead's absence teaching (and so Russell may be more responsible for the humour than Whitehead, whose *Principia* correspondence is nevertheless very witty). Russell later confessed: "the difficulty and the labour were too great for any pleasure to be possible."[10] "[M]y intellect never quite recovered from the strain."[11] To a young person some 25 years later he reportedly remarked that, due to the level of concentration involved, he "had actually damaged his brain" doing *Principia*.[12] Russell so dedicated

[3] Russell, "The Use of Books" (1951), in *B&R* 1: xlviii–xlix.   [4] *MPD*, p. 75.
[5] Russell to Lucy Martin Donnelly, 18 Oct. 1909 (RA1 710); *SLBR* 1: #152; quoted in *Papers* 6: xiv.
[6] Russell to Donnelly, 18 March 1908 (RA1 710); quoted in *Papers* 6: xiv.
[7] "Some Personal Impressions of Russell as a Philosopher", in Schoenman, ed., *Bertrand Russell: Philosopher of the Century* (London: Allen and Unwin, 1967), p. 108.
[8] *Papers* 6: xxiv.   [9] *Ibid.* 6: xxi.   [10] *Auto.* 1: 156.   [11] *Ibid.* 1: 153.
[12] The young person was budding film director Richard Leacock, who quotes the

himself to the task that when it came to the supreme difficulty, the Paradox, he was prepared, he says, to spend "the whole of the rest of my life" in solving it.[13] During the years of writing *Principia* he would "stand on the footbridge at Kennington, near Oxford" and feel close to putting himself under the trains.[14] It was during this period that he "gave it [chastity] a good try once, but never again!"[15] No wonder, then, that when he had packed up the manuscript, he felt "more or less as people feel at the death of an ill-tempered invalid whom they have nursed and hated for years."[16] He was not living a life conducive to humour.

Yet Russell had already made a good start in rendering mathematics mirthful, or at least witty. In *The Foundations of Geometry*, in discussing the different spaces of the Flatlanders and the Spherelanders and the possibility of a fourth dimension for us, he commented:

The only people, so far as I know, who have used this analogy, are Dr Abbot and a few Spiritualists—the former in joke, the latter to explain certain phenomena more simply explained, perhaps, by Maskelyne and Cooke.[17]

Maskelyne and Cooke are worth knowing about. The former, author of a best-seller on card sharping, combined with the latter, a carpenter who built a "spirit cabinet", to expose a number of frauds perpetrated by spiritualists.

Volume 1 of *Principia* was published in December 1910. Hardy later reviewed it on the front page of *The Times Literary Supplement*. He compared *Principia*'s humour with that of *The Principles of Mathematics*:

We may perhaps venture to pick out a minor feature of the book for commendation. It is easy to think, but hard to joke, in symbols; and this volume has not the consistent humour of the *Principles of Mathematics*. Still, considering the difficulty of the medium, some of the jokes are very good. The best is that per-

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remark in his forthcoming *The Feeling of Being There: a Filmmaker's Memoir* (Paris: Semeïon Editions, 2011). To J. E. Littlewood Russell said the book "had taken so much out of him that he sometimes doubted whether he would ever be the same man again" (Wood, p. 69).

[13] *MPD*, p. 79; *Auto*. 1: 152.   [14] *Auto*. 1: 152.

[15] Russell to Irina Wragge-Morley (later Stickland), 5 March 1947, RA rec. acq. 921; quoted in Clark, p. 166.

[16] *SLBR* 1: #152; quoted in *Papers* 6: xiv.

[17] *An Essay on the Foundations of Geometry* (Cambridge, UK: Cambridge U. P.), p. 105.

petrated at the expense of the law of contradiction. But it would be unfair to the circulation of the book that a reviewer should repeat them; and we leave the reader to discover them for himself.[18]

The authors long ago[19] were repaid their £50 subsidies for *Principia*'s publication, so I will proceed without fear of harming the book's circulation. As for the consistent humour of the *Principles*, one joke Hardy may have had in mind is the famous obituary of the concept "a man"— not, as Russell says, "an actual man with a tailor and a bank-account or a public-house and a drunken wife".[20] Concepts don't have attributes similar to the things, if any, of which they are concepts. Russell proceeds to the obituary:

… we should be surprised to find in the *Times* such a notice as the following: "Died at his residence of Camelot, Gladstone Road, Upper Tooting, on the 18th of June 19—, Man, eldest son of Death and Sin."   (*PoM*, p. 54)

Hegelians fared poorly in the *Principles*: "as to what they meant by continuity and discreteness, they preserved a discreet and continuous silence",[21] a remark Russell recalled in the film interview. Monists were not exempted from being the butt of humour in *Principia*, but let us recall that Russell praised Chinese humour for its "restraint and understatement".[22] Gregory Landini finds[23] "a logician's joke" in this passage:

We might, of course, have included among our primitive propositions the assumption that more than one individual exists, or some assumption from which this would follow, such as

$$(\exists \phi, x, y) \,.\, \phi \,!\, x \,.\, \sim \phi \,!\, y.$$

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[18] *Times Literary Supplement*, 7 Sept. 1911, p. 322; reprinted in Hardy's *Papers*, 7: 862.

[19] Though not until 1949. See Russell's correspondence with Cambridge University Press (RA rec. acq. 25). He raised the matter of their subsidy of Volumes I–III when CUP was on the verge of reprinting the second edition. CUP replied that the authors had contributed to the expenses of Volume I only. This distinction, which Russell accepted in his next letter, is so nice that it is *ipso facto* amusing.

[20] *PoM*, p. 53. Possibly the second actual man (and woman) was a stock example from the temperance movement, which Russell supported at the time.   [21] *PoM*, p. 287.

[22] *The Problem of China* (London: Allen and Unwin, 1922), p. 188.

[23] Landini, *Russell* (London: Routlege, 2011 [2010]), p. 99.

But very few of the propositions which we might wish to prove depend upon this assumption, and we have therefore excluded it. It should be observed that many philosophers, being monists, deny this assumption.   (*PM* 1: 335)[24]

In *24, the authors had verged upon outright flippancy. The proposition

*24.1 $\vdash . \Lambda \neq V$

is translated as "nothing is not everything" (1: 216). Formally it means that the null class is not the universal class—in other words, there is (at least) one individual. It's amusing to find that in this, *Principia* and monism have a slender basis of agreement.

Russell's mathematical humour didn't cease with *Principia*. Sometimes it was even topical. Now, wartime brought restrictions on Britain's paper supply. This was the case in 1918, and again in 1945, when the publication of Russell's own best-seller, *A History of Western Philosophy*, was delayed. In 1918 he was writing the manuscript of *Introduction to Mathematical Philosophy* in prison. The handwriting is tiny, the better to squeeze more words on a page. He came to a point in discussing the number of classes of classes in a world with just nine individuals. The number is 2^{512}. This number, he says, will have "about 153 digits". (Kevin Klement tells me the exact number is 155.) He moves on to classes of classes of classes. Here the number of digits will be "about three times 10^{152}". Russell comments: "In a time of paper shortage it is undesirable to write out this number...."[25] In this remark he combined mathematical acumen, a topical reference, and a gesture of faux patriotism.

We reach at last the collected wit and humour of *Principia Mathematica*. As we saw, Hardy lowered the reader's expectations. Philip Jourdain, who knew a mathematical joke when he saw one, thought the famous remark about 1 + 1 = 2 might be "the only joke in the book" when he reviewed Volume II.[26] Hardy had only read Volume I by the

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[24] Allowance for monists, or monistic philosophers, is made three other times (*PM* 1: 216, 2: 8, 325). At the last passage the authors say that "Our primitive propositions do not suffice to disprove this supposition" (that there is more than one individual, or [at 2: 8] that "the whole universe" consists of a single individual).
    All page references to *Principia Mathematica* are to the second edition (1925–27).
[25] *IMP*, p. 133.
[26] P. E. B. Jourdain, "*Principia Mathematica*", *The Cambridge Review* 33 (25 April 1912): 381.

time of his review, and what is perhaps the wittiest passage doesn't come until near the end of Volume III. I'll leave the best for the last.

Readers of the Descriptions section of the Introduction to the first edition will notice the odd example proposition, "the author of *Slawken-burgius on Noses* was a poet"[27] (1: 68). The proposition is false, in Russell's terms, because, as Nick Griffin and Alasdair Urquhart pointed out to me long ago, there is no such book outside of its fictional reference in *Tristram Shandy*. If you read Lawrence Sterne, you'll find that the non-existent book has a Latin title, three pages of "original" Latin text, and several more pages of English translation. Perhaps Russell enjoyed the idea that parts of a non-existent book could be read.

There is an element of self-reference in this passage, but it is not what William F. Fry, Jr. in his article "Humor and Paradox", identified as a dominant theme in humour, instancing *Principia*'s attempt to avoid self-referential (or vicious-circle or illegitimate) totalities.[28] So far as I can discover, *Principia* avoids self-referential humour. Not a single joke comes at the expense of illegitimate totalities, despite the lure of such fun. Yet this kind of joke has been dubbed "Russell jokes" in *Mathematics and Humor* by John Allen Paulos (who was a late correspondent of Russell's).[29] Russell jokes are defined as those "whose logical underpinning is some version of Russell's paradox or its resolution". In fact—perhaps in deference to the seriousness of the theory of types—Russell very seldom indulges in illegitimate totalities. But the lure was irresistible when it came to reviewing Bergson's little book, *Laughter*, before Volume II of *Principia* appeared. Russell declined to extend the theory of types to Bergson's laughter formula about humans behaving mechani-

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[27] Russell was fond of nasal examples. In "Mathematical Logic as Based on the Theory of Types" (written in 1907), he maintained that you couldn't avoid mentioning a topic by mentioning that you won't mention it: "One might as well, in talking to a man with a long nose, say 'When I speak of noses, I except such as are inordinately long' …" (*LK*, p. 63; *Papers* 5, forthcoming). Thanks to Landini for bringing this example to my attention from Jourdain's *The Philosophy of Mr. B*rtr*nd R*ss*ll* (London: Allen and Unwin, 1918), p. 77.

[28] *American Behavioral Scientist* 30 (1987): 42–71 (at 47–8, 55). Fry maintains that "certain self-referent paradoxes are central to the nature of humor", and that it's a good thing that Gödel showed "that these paradoxes are inherently unresolvable" (p. 56).

[29] Chicago and London: U. of Chicago P., 1980, p. 44. Don't miss Paulos' dialogue, "Groucho Meets Russell", in his *I Think, Therefore I Laugh* (New York: Columbia U. P., 1985), pp. 10–12, 154–5.

cally, when he concluded his review as follows: "Every formula treats what is living as if it were mechanical, and is therefore by his own rule itself a fitting object of laughter."[30] The application of the theory of types would make such logical humour impossible.

Nevertheless, Jourdain had his alter ego for Russell indulge in a superb "Russell joke", namely the multi-level jest involving several Scotsmen in the short chapter, "The Hierarchy of Jokes".[31] There is surely sufficient reason to credit Russell with embellishing this joke about the 37 proto-Aryan jokes and the means of avoiding thereby a vicious-circle fallacy. Russell had privately in 1911 caricatured some Scots for an alleged dull sense of humour when he encountered them in his classes.[32] There's more substantial evidence. The first-level basis of the joke came from Gilbert Murray. "He assured me once", Russell recollected in 1958, "that there was an Oxford don who had reduced all jokes to 37 proto-Aryan originals and, when anybody made a joke in his presence, he would say, 'Yes. There is that joke.'"[33] The story is a joke at the next level. Jourdain's wittily phrased rendition has four levels, with the first three involving the unfortunate Scotsmen and the fourth being about the first three, all suitably nested. The story ends with an analysis of the topics concerned in the lower levels and of who enjoys such jokes. Jokes of

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[30] *Papers* 12: 386; Jourdain, *The Philosophy of Mr. B*rtr*nd R*ss*ll*, p. 87.

[31] *The Philosophy of Mr. B*rtr*nd R*ss*ll*, Chap. 39. Irving Copi liked the chapter so much that he appended it to *The Theory of Logical Types* (London: Routledge & Kegan Paul, 1971). (He altered "Scotchmen" to "Scotsmen".) Russell had recommended the book to his publisher as "very amusing, and at the same time by no means uninstructive" (Russell to Allen and Unwin, 10 June 1918, RA rec. acq. 70). He later described it as "a curious work containing many direct statements of mine" (Lester E. Denonn, "Recollections of Three Hours with Bertrand Russell", *Correct English* 44 [Dec. 1943]: 14–19 [at 14]). I. Grattan-Guinness points out other jokes taken from the *Principia* period in *Dear Russell—Dear Jourdain* (London: Duckworth, 1977), pp. 140–1, and in his introduction to Jourdain, *Selected Essays on the History of Set Theory and Logics (1906–1918)* (Bologna: Editrice CLUEB, 1991), pp. xxxvii ff. Almost as amusing as Jourdain's book, and replete with insider information, is Broad's review in *Mind* 28 (1919): 485–6. Broad considered the proof that "jokes form a hierarchy in the sense of the Theory of Types" an important novelty of Mr. R*ss*ll's in logic!

[32] Russell to Lady Ottoline Morrell, no. 214, 11 Oct. 1911 (Morrell Papers, Texas; RA rec. acq. 69); quoted in *Papers* 6: xxiv.

[33] "A Fifty-Six Year Friendship", in Jean Smith and Arnold Toynbee, eds., *Gilbert Murray: an Unfinished Autobiography* (London: Allen and Unwin, 1960), p. 207. In the same work Russell identifies the maid-servant of Jourdain's Chapter 11 ("Objective Validity of the 'Laws of Thought'") as the Murrays' maid-servant.

higher levels require "a sound logical training", "while jokes of transfinite order presumably only excite the inaudible laughter of the gods."[34] We may conclude that Russell was not above Russell jokes, at least during the euphoria of publishing *Principia.*

The joke on the law of contradiction,

$$*3.24 \qquad \vdash . \sim (p . \sim p)$$

is in the summary on page III of Volume I: "in spite of its fame", the authors comment, "we have found few occasions for its use." Maybe the *PM*@100 conference will tell us why that is. Incidentally, Russell maintained an immense index of where propositions were used in *Principia.* It is easy to confirm how seldom the law of contradiction's number was employed—only at *22.89 and in the demonstration of *60.33.[35] Jourdain, in his joke-book on Russell, also noticed the passage.[36]

Wit and humour give a book life, but not all that gives a book life is witty or humorous. In a work of the most rigorous symbolic reasoning, this reason is offered once: "because life is too short." That is why a person cannot assert all the non-intensional functions of $\phi!\hat{z}$ (1: 73). *Principia* is populated by polar explorers, the featherless biped of the *Principles* ("x has two legs and no feathers" [1: 23, 73]), Adam and Eve (1: 546, 579), Christians and Mohammedans and their respective sets of wives (1: 299), Socrates, Plato and Aristotle (*passim*[37]), an important "imaginary sceptic" (1: 38), white employers in relation[38] to "coloured" employees (1: 265, a political remark dating from the Wimbledon by-election?), rich fathers of Etonian sons (1: 281), and the sons of Cabinet ministers (α) and sons of foolish male parents (β). The emptiness of their intersection is symbolized in the summary of *72.411: $\alpha \cap \beta = \Lambda$. Mysteriously (the relation of son to father being many-one),

If we make R the relation of son to parent (which is not many-one), it no longer

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[34] *The Philosophy of Mr. B*rtr*nd R*ss*ll*, Chap. 39.

[35] "Props Where Used", RA1 230.031270 (260 leaves). The index stops mid-way through Volume III, at *276.43.

[36] *The Philosophy of Mr. B*rtr*nd R*ss*ll*, p. 21.

[37] *Principia*'s first edition can be searched digitally in the University of Michigan Historical Mathematics Collection (http://quod.lib.umich.edu/u/umhistmath/).

[38] "[A] relation limited both as to its domain and as to its converse domain": $\alpha \uparrow R \upharpoonright \beta$.

follows that the sons of Cabinet Ministers and the sons of fools have no common member.   (*PM* 1: 441)

Since there could not have been a woman in the relevant Liberal Cabinets of 1906 and 1910, the example's presence and its conclusion suggest (but do not imply) that there was a son of a Cabinet minister and a foolish mother. Political research, inside knowledge of the English establishment, or both, might confirm a possibly libellous joke here. This must count as one of the book's "obscure corners".

Dr. Cook and Commander Peary are a topical reference in a work that is not supposed to involve more than one existing thing. They are the arguments that satisfy the function "$\hat{x}$ is a white man who claims to have reached the North Pole" (1: 73).[39] Apt fauna such as a horse's head, an oyster and a hydra (1: 291), half a pound of cheese (3: 407), and "the manor of East Greenwich" (1: App. C, 664) further enliven the book.

Tradition has it that in lecturing on *Principia* Russell "invented his own pet names for many symbols".[40] It's a pity he wasn't asked to tape-record a page. The only pet names that have come down to us are "E Shriek", as in "E!" and "∃!", and "R. Hook" ("right hook"), for "⊃".[41]

Volume II has but a single humorous passage, though it is doubly funny. It follows the demonstration of

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[39] When Russell wrote that sentence, in his 1909 reply to Poincaré ("The Theory of Types", *Papers* 6: 27) that explicated the theory of types, he didn't say "claims to have reached" but "has reached". Between drafting the reply in September 1909 and passing the proofs of Volume 1 in spring 1910, controversy arose as to whether Dr. Cook reached the North Pole, and *Principia*'s text was updated for this topical reference.

[40] Wood, p. 49.

[41] As H. M. Sheffer wrote the names in a letter of 2 March 1911 to Russell, after attending his classes at Cambridge (RA1 710). Russell's teaching assistant at Harvard, Harry T. Costello, in reviewing the second edition of *Principia*, had no new pet names but wrote fondly that "We came rather to like the language, with its *p*'s and *q*'s, and its *phi-x-caps* and *R-backward-arrows*, and *existences-with-a-shriek*, and *existences-upside-down*, and so on, even up to the mysterious generalization called the *female-relation*, and the hypothetical majesty of *Aleph-sub-zero*. We tried catching one another on such puzzlers as ... *litmax* and *multax* ...'" (*Journal of Philosophy* 25 [1928]: 438–45 [at 438–9]). Later Costello recalled Russell's British pronunciation of ϵ: "ep-sigh-lon" ("Logic Then and Now", *Journal of Philosophy* 54 [1957]: 245–64 [at 255]).

A passage in Broad's "A General Notation for the Logic of Relations" (*Mind* 27 [July 1918]: 284–303 [at 299]) suggests that Whitehead was the originator of "shriek". Russell praised the article from his prison cell but said of an innovation that he didn't much like shrieks upside down (message to Broad, 17 July 1918, RA rec. acq. 17c).

$$*110.643 \qquad \vdash . \; 1 +_c 1 = 2.$$

The authors' complete comment is: "The above proposition is occasionally useful. It is used at least three times", and they give all three numbers from Russell's "Props Where Used" index,[42] where he annotated the entry with "$1 +_c 1 = 2$", including the cardinality subscript. There's meta-humour to the passage. It took 799 pages of the first edition to reach this proof. Many have pointed out the passage, although it seems that the ethicist A. I. Melden did not read this far. He wrote that it is as absurd to derive an *ought* from an *is* "as it would be to try to squeeze out humor from any page of *Principia Mathematica*."[43] It is popularly believed that $1 + 1 = 2$ is proved at *54.43,[44] but that passage isn't *the* proof because the authors comment, "From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$" (1: 362).

The last bit of wit in *Principia* is so good that no wonder Whitehead and Russell saved it for almost the last page. Wittgenstein had asked how the authors planned to end *Principia*. Russell reported his response to Lady Ottoline, that "we should have no concluding remarks, but just stop with whatever formula happened to come last."[45] The authors end Section D, Cyclic Families, of Part VI, Quantity, with this sentence:

We have given proofs rather shortly in this Section, particularly in the case of purely arithmetical lemmas, of which the proofs are perfectly straightforward, but tedious if written out at length.[46]   (*PM* 3: 461)

Thus the work comes to an end, not with moral uplift or even an explicit conclusion about logicism, but with an expression of relief.[47]

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[42] The index excludes the proposition's *mention* in the summary at 2: 73.

[43] A. I. Melden, "Reasons for Action and Matters of Fact", *Proceedings and Addresses of the American Philosophical Association* 35 (1961–62): 45–60 (at 45–6).

[44] E.g. *Logicomix*, by Apostolos Doxiadis and Christos H. Papadimitriou (New York: Bloomsbury, 2009), pp. 184–5, and *The Number Devil: a Mathematical Adventure*, by Hans Magnus Enzenberger (New York: Henry Holt, 2000), p. 227.

[45] Russell to Ottoline Morrell, no. 427, *c*. 30 April 1912.

[46] "It is surprising to read such a statement in the Bible of logicism", comments Grattan-Guinness in *The Search for Mathematical Roots, 1870–1940* (Princeton and Oxford: Princeton U. P., 2000), p. 410. He explains it thus: "doubtless it was motivated by the role of the theory [in question] as a tool for use in the Volume 4 to come."

[47] Acknowledgements: Dennis Darland, Ivor Grattan-Guinness, Nicholas Griffin, Kevin Klement, Gregory Landini, Bernard Linsky and Alasdair Urquhart.