LOGICAL FORM IN *PRINCIPIA MATHEMATICA* AND ENGLISH

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The theory of descriptions, presented informally in “On Denoting” and more formally in *Principia Mathematica*, has been endorsed by many linguists and philosophers of language as a contribution to natural-language semantics. However, the syntax of *Principia*’s formal language is far from ideal as a tool for the analysis of natural language. Stephen Neale has proposed a reconstruction of the theory of descriptions in a language of restricted quantification that gives a better approximation of the syntax of English (and, arguably, of other natural languages). This has led to resistance from some Russell scholars who object to the identification of descriptions with quantifiers at the level of logical form in this new language on the grounds that the identification fails to respect the Russellian conception of descriptions as incomplete symbols. I defend Neale’s reconstruction of the theory and argue that he has preserved everything essential to the theory, including the notion of an incomplete symbol. However, I then go on to argue, contrary to Neale and his objectors as well as Russell himself, that the doctrine of incomplete symbols is a superfluous and undesirable element of the theory that is best jettisoned from the theory.

INTRODUCTION

Stephen Neale (1990) provides a full-scale reconstruction, elaboration, and extension of Russell’s famous theory of descriptions. The reconstructive element of the project involves a restatement of the theory in a language of restricted quantification (rq) that is designed to show the independence of the theory as a philosophical doctrine from the syntax of the language (pm) of *Principia Mathematica* in which Russell couched the theory. This is desirable because it shows that the theory

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is more relevant as a theory of natural language quantification than it may have appeared in Russell’s hands. In particular, RQ does not inflict as much violence on the surface syntax of the natural language it is applied to beyond that independently required for capturing semantically relevant features like scope. Despite the snug fit with surface syntax, however, use of RQ is not intended to signify a retraction of Russell’s doctrine that descriptions are “incomplete symbols”. The preservation of a discrete expression (namely a restricted quantifier) corresponding to a quantified noun-phrase (NP) in the translation into RQ does not mark a commitment to there being a specific thing that the restricted quantifier stands for. The failure of an NP to stand for an object or, in Neale’s more favoured terminology, to contribute an object to truth-conditions, is what makes the NP an incomplete symbol. Thus NPs divide into those that refer and those that quantify. One of Neale’s many extensions of the theory, accordingly, is an attempt to demonstrate that every NP of English is either a quantifier or a referring expression.

Neale’s claim that his reconstruction of the theory of descriptions preserves Russell’s insight that descriptions are “incomplete symbols” has met with some resistance, apparently to his surprise.1 The resistance arises from a concern that the smooth notational transition from clunky old PM formulas to their elegant RQ replacements conceals a more substantial replacement of some of Russell’s philosophical motivations for the theory that leave Neale’s version bereft of the original resources that justified calling descriptions incomplete symbols. In particular, it is argued, Neale’s attempt to conjoin the theory with a Chomskian syntactic theory inevitably discards Russell’s original conception of logical form, replacing it with the LF representations provided by best current syntactic theory. Aspects of Russell’s conception of logical form such as the role it played in his metaphysics are thereby without a home in Neale’s theory of descriptions. These aspects, however, are claimed to be central to the theory of incomplete symbols.

In what follows, I will both defend and attack Neale. I will defend his claim that he has preserved Russell’s doctrine that descriptions are incomplete symbols. However, I will go on to argue, with a qualification, that he is mistaken to do so. I will argue that Neale follows Russell too closely by fallaciously inferring that because an expression does not con-

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1 See, e.g., the appendix to Neale 2001.
tribute an object to the truth-conditions of a sentence containing it, it
does not have a unitary meaning that is grasped by one who understands
it, or sentences containing it.

1. DESCRIPTIONS AS QUANTIFIERS AND THE LF HYPOTHESIS

Neale’s reading of the theory of descriptions is that descriptions are sim-
ply a species of quantifier. This view has been common in formal sem-
antics at least since work by Montague and later by Barwise and Cooper
on generalized quantifiers. Neale’s preferred language for representing
logical form is the language \( \text{rq} \) of restricted quantification. A restricted
quantifier directly represents what Russell called a “denoting phrase”,
that is a determiner phrase formed by attaching a determiner to a nom-
inal, as in “every man”, “some man”, “a man”, “the man”, etc. A deter-
minder phrase of the form “\( \text{df} \)”, where \( \text{d} \) is a determiner and \( \text{f} \) a nominal
will be translated into \( \text{rq} \) as the restricted quantifier expression “[\( \text{Dx}: Fx \)]”. This quantifier expression attaches to a formula containing the
variable \( x \), which is bound by the quantifier. Thus “every \( F \) is \( G \)”
becomes “[every \( x : Fx \)](Gx)”.

\( \text{rq} \) is superior to \( \text{pm} \) in terms of elegance and, more importantly, its
relationship to English. Translation of “every \( F \) is \( G \)” into \( \text{pm} \) yields the
formula

\[(1) \quad \forall x(Fx \supset Gx)\]

If we take \( \text{pm} \) to reveal the underlying logical form of the original sen-
tence, we must conclude that the logical form is radically different to the
surface syntax of the sentence. This is even more evident if we accept the
analysis proffered by the theory of descriptions of “the \( F \) is \( G \)”. Our \( \text{pm} \)
translation is

\[(2) \quad \exists x((Fx & \forall y(Fy \supset x = y)) & Gx)\]

The original syntax of the sentence has been obliterated. In terms of
surface syntax, there is no difference between “every \( F \) is \( G \)” and “the \( F \)
is \( G \)”. Both have a structure that we could represent by a conventional

\[\text{See, e.g., Montague 1973 and Barwise and Cooper 1984.}\]
phrase-structure tree as follows:

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S
   /\    /
 NP  VP
    /   /
 Det  N  V  A
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Yet, after the translation into PM, the two sentences are transformed into formulas that have a very different structure to each other, to the extent that they even contain items of different lexical categories. Does this matter? It may not have troubled Russell—he took the theory of descriptions to provide evidence that grammar was a misleading guide at best to logical form. However, the claim that grammar conceals logical form is a somewhat complicated one. It would be over-simplistic to take Russell as just abandoning all hope of ever discovering facts about logical form from facts about grammar. Furthermore, regardless of the question whether Russell himself saw this as a problem (a question we will address in due course), there is the more pressing question of whether he should have done. If the syntactic features of (1) and (2) that make them seem similar are illusory and do not survive translation into the language that renders perspicuous their logical forms, then those features cannot guide us in locating their logical forms. But in that case, what does guide us? Unless some feature or function of the sentence can be located that guides us in establishing its logical form, there will be no systematic means of recovering a sentence’s logical form.

One natural suggestion, perhaps the one that Russell would have offered if the question had been posed to him in this way, is that the systematic means of recovering a sentence’s logical form lies not with its syntax but with its semantics. Certainly one of Russell’s most elegant arguments for the analysis of definite descriptions given by the theory of descriptions makes a direct appeal to the semantics of a range of sentences that is supposed to demonstrate that descriptions cannot be singular terms. For example, the fact that (3) is ambiguous, and that this ambiguity cannot be lexical, is provided as ground for thinking that definite descriptions must have sufficient (grammatically concealed) structural complexity to deliver two contrasting logical forms ((4) and (5)).
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3 Sainsbury 2004 reiterates Evans’s argument to make the same point.

4 It should be noted that the terms “surface structure”, “deep structure”, “phonetic form”, and “logical form” carry connotations that are not intended by Chomsky. For this
prior to any transformations licensed by particular conventions of one’s language. A simple example is the movement of a determiner phrase (dp) in passivization. So, for example in (6), the dp “the book” has been moved from object position (in (7)) to subject position (which is vacant in (7)):

(6)  ss: The book was written.
(7)  ds: … was written the book.

Of course, the ss representation associated with (6) contains more information than the English sentence just provided. It is a syntactic structure making the structural information of the sentence’s surface syntax explicit. The ss representation for (6) contains tree nodes, branches, and so on. This syntactic information is not made explicit in speech. Accordingly the pronunciation of the sentence is associated with a level of representation distinct from either ds or ss, namely pf (phonetic form). However, neither ss nor pf displays the structural detail required for semantic evaluation. The common phenomenon of syntactic ambiguity demonstrates the need for a further level of syntactic representation. This level must make explicit all of the syntactic data required for semantic evaluation. For example, it must disambiguate the ss associated with (3) into two lfs associated with (4) and (5) respectively.

The suggestion we considered above was that the logical forms in (4) and (5) might be arrived at by direct consideration of their truth-conditions. This would have been evidence that logical forms were semantic, not syntactic, items. In addition to the objections considered above, the Chomskian model offers some independent reason for thinking that logical forms are (at least partially) syntactic structures. For one thing, although scope phenomena are not commonly made explicit in the syntax of English, this failure is not uniform across all languages. Addition-
ally, scope phenomena do interact with grammatical features of English, albeit to a lesser extent than in other languages. Cook and Newson (2007) compare the following two examples as evidence of this:

reason, it is now common to replace the first two by the more neutral “s-structure” and “n-structure”. For reasons that will become apparent, lf representations (lfs for short) differ quite radically from logical forms as traditionally conceived by philosophers.

1 Hungarian, for example, makes some scope relations between quantifiers explicit in ss. See Cook and Newson 2007, p. 179.
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(8) Some professor believes [every student to have failed].
(9) Some professor believes [every student failed].

(9) is not ambiguous, but (8) is. It thus appears that, unlike the subject of a non-finite clause, the subject of a finite clause cannot have wide scope over a higher subject. LF is the level of syntactic representation that makes scope phenomena, and any other additional syntactic information that is not made apparent in the SS of the language in question, but is demanded for semantic evaluation, fully explicit.

Neale’s proposal, made explicit in Neale (1993), is that there is much to be gained from “exploring the view that a fully worked out theory of LF will be a fully worked out theory of logical form” (ibid., p. 95). Adopting LF representations as logical forms marks a substantial departure from the traditional philosophers’ conception of logical form. LF representations (“LFS” for short) are syntactic representations. They present a grammatical constraint on what philosophical logicians are licensed to postulate as logical forms. Admittedly, this grammatical constraint is not as simplistic as the constraints ordinary-language philosophy may have imposed, but all the same, it means that logical forms are constrained by syntactic theory, and as these are taken as the objects of semantic evaluation, we have a firm constraint on the semantics of the language studied. The claim that LFS are the vehicles for semantic evaluation in natural-language semantics is often summarized as the “LF hypothesis”, stated by Larson and Segal (1995, p. 105) in this form:

**LF hypothesis** The level of logical form is where syntactic representation is interpreted by semantic rules.

In other words, as they put it, the level of LF is the “interface between syntax and semantics” (ibid.). Aside from imposing constraints on the semantics, this means that the syntax must also be constrained to the extent that it must deliver something suitable for semantic evaluation. In the context of a truth-conditional semantic theory, this means that the level of LF representation must be “suitable for compositional interpretation: it must provide an articulation of constituents allowing the truth-relevant contribution of the whole to be calculated from the truth-relevant contributions of its parts” (ibid.). Russell’s incomplete symbols, interpreted as restricted quantifiers in LFS, must conform to these constraints also.
The previous discussion demonstrates that Neale’s proposed relocation of logical form within syntactic theory, whether he intends it to or not, lends support to his defence of the theory of descriptions. With this proposal in place, Neale can appeal to independently motivated syntactic evidence to support the claim that descriptions are quantifiers. As we have seen, if one looks only to the semantic evidence, it is possible to arrive at Russellian truth-conditions without accepting this claim. The challenge posed to the theory of descriptions by a referential theory of descriptions coupled with a negative free logic, for example, can be easily diverted if one can point to syntactic facts which demand we treat descriptions as quantifiers before we even confront the semantic data. However, by locating logical forms in a level of syntax constrained in the ways just mentioned, Neale invites the accusation that he has abandoned Russell’s central thesis. We will now turn to consider that accusation.

2. **INCOMPLETE SYMBOLS—THE OBJECTIONS**

There can be no doubt that Russell’s himself understood logical form in a far more ontologically loaded sense than the sense one would want to apply to LF structures. For example, he writes: “The study of logic … is concerned with the analysis and enumeration of logical forms, i.e. with the kinds of propositions that may occur, with the various types of facts, and with the classification of the constituents of facts” (Russell 1914, p. 109; Papers 8: 65). Far from being the structures of sentences, on this account, logical forms are the structures of the facts that sentences represent.6 The claim that descriptions are “incomplete symbols” has immediate ontological implications once it is coupled with this notion of logical form: put simply, descriptions do not survive the analysis of sentences containing them into their logical forms. The analysis of descriptive sentences into their logical forms eliminates the descriptions. This was the desired result that motivated the theory for Russell in the first instance.7 A number of commentators on Russell’s theory have objected to Neale’s revamping of it on the grounds that, by locating the arguments for it in the realm of syntax, Neale has effectively divorced the

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6 It should also be noted that this claim is made some time after Russell had ceased to identify facts with true propositions, so the logical forms of facts that Russell alludes to here cannot be the logical forms of propositions in his view.

7 See the various manuscripts in Papers 4.
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theory from its original (ontological) motivations. To remain faithful to Russell’s theory, an account should issue logical forms for descriptive sentences that do not contain any constituents corresponding to the descriptions that feature in the original sentences, it is argued, and Neale is alleged to have failed to meet this requirement. Such is the conclusion reached, for example, by Bernard Linsky, who takes it as evidence that Neale is not fully endorsing Russell’s theory after all: “Neale asserts, then, that Russell was right about the truth-conditions for sentences with descriptions, but wrong about their logical form, since he denied that descriptions were constituents of logical form” (Linsky 1992, p. 681). 8

Is Neale disagreeing with Russell on the logical form of descriptive sentences? This seems to flatly contradict Neale’s insistence that “[the \(x : Fx\)(Gx)\] is *definitionally equivalent to* \((\exists x)((\forall y)(Fy \equiv x = y) \& Gx)\)” (Neale 1990, p. 45). Neale himself dismisses Linsky’s objection as either an obviously false claim that Neale disagrees with Russell’s account of the truth-conditions of descriptions (an interpretation which clearly cannot be supported in light of the above quotation from Linsky), or a “trivially true” but “uninteresting” complaint that Neale has abandoned PM in favour of RM (Neale 2001, p. 231). This is unfair to Linsky, however. Linsky’s point is that Neale is invoking a purely syntactic notion of logical form that cannot be easily substituted for Russell’s notion without altering the content of Russell’s theory. I suggested above that Russell (mistakenly) construed logical forms as recoverable by direct examination of the truth-conditions of sentences. Linsky appears to be in partial agreement with this claim, but he also perceives an equally crucial ontological component to Russell’s notion of logical form:

The formalizations in the logic of *Principia Mathematica* provide the closest approach that Russell, long before Tarski’s definition of truth, could have given to an account of truth-conditions. But there is much more to Russell’s discussion of logical form than that. It is the other, more ontological, aspects of logical form, having to do with the form and unity of propositions and facts, that are of most significance for interpreting some of Russell’s other notions such as that of an “incomplete symbol”. (Linsky 2002, p. 396)

We can extract from Linsky’s interpretation of Russell two reasons why he objects so strongly to Neale’s account of logical form: (1) Russell’s

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account of logical form is independent of the syntax of natural language, hence it cannot be assimilated to $LF$; and (2) the ontological aspects of Russell’s notion of logical form are not captured by Neale’s analysis: “Russell’s notion of logical form is clearly directly concerned with the truth-conditions, logical powers, and existential import of propositions, and indirectly if at all with the syntactic features of sentences expressing those propositions” (*ibid*).

We have already seen, in the previous section, a good reason for abandoning (1) on Russell’s behalf. Without an appeal to syntactic features of descriptions, and natural-language quantifiers more generally, we are stripped of any knock-down objection to the view that descriptions are singular terms. To reiterate: the notion of scope will not do the work Russell wants it to do if it is drawn from semantics alone; unless scope is construed as a syntactic phenomenon, Russell’s quantificational analysis of descriptive sentences will not be the only analysis capable of yielding the Russelian truth-conditions, as a negative free logic will also contain sentences displaying the same scope ambiguities as Russell’s analysis predicts, despite taking descriptions to be singular terms.

It is not at all clear that if we follow Neale in abandoning (1) we are departing from the spirit as well as the letter of the theory of descriptions. The theoretical background to the postulation of $LF$ as a level of syntactic representation was simply unknown to Russell. It is tempting to think that Chomsky’s location of grammar within the actual psychology of speakers would have appealed to Russell’s later attitude towards language. It does not fit well with the Platonist conception of logic and logical form Russell held in his early work. Nonetheless, the basic idea that sentences have different levels of syntactic structure which are to be uncovered by analysis is certainly not in any way incompatible with Russell’s attitude towards language at this time. Ultimately, it is futile to speculate too far on how Russell might have responded to these doctrines that he was in fact unaware of. This leaves the question of how to respond to (2).

Does Neale’s presentation of the theory of descriptions divorce the theory from its ontological consequences? Certainly Neale is quick to distance himself from particular features of Russell’s ontology and epistemology, insisting on the independence of the theory of descriptions from

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9 See, e.g., his *An Inquiry into Meaning and Truth* (1940), *Human Knowledge* (1948).
those features. For example, he states that the theory has no essential connection to Russell’s sense-data epistemology and associated ontological commitments. It is hard to object to this. The theory of descriptions is a theory about which terms refer, not what they refer to. Similarly, Neale is surely right when he points out that the theory of descriptions is independent of Russell’s doctrine that names are disguised definite descriptions. Linsky’s objection, however, is not directed at Neale’s departure from these particular ontological doctrines of Russell’s. Rather, the heart of his objection is that logical forms themselves, and their constituents, both are, and represent, elements of Russell’s ontology:

The lexical items to be inserted at the leaves of trees in logical form have significant ontological status for Russell. In contemporary LP they are simply the primitive words of the language. No assumption is made that each word corresponds to something in the world…. I propose that it would be important to him to choose the right logical terms to be primitive, as they would have to reflect elements of logical form in the world: “the” does not name any such constituent. Neale rejects these aspects of Russell’s view, thinking that he is dropping something incidental to his purposes. Instead, the difference between a syntactically primitive lexical item and an item corresponding with a genuine constituent of the world is a large one, and not one to be glossed over.

(Linsky 2002, p. 404)

Neale’s dismissal of Linsky’s objection as founded on a confusion about the relative merits of the syntax of PM and Q will is therefore unfounded. However, in the next section, I shall argue that Linsky’s claim that Neale is guilty of glossing over the difference between syntactic primitiveness and ontological import is also unfounded.

3. **INCOMPLETE SYMBOLS—RESPONDING TO THE OBJECTIONS**

If one accepts Linsky’s claim that Russell’s logical forms carry ontological commitments, then it would seem to follow that Neale’s attempt to assimilate Russell’s logical forms to LP will only be successful if it retains the same feature. In other words, if Linsky is right about Russell’s conception of logical form, the postulation of restricted quantifiers in logical forms will entail a commitment to entities in the world corresponding to them so long as Russell’s conception of logical form has been retained. If it has not been retained, there will be no essential connection between Q and the theory of descriptions.
Neale insists that quantifier expressions do not stand for entities of any kind: “To say that a sentence $S$ expresses a general proposition is just to say that the grammatical subject of $S$ is not the sort of expression that stands for an object and does not contribute an object to the proposition expressed by (or the truth conditions of) an utterance of $S$” (Neale 1993, p. 89). It should be noted that, although he draws extensively on the insights of generalized quantifier theory as a means of extending quantification theory beyond the first-order quantifiers so as to capture natural-language determiner phrases like “most $F$’s”, “few $F$’s”, etc., Neale does not assign objects to the quantifier phrases of $\text{rq}$ in the semantics in the way that is common. It is common in work on natural-language semantics to interpret restricted quantifiers by assigning sets of sets to them known as generalized quantifiers.\textsuperscript{10} For example, the semantic value of the restricted quantifier “every $F$” is the set of all supersets of the set of $F$’s, and that of “the $F$” is the set of all supersets of the singleton set of the unique object that is $F$:\textsuperscript{11}

\begin{align*}
v(\text{“all $F$’s”}) &= \{X \subseteq U : \{x : Fx\} \subseteq X\} \quad \text{Df.} \\
v(\text{“the $F$”}) &= \{X \subseteq U : \text{for some } u \in U, \{x : Fx\} = \{u\} \text{ and } u \in X\} \quad \text{Df.}
\end{align*}

This greatly simplifies the statement of truth conditions for sentences containing restricted quantifiers. It is important to note, however, that Neale does not endorse this interpretation of the quantifiers of $\text{rq}$. This is made explicit in Neale (2001, pp. 229–30). There he notes that, while one is at liberty to employ the resources of generalized quantifier theory to provide a semantics for $\text{rq}$, one is not obliged to. Furthermore, Neale himself adopts a Tarskian semantics for the quantifiers of $\text{rq}$, giving axioms such as:

\[(\forall s)(\forall k)(\forall \phi)(\forall \psi) \left( s \text{ satisfies } \Gamma [\text{some } x : \phi] \psi \equiv \text{some sequence satisfying } \phi \text{ and differing from } s \text{ at most in the } k\text{th place also satisfies } \psi \right). \quad (\text{Ibid.}, \text{ p. 43})\]

Use of this axiom for the $\text{rq}$ expression [some $x : \phi$] does not invoke the set-theoretic apparatus described above. Thus, there is no sense in which a set-theoretic, or any other kind of, object is being assigned to the ex-

\textsuperscript{10} See Barwise and Cooper 1981.

\textsuperscript{11} The definitions are restricted to a given domain $U$. I am using the expression “$v(e)$”, to denote an expression $e$’s semantic value.
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Of course, the interdefinability of the universal and existential quantifiers with negation means that the universal quantifier could be replaced by an existential one. For the sake of argument, I here assume that the universal quantifier is primitive, with the existential quantifier introduced as a defined sign. The same remarks will apply to the existential quantifier if we take that as primitive instead.
account of definite descriptions from his wider theory of denoting in a way that does not appear to fit with Russell’s statement of the theory. The theory of descriptions is not just a theory of definite descriptions, but also a general theory of denoting within which the account of definites is one component. According to “On Denoting”, universally and existentially quantified phrases are also included on the list of denoting phrases:

By a “denoting phrase” I mean a phrase such as any one of the following: a man, some man, any man, every man, all men, the present king of France, the present king of England, the centre of mass of the solar system at the first instant of the twentieth-century, the revolution of the earth round the sun, the revolution of the sun round the earth.

(Russell 1905, p. 41; Papers 4: 415)

The claim made in “On Denoting”, and by the theory of descriptions more generally, is that all denoting phrases are incomplete symbols: “Everything, nothing, and something, are not assumed to have any meaning in isolation, but a meaning is assigned to every proposition in which they occur” (ibid., p. 42; 4: 416). None of the above phrases, including the universal quantifier expressions, has any meaning in isolation. As the universal quantifier clearly does occur in the logical forms of certain propositions on Russell’s theory, Linsky’s requirement that an incomplete symbol should not feature in any Russellian logical form is overly restrictive.

What does Russell mean when he calls an expression an incomplete symbol, if not that it does not occur in logical form? The only plausible explanation remaining is that the expression does not contribute an object to the propositions (or truth-conditions) that sentences in which it features are used to express. Indeed, this is the most obvious interpretation of many of the arguments Russell gives to show that descriptions are incomplete symbols. Consider, for example, the following argument from Principia, which is worth quoting in full:

[I]t can easily be shown that $(\forall x)(\phi x)$ is always an incomplete symbol. Take, for example, the following proposition: “Scott is the author of Waverley.” [Here the author of Waverley” is $(\exists x)(x \text{ wrote Waverley}).] This proposition expresses an identity; thus if “the author of Waverley” could be taken as a proper name, and supposed to stand for some object $c$, the proposition would be “Scott is $c$.” But if $c$ is any one except Scott, this proposition is false; while if $c$ is Scott, the proposition is “Scott is Scott,” which is trivial, and plainly different from “Scott
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is the author of Waverley.” Generalizing, we see that the proposition

\[ a = (\forall x)(\phi x) \]

is one which may be true or may be false, but is never merely trivial, like \( a = a \); whereas, if \((\forall x)(\phi x)\) were a proper name, \(a = (\forall x)(\phi x)\) would necessarily be either false or the same as the trivial proposition \(a = a\). We may express this by saying that \(a = (\forall x)(\phi x)\) is not a value of the propositional function \(a = y\), from which it follows that \((\forall x)(\phi x)\) is not a value of \(y\). But since \(y\) may be anything, it follows that \((\forall x)(\phi x)\) is nothing. Hence, since in use it has meaning, it must be an incomplete symbol. (PM I: 67)

Turning a blind eye to the frequent use/mention slips made in this passage, the only way to make sense of the argument is as an argument about whether or not a description that Scott answers to contributes him to propositions expressed by sentences containing it. All that the argument (if indeed it is sound) establishes, is that such descriptions do not contribute Scott to those propositions. In short, the argument merely concludes that descriptions do not contribute objects to propositions: descriptive sentences are object-independent. But this is precisely what Neale takes the mark of an incomplete symbol to be. No objects are required by the truth-conditions of quantificational sentences. This much is true of sentences of both the form “all \(F\)’s are \(G\)’s” and the form “the \(F\) is \(G\)”. Russell, it is true, never went so far as to explicitly state that “the” is a quantifier, like “all”. But, in light of the above discussion, this addition to the theory seems entirely appropriate: if expressions that Russell recognized as quantifiers were included in his list of incomplete symbols, it cannot be distorting his claim that definite descriptions are incomplete symbols to assimilate them also to quantifiers. Assimilating definite descriptions to quantifiers serves to clarify the theory of descriptions, not to depart from it.

Thus far, I have argued that Neale’s extension of the theory of descriptions serves to both clarify and strengthen the theory in comparison to Russell’s original statement of it. It clarifies the theory by assimilating descriptions to quantifiers. It also strengthens the theory by identifying logical forms with \(Ls\) and thus relocating the philosophers’ notion of logical form in the domain of syntax. This strengthens the theory, I have argued, because the semantic data alone are not sufficient to demonstrate the correctness of a quantificational analysis of descriptions. In the next section, I will argue that other considerations regarding the syntax of
descriptive sentences provide compelling grounds for making a further modification of the theory. Ironically, the modification in question is precisely that which Linsky accuses Neale of making, namely the assignment of unitary meanings to definite descriptions (and the other quantifiers). I have just argued that Linsky’s accusation is unfounded: Neale does not modify the theory in this way. I will now argue, however, that Neale should have made the modification, and that this would also serve only to improve, not to distort, the theory of descriptions.

4. INCOMPLETE SYMBOLS, SEMANTICS, SYNTAX, AND UNDERSTANDING

The motivation for the shift from PM to RQ, as stated in Neale (1990), is the need to forge a closer connection between logical form and “the superficial syntactical structures of English sentences” (ibid., p. 40). Two arguments are given there to show that the relationship between the syntax of PM and English is problematic. First, as discussed in section 2 above, PM introduces new lexical items into a sentence’s translation that do not correspond to any existing lexical items in the original sentence. This is symptomatic of a lack of any systematic mapping from the syntax of English to PM that is especially vivid, as we saw above, if we compare the translations of, say, “every king of France is bald” and “the king of France is bald”, each of which has the same phrase structure in English but differs strikingly when translated into PM. Secondly, there are natural-language quantifiers that cannot be translated into PM. Take, for example, the English quantifier word “most”. Not only can this not be translated as a complex quantifier composed of a combination of universal and existential quantifiers (as “the” is in PM), but even if we added the quantifier expression “most” to PM, giving formation rules to permit formulas of the form (most x)(Fx), which are to be interpreted as true if and only if more than half the objects in the domain of quantification are F; we will still find that many English sentences resist translation into the resulting language. Consider:

Most Manchester United players are wealthy.

The quantifier (most x) must bind the variables in an open sentence which translates both the predicates “is a Manchester United player” and “is wealthy”. The syntax of PM dictates that a logical connective is re-
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required to effect this. Let “©” be such a connective, yielding:

\[(\text{most } x)(\text{Manchester United player } x \text{ © wealthy } x)\].

The problem is that none of the existing connectives of \(\text{PM}\) is adequate to play the role of “©”. The connective “&” cannot be “©” as this would make the above sentence true if and only if most things are both Manchester United players and wealthy, which is not what the original sentence means. Nor can “∀” be “©”, as this would mean that the sentence is true if and only if most things are wealthy if they are Manchester United players, but as most things are not Manchester United players, most things are wealthy if they are Manchester United players, on the classical semantics bestowed on \(\text{PM}\)’s “∀” (see Neale 1990, p. 40; also Wiggins 1980).

The source of both of these problems is the same: that quantifiers are unrestricted in \(\text{PM}\) whereas they are typically restricted in English. “Most Manchester United players are wealthy” makes a claim about most Manchester United players, not most things. This is why the additional lexical items are required in the \(\text{PM}\) translations, to connect together the open sentences that provide the necessary restrictions and that are then bound by the quantifiers in an attempt to capture the right truth-conditions. This is truth-conditionally adequate for the classical quantifiers (i.e. the universal, existential, and other quantifiers defined as complexes of them), but it fails for others, like “most”. The solution to both problems, therefore, is to employ the restricted quantifiers of \(\text{RQ}\). The following formula of \(\text{RQ}\) is true just in case the number of \(F\)’s which are \(G\)’s exceeds the number of \(F\)’s which are not \(G\)’s:

\[[\text{most } x: \text{Fx}](Gx)\].

It should be noted that, while the issue here is one about the syntactic relationship between the language employed for the statement of logical forms and the language whose sentences’ logical forms are being captured, the cause of the concern over this is, at least partially, semantic. It is because \(\text{PM}\) does not contain any formula that can be interpreted as having the same meaning as “most Manchester United players are

\[\text{©}\] Or binary quantifiers, although this is slightly more complicated. See Neale 1990.
wealthy” that its syntax is deemed inadequate. One of the great benefits of generalized quantifier theory is that it reconnects the semantics of quantifier phrases with their syntax, by respecting the common occurrences of quantifiers (and determiners in general, if indeed any determiners are not quantifiers) as constituents of determiner phrases, rather than lexical items that occur in isolation. Furthermore, we can even go on to provide precise semantic values for the determiners themselves, in the form of functions from sets of individuals to generalized quantifiers. For example, for a domain \( U \), the semantic value of the determiner “all” is a function from any set of individuals \( X \) to the set of all subsets of \( U \) which are supersets of \( X \).

Why, then, does Neale refuse to assign generalized quantifiers as the semantic values of the restricted quantifier expressions of \( rQ \)? The only plausible answer, so far as I can tell, is that he is not prepared to abandon the doctrine that quantifiers, descriptions in particular, are incomplete symbols. The discussion conducted in the previous sections of this paper shows, however, that there is nothing to be gained by refusing to admit generalized quantifiers as the semantic values of the quantifier expressions of \( rQ \). Furthermore, as we have just noted, there is much to be lost by the refusal.

There is nothing to be gained by refusing to assign generalized quantifiers to the quantifier expressions of \( rQ \) because, as we saw in the last section, the key insight motivating Russell’s talk of “incomplete symbols” is the realization that descriptive (like other quantificational) sentences do not express object-dependent propositions. Quantifiers are incomplete symbols because they do not contribute objects to propositions (or truth-conditions). But Russell is mistaken, and Neale follows him in making the same mistake, in thinking that this means that quantifier expressions, including descriptions, do not have meanings. These two notions of object-independence and meaninglessness are quite distinct from one another. This is obvious enough from an examination of the passage in Frege’s *Grundlagen der Arithmetik* often cited as the first suggestion of the analysis of quantifiers as generalized quantifiers. Frege suggests that the sentence “All whales are mammals” should be understood as being about the concepts *whale* and *mammal*, and that the universal quantifier expresses a relation between these concepts, namely that of *subordination*: the sentence says that the concept whale is subordinate to the concept mammal (by which Frege simply means that \( \{ x : x \text{ is a whale} \} \subseteq \{ x : x \text{ is a mammal} \} \)). Having made this point, he immediately goes on
to lucidly illustrate the object-independence of the proposition expressed by the sentence:

It is true that at first sight the proposition “All whales are mammals” seems to be not about concepts but about animals; but if we ask which animal then are we speaking of, we are unable to point to any one in particular. Even supposing a whale is before us, our proposition still does not state anything about it…. As a general principle, it is impossible to speak of an object without in some way designating or naming it; but the word “whale” is not the name of any individual creature…. However true it may be that our proposition can only be verified by observing particular animals, that proves nothing as to its content. (Frege 1884, pp. 60–1)

Frege, of course, did not assimilate descriptions to quantifiers and held them to be expressions which do “designate” individuals as their semantic values. Hence, he would not have made the same points about descriptive propositions. But that is beside the point: the point made in the passage is a general one about the object-independence of propositions expressed by quantificational sentences. If one does think that descriptive sentences are quantificational, the same point will hold of them. Obviously, however, this does not exclude the possibility of analysing “all whales” as expressing a generalized quantifier, as Frege is suggesting that very possibility in the same passage.

5. CONCLUSION

Neale’s outright dismissal of Linsky’s criticisms is perhaps evidence of the dramatic departure that current philosophers of language have taken from Russell’s original conception of logical form. Linsky is certainly correct to point out the extent to which a Chomskian conception of logical form differs from a Russelian one. Nonetheless, the central argument of this paper has been that the shift from a conception of logical form derived from ontological and semantic considerations, such as Russell’s, to a syntactically driven one of the sort proposed by Chomsky and endorsed by Neale is to the benefit, not detriment, of the theory of descriptions. Furthermore, contrary to what Russell, Neale, and Linsky maintain, adopting a semantic theory that treats quantifiers as meaningful in isolation is perfectly compatible with the most important insight behind Russell’s claim that quantifiers are incomplete symbols.
REFERENCES


