ON CHWISTEK'S PHILOSOPHY OF MATHEMATICS

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The paper is devoted to the presentation of Chwistek's philosophical ideas concerning logic and mathematics. The main feature of his philosophy was nominalism, which found full expression in his philosophy of mathematics. He claimed that the object of the deductive sciences, hence in particular of mathematics, is the expression being constructed in them according to accepted rules of construction. He treated geometry, arithmetic, mathematical analysis and other mathematical theories as experimental disciplines, and obtained in this way a nominalistic interpretation of them. The fate of Chwistek's philosophical conceptions was similar to the fate of his logical conceptions. The system of rational meta-mathematics was not developed by him in detail. He worked on his own ideas without any collaboration with other logicians, mathematicians or philosophers. His investigations were not in the mainstream of the development of logic and philosophy of mathematics.

Leon Chwistek (1894–1944) is known mainly for his logical works, in particular for his simplification of Whitehead and Russell's theory of types. His logical investigations, however, were—as was the case with some Polish logicians, e.g., Stanisław Leśniewski—connected with his philosophical ideas concerning logic and mathematics. Moreover, they were in a sense motivated by those ideas. Building semantics, he wanted to overcome philosophical idealism and was against the conception of an absolute truth. He did not content himself with solving particular definite fragmentary problems but—similarly to Leśniewski—attempted to construct a system containing the whole of mathematics.

[1] The financial support of Foundation for Polish Science [Fundacja na rzecz Nauki Polskiej] is acknowledged.

Chwistek's interest in logic dates from his studies in Göttingen, in particular from the moment he attended a lecture by Poincaré in 1909. He decided then to unify the ideas of Russell and Poincaré and to reform the theory of logical types by eliminating non-predicative definitions.

He decided to rebuild the system of Whitehead and Russell and did it in a nominalistic way by constructing a simple theory of types rediscovered later by F. P. Ramsey. Its foundations were formulated in his papers from the 1920s, in particular in Chwistek (1921a, 1922a and 1922b). Russell mentioned Chwistek and his contribution in the Introduction to the second edition of *Principia Mathematica*:[2]

> Dr Chwistek took the heroic course of dispensing with the axiom [of reducibility] without adopting any substitute; from his work, it is clear that this course compels us to sacrifice a great deal of ordinary mathematics. (*PM* 1: xiv)

In Chwistek (1924 and 1925) he formulated a pure theory of logical types—a theory of constructive types. In this theory non-constructive objects are rejected, but the price for that is the greater formal complexity of the system.

Those investigations led Chwistek to the construction of a full theory of expressions and—on the basis of it—his so-called rational meta-mathematics. This would be a system more fundamental than logic, and it should enable the reconstruction of a classical logical calculus and of Cantor's set theory. It should fulfil nominalistic assumptions, hence in particular it should be free of any existential axioms, first of all the reduction axiom and the axiom of choice. All this was based on the assumption that the theorems of the system being constructed, and consequently of classical logic and of set theory, refer only to expressions or inscriptions that can be obtained in a finite number of steps by a rule of construction fixed in advance, and not to the reference of those expressions. Moreover, those expressions or inscriptions were understood as physical objects.

Those ideas were developed by Chwistek later as part of his philosophy of logic and mathematics, in particular as a part of his ideas concerning the methodology of the deductive sciences. He developed them mainly in his 1935 book, *Granice nauki; Zarys logiki i metodologii nauk*

[2] *Cf.* also the correspondence between Russell and Chwistek (see Jadacki 1986).

ścisłych. The English revision and translation, *The Limits of Science: Outline of Logic and of the Methodology of the Exact Sciences*, appeared in 1948.

According to Chwistek, human knowledge is neither complete nor absolute. It cannot be complete because statements concerning the totality of objects lead to inconsistencies. It cannot be absolute because there is no absolute reality. In *The Limits of Science* he wrote:

> It follows from these considerations that the principle of contradiction does not permit complete knowledge, i.e. knowledge which includes the answer to all questions. The attempt to secure such knowledge will sooner or later conflict with sound reason. (1948, p. 42)

And common sense is, according to him—beside the admission of experience as a fundamental source of knowledge and of the necessity of schematization of experienced objects and phenomena—a factor common to all correct cognitive processes. It consists in rejecting all assumptions that cannot be experimentally checked, are inconsistent with experiments, are not based on reliable and certain statements concerning simple facts, or cannot be logically reduced to such statements. Both empirical and deductive knowledge are relative. The first is relative because there are various types of experiments corresponding to various realities, and the second because it depends on the accepted system of concepts. Chwistek refers here to rational relativism.

Chwistek accepted the principle of the rationalism of knowledge and was decidedly against irrationalism. Rationalism consists in accepting only two sources of knowledge, namely experience and strict reasoning. It concerns not only mathematics and the exact sciences but experimental sciences and philosophy as well. He wrote: "… the point of departure in constructing a world view should not be a confused metaphysics, but simple and clear truths based upon experience and exact reasoning" (1948, p. 3). Consequently he was against irrationalism, metaphysics and idealism in philosophy and mathematics.[3] He sharply criticized Plato, Hegel, Husserl and Bergson. Seeing the defects of positivism, he nevertheless appreciated its epistemological conceptions. In addition Chwistek highly appreciated dialectical materialism, seeing in fact almost no fun-

[3] It is worth noting here that Chwistek was against irrationalism and idealism not only because they are—in his opinion—incorrect philosophical theories but also because they are the source of human suffering, social injustice, cruel excesses and wars.

damental conflicts between it and positivism. His own epistemological conceptions he described as critical rationalism, which he set against dogmatic rationalism.[4]

A way out of the difficulties caused by irrationalism, and simultaneously a weapon in a struggle against it, was formal logic, in particular rational meta-mathematics founded by him. Chwistek begins his *Limits of Science* by writing in the first sentence: "We are living in a period of unparalleled growth of anti-rationalism" (1948, p. 1). And he ends the Introduction thus: "History teaches that ultimately victory has always been the destiny of societies who employ the principles of exact reasoning" (1948, p. 23). He writes also in the Introduction:

> When this new system [i.e., the system of rational meta-mathematics] is completely worked out, we will be able to say, that we have at our disposal an infallible apparatus which sets off exact thought from other forms of thought.
> (1948, p. 22)

Chwistek's epistemological views were close to neo-positivism. He claimed that an object of scientific knowledge can be only what is or can be given in experience, hence only what can be seen or experienced by the senses eventually assisted by instruments. He wrote: "… in speaking about reality we have in mind not some ideal object but the patterns which must be employed in dealing with a given case" (1948, p. 261).

Both in science and philosophy one should—according to Chwistek—use a constructive method. He explained this in his paper "Zastosowanie metody konstrukcyjnej do teorii poznania" [Application of a Constructive Method to Epistemology] (1923). Though one can apply the constructive method in a complete form mainly in deductive sciences, it can be used also in empirical sciences and in philosophy. It is based on the analysis of the intuitive concepts used in a given discipline. It enables the separation of primitive notions whose meaning is characterized in axioms. On the basis of axioms one now obtains theorems with the help of laws of (formal) logic. Later Chwistek came to the conclusion that constructing deductive systems in philosophy is useless—in fact, such a system cannot be constructed because philosophical investigations

[4] Notice a certain difficulty in interpreting Chwistek's philosophical view. In fact he often used classical philosophical notions but gave them a special meaning which he never explained or explained in an insufficient way.

are too complicated.

We said above that, according to Chwistek, only what is given in an experience can be an object of a cognition. There are, however, various types of experience. In this way we come to the best-known original philosophical conception of Chwistek, namely his theory of the plurality of realities.[5] He explained it for the first time in the paper "Trzy odczyty odnoszące się do pojęcia istnienia" [Three Lectures concerning the Concept of Existence], claiming that "the intuitive belief in one reality seems to be a superstition" (1917, p. 145), and suspecting that the concept of many realities was already present in Pascal and Mach (*cf.* pp. 149–50). He developed his conception in the book *Wielość rzeczywistości* [Plurality of Realities] (1921b), and his final version can be found in *Granice nauki* (1935). Its foundations were explained once again in the English edition of the book published in 1948—hence already after his death—but this edition contains nothing about this that is not already in the 1935 edition.

In the first period (i.e., until 1925) Chwistek distinguished the meaning of the concepts "reality" and "existence". The latter, according to him, has a more general character because it concerns not only objects of reality but abstract objects such as objects of mathematics. He wrote: "If we assumed that everything that exists is in fact real, then we should accept as real all mathematical relations together with elements of experience" (1917, p. 145).

In his (1917) Chwistek distinguished three possible positions concerning the problem of existence: nominalism, realism and hyper-realism. According to him, "nominalists demand descriptions by words excluding inconsistencies"; realists do not demand descriptions by words, but they "exclude inconsistent objects"; and hyper-realists "do without descriptions by words and do not exclude inconsistent objects" (1917, p. 126).

In the beginning he accepted only two realities and attempted to formalize his theory. In *Granice nauki* he abandoned the attempt at formalization and accepted four types of reality corresponding to four possible types of experience. Hence we have the reality of impressions, the reality of images, the reality of things (i.e. everyday life), and physical reality (as constructed in the exact sciences). He attributed independent existence and full equality of theoretical rights to each kind of reality.

Having presented Chwistek's general methodological and ontological

[5] This theory is sometimes compared with Popper's conception of three worlds.

ideas, let us turn to his views connected directly with the philosophy of mathematics. In fact we have already mentioned some of his views of mathematics lying at the basis of his logical conceptions. Now we shall consider his nominalism, which found full expression in his philosophy of mathematics.

Chwistek claimed that the objects of the deductive sciences, hence in particular of mathematics, are expressions constructed according to accepted rules of construction. Consequently objects of mathematics are not ideal objects such as points, lines, numbers or sets. Objects of mathematics are, in fact, expressions which are physical objects given to us in experience. They can be transformed according to accepted rules. In every given system one accepts such rules as well as some expressions that play the role of axioms and form the basis on which one deduces theorems. Rules of transformation and axioms are chosen in such a way that the expressions can be interpreted as descriptions of particular states of things. To be able to apply deductive theories to different disciplines, and generally to get to know particular domains of reality, one should schematize elements of the latter.

Geometry is, according to Chwistek, an experimental discipline. In Chapter IX of *The Limits of Science* he wrote:

> Geometry is an experimental science. It depends upon the measurement of segments, angles, and areas. The Egyptians conceived it in this way and it has remained essentially the same up to this very day. To-day what is generally regarded as geometry, i.e. what is included in textbooks, is the peculiar mixture of experimental geometry and the geometrical metaphysics which was inherited from the Greeks as Euclid's *Elements*. (1948, p. 170)

The development in the nineteenth century of the systems of non-Euclidean geometry of Bolyai, Gauss and Lobatchevsky—which Chwistek considered to be the most important achievement in the exact sciences—in his opinion refuted Kantian idealism.[6] Those geometries

[6] The claim that non-Euclidean geometries refuted Kant's philosophy of geometry seems to be not fully justified. In fact, one should take into account that Kant distinguished postulating the existence of an object and its construction. For a postulation of the existence of an object the inner consistency of the given concept is enough, but a construction of it presupposes a certain structure of perceptual space. Hence one can postulate the existence of a five-dimensional sphere because this concept is consistent, but one cannot construct such a sphere because the perceptual space has only three dimen-

have shown that, e.g., the concept of a line has no objective character but depends on adopted axioms. This suggests that a proper philosophy for geometry is conventionalism. Indeed, in his first papers, e.g. in "Trzy odczyty odnoszące się do pojęcia istnienia" [Three Lectures concerning the Concept of Existence] (1917), Chwistek states explicitly that the existence of consistent systems of non-Euclidean geometries refutes the thesis of the *a priori* character of geometry. It seems that he would be ready to accept conventionalism, though he never stated it explicitly. He wrote:

> Both systems [i.e., the system of Euclidean geometry and systems of non-Euclidean geometries] are free of inconsistencies—in fact they can be reduced to analytic geometry. Hence there are almost no fundamental differences between them from the theoretical point of view. Intuition easily accepts Lobatchevsky's theorems that only at first glance seem to be paradoxical.... So we come to the conclusion that both geometries are equally true, each of them refers to different lines; the differences between those two types of lines can be caught neither by experimental means nor by intuitive ones, hence a segment of a line we draw or think can serve as an illustration of one or another type depending on our will. (1917, pp. 144–5)

In *The Limits of Science*, however, Chwistek clearly and categorically rejected conventionalism, claiming that geometry—like all other fundamental experimental sciences—should be based on a theory of expressions. In fact conventionalism introduces hypothetical entities—as was the case already in J. S. Mill or later in Poincaré, the propagator of this tendency.[7] Chwistek wrote:

> It seems that it is impossible to attain a general concept of geometry without using formulae. It is therefore clear that the conception of geometry as the science of ideal spatial constructions must be nullified.... To speak of different four-dimensional space-times it is necessary to employ five-dimensional space-time. It is clear that all this has only as much meaning as do mathematical formulae. (1948, pp. 186–7)

sions. In fact, Kant claimed nothing that would exclude the possibility of consistent systems of geometry other than Euclidean one.

[7] It is worth adding here that Chwistek claimed that conventionalism also became a source of reactionary social views and tendencies by reducing truth to efficiency and thus leading to the reinforcement of the ruling classes, writing: "It should be observed that idealism clothed in the feathers of conventionalism became a very dangerous instrument in the hands of those who were reacting against the old dogmatic idealism" (1948, p. 234).

One should also treat arithmetic, mathematical analysis and other mathematical theories in the same way as geometry, obtaining in this way a nominalistic interpretation of all of them.

The fate of Chwistek's philosophical conceptions was similar to the fate of his logical conceptions. The system of rational meta-mathematics was not developed by Chwistek in detail. It could not be done by his collaborators (among them Jan Herzberg, Władysław Hetper and Jan Skarżyński), or by his students (Wolf Ascherdorf, Celina Gildner, Kamila Kopelman, Abraham Melamid, Józef Pepis and Kamila Waltuch), because all of them were killed during the Second World War. Chwistek went alone down his own paths. His investigations were not in the mainstream of the development of logic and philosophy of mathematics. Like Leśniewski (*cf.* Murawski 2004), Chwistek worked on his own conceptions and ideas without any collaboration with other logicians, mathematicians or philosophers. Despite being a professor of Lvov University, he had no close contacts with the Lvov–Warsaw philosophical school (*cf.* Woleński 1989). His ideas have often been sharply criticized. He wrote of his professional reception: "the circles of professional philosophers reacted to the idea of the plurality of realities either with contempt or with an unparalleled resentment being close to a wild fury" (1933, p. 56).

What were the reasons for such reactions? In fact Chwistek's philosophical investigations had no systematic character, and it seems that they were not treated by him with a full sense of responsibility (*cf.* Chwistek 1961, Preface, p. vii). He did not explain many of concepts he used, and his conceptions had been "earlier proclaimed than checked" (*ibid.*). He did not develop his systems in detail, being content to sketch them. His works did not stimulate interest among logicians and philosophers (with the exception of his version of type theory). It was only after 1945 and the growing the interest in nominalism in the philosophy of mathematics that some of his ideas found recognition. In particular, J. R. Myhill published a series of papers devoted to the problem of whether, and to what extent, Chwistek's systems of rational meta-mathematics can be applied to obtain a proof of the consistency of set theory in the version proposed by the Bourbakists (*cf.* Myhill 1950, 1951a, 1951b). A recognition of Chwistek's contribution to nominalism in the philosophy of mathematics is found also in Henkin's paper, "Nominalistic Analysis of Mathematical Language" (1962), where he wrote:

While the nominalistic tradition in philosophy is of course very ancient, a specific concentration of interest in this viewpoint, as applied especially to the analysis of *mathematical* language, can be clearly discerned in the work of the Polish school of logicians of the early decades of this century. The names of Lesniewski, T. Kotarbinski, Chwistek and his student Hepter, and Tarski are all associated with this activity. Among the best known of this work is Kotarbinski's theory of Reism ("Everything that is, is a person or an object") and his efforts to define all categories in terms of one lowest category, and Chwistek's efforts to establish the consistency of mathematics by providing an interpretation wherein all of its assertions could be shown to be truths about physical objects.

Aside from some writings of Russell this interest of the Polish logicians did not seem to be reflected outside of their own country. But with the transplantation of Tarski to the United States in 1938[8] the concern with nominalism made itself evident in this country, and subsequently in countries of western Europe. From 1940 on there has been a steady series of publications in this area, including works by Quine, Goodman, Martin, Woodger, Church, Wang, G. Bergmann, Lejewski, Scheffler—and even myself. Nominalistic considerations have made themselves congenial both to some holding a constructivist viewpoint in the foundations of mathematics, and to others who have advanced a formalistic position. (Henkin 1962, pp. 187–8)

Note also that recently R. Sylvan, the Australian philosopher, made favourable reference to Chwistek's pluralism in his book, *Transcendental Metaphysics* (1997).

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[8] [In fact Tarski left Poland in August 1939.—R. M.]

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