Articles

LOGIC, TOPOLOGY AND PHYSICS: POINTS OF CONTACT BETWEEN BERTRAND RUSSELL AND MAX NEWMAN

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This article reviews the interactions between Russell and the English mathematician Max Newman. The most substantial one occurred in 1928, when Newman published some penetrating criticisms of Russell's philosophy of science, and followed up with two long letters to Russell on logical knowledge and on the potential use of topology in physics. The exchange, which opened up some issues in Russell's philosophy that he did not fully cope with either at the time or later, is transcribed here. Their joint involvements with the Royal Society of London are also recorded.

I. RUSSELL'S PHILOSOPHIZED LOGIC

Bertrand Russell (1872–1970) wrote his autobiography at various times from 1931 onwards, intending it to be published posthumously. However, the financial demands of his many activities and organizations of the 1960s prompted him to have it published in his lifetime, padded out into three volumes (1967–69) by extensive transcriptions of letters to and from his many acquaintances. The gain in information is offset by a loss in cohesion, since many letters involve activities and relationships that are not discussed in the text.

A particularly striking example is a letter that Russell wrote in April

1928 to "Max Newman, the distinguished mathematician", in which he agreed with some criticisms of his philosophy that Newman had made in an article; but, typically, there is no explanation of the source of the issues.[1] In fact, Newman had pointed to some major difficulties in Russell's philosophy that Russell never fully sorted out either then or in his later philosophical writings. Nor did he reply to two long and remarkable letters that Newman sent him later in 1928. The full exchange, two letters each, is transcribed at the end of this article, after an explanation of the contexts and other aspects of their relationship.[2]

Russell's logical and philosophical career has been well documented, including in the autobiography, and a few details suffice here.[3] Graduating in mathematics and philosophy in 1894 from Trinity College Cambridge, he soon won a research fellowship to seek a new philosophy of mathematics. Around 1900 he learnt of the mathematical logic of Giuseppe Peano, and bolted onto it his own logic of relations and Georg Cantor's theory of sets to create a body of knowledge sufficient to fulfil all mathematical needs: not just proof methods and derivations but also mathematical objects, starting out with definitions of the finite cardinal integers as certain sets of sets. However, he soon also found that both logic and set theory admitted serious paradoxes, which rendered the reconstruction of mathematics much more difficult. Securing the cooperation of his former tutor A. N. Whitehead, they put forward their case in *Principia Mathematica* (three of an intended four volumes, 1910–13).

Following the philosopher Rudolf Carnap, their position is now called "logicism". It was the most prominent general philosophy of mathematics in the 1910s but not the only one. Around 1900 David Hilbert had started a programme later called "metamathematics", in which axiom-

[1] *Auto.* 2: 176–7. In a later letter (25 Aug. 1940 to L. Silcox) Russell praised Newman to a correspondent as "a very valuable critic" (*Auto.* 3: 39).

[2] Newman's modest archive is kept in St. John's College Cambridge; thanks to David Anderson, much of it is available in digital form at http://www.cdpa.co.uk/Newman/, and individual items are cited in the style "NA, [box] a- [folder] b- [document] c". The Russell file, 2/15, contains both sides of the exchange and several other pertinent documents. The signed holograph originals of letters **10.2** and **10.4** are in the Russell Archives.

[3] On the mathematical sides of Russell's mathematical logic see I. Grattan-Guinness, *The Search for Mathematical Roots, 1870–1940: Logics, Set Theories and the Foundations of Mathematics from Cantor through Russell to Gödel* (Princeton: Princeton U. P., 2000). On the philosophy, refreshing is P. Hager, *Continuity and Change in the Development of Russell's Philosophy* (Dordrecht: Kluwer, 1994), although Newman is not noted.

atized mathematical theories, and also formal logic, were studied for their properties of consistency, completeness and the independence of axioms. By the mid-1920s it was eclipsing all other foundational studies among the few mathematicians and philosophers who took any interest in them at all. Common to logicism and metamathematics was the "classical" two-valued logic covered by law of the excluded middle, in which an asserted proposition is either true or untrue. Some attention was given at that time to other logics, either to complement classical logic or, in the case of the Dutchman L. E. J. Brouwer, to replace it with his own intuitionistic logic in which the law was rejected, along with all indirect proof methods in mathematics.

2. NEWMAN'S UNUSUAL "BILDUNG"

Born in London in 1897 to a German father and an English mother, and their only child, Maxwell Hermann Alexander Neumann gained a scholarship to St. John's College Cambridge in 1915 and took part I of the mathematics tripos in the following year. However, carrying the surname "Neumann" in Britain during the Great War was a handicap sufficient for his father to be interned as an enemy alien; the family changed its surname to "Newman", and Max had to leave his college until 1919, when he returned and completed part II of the tripos in 1921, with a distinction in the advanced topics.

The next step of Newman's career was very unusual: he spent much of the academic year 1922–23 at Vienna University. He did not go alone, but went with two fellow Johnians.

As an undergraduate enrolling in 1919, Lionel Penrose had been interested in studying Russell's mathematical logic; upon finding that Russell had been dismissed by Trinity College for his pacifist activities in 1916, he specialized in traditional versions of logic as taught in the moral science(s) tripos by W. E. Johnson. He and Newman quickly became close friends, and doubtless he drew Newman's attention to mathematical logic, which was absent from the mathematics tripos(!). He became interested in the bearing upon logic and mathematics of psychology and psychoanalysis, subjects on which the tripos offered several courses. He decided to go to Vienna to meet Sigmund Freud and Karl Bühler and their colleagues; it seems likely that he *initiated* the visit, and asked friends if they wanted to go also.

The other Johnian was Rolf Gardiner, then one year into the languages tripos. Later he became well known in ecology, organic farming and as an enthusiast for the Nazis,[4] and also the father of the conductor Sir John Eliot Gardiner. The three young men were accompanied by Gardiner's younger sister Margaret, who was to become an artist and also a companion to the biologist Desmond Bernal; in her autobiography she reminisced a little about that time in "the still deeply impoverished town" of Vienna, where Penrose and Newman would walk side-by-side down the street playing a chess game in their heads.[5]

Of Newman's contacts with the mathematicians in Vienna we have only a welcoming letter of July 1922 from ordinary professor Wilhelm Wirtinger;[6] but it seems clear that Newman's experience of Viennese mathematics was decisive in changing the direction of his researches. His principal research interest was to turn to topology, which was *not* a speciality of British mathematics. By contrast, it was quite respectably represented at Vienna: some of Wirtinger's own work interacted with the topology of surfaces; in 1922 the university gave an extraordinary chair to Kurt Reidemeister, who was to become a specialist in "combinatorial" (now "algebraic") topology, like Newman himself; the junior staff included Leopold Vietoris; and the student body included Karl Menger, who however was rather ill at the time and may not have met Newman.

But the outstanding figure was ordinary professor Hans Hahn: not only a specialist in the topology of curves, he was also one of the very few mathematicians of his day who took formal logic seriously, and moreover not only as a matter of research but also a topic for undergraduate education. In particular, during Newman's time in town he ran a preparatory seminar on "algebra and logic", and in later years held two full seminars on *Principia Mathematica*. Soon he was to have Kurt Gödel as a doctoral student working on the completeness of the first-order functional calculus with identity, and as editor of the *Monatshefte für Mathematik und Physik* he published both that paper and a sequel of 1931 on the incompletability of first-order arithmetic (which was to be registered

[4] R. J. Moore-Colyer, "Rolf Gardiner, English Patriot and the Council for the Church and Countryside", *The Agricultural History Review* 49 (2001): 187–209.

[5] M. Gardiner, *A Scatter of Memories* (London: Free Association Books, 1988), pp. 61–8.

[6] The Wirtinger letter is at NA, 2–1–2. For more details of Newman's emergence as a logician see I. Grattan-Guinness, "Discovering the Logician Max Newman (1897–1984)", forthcoming.

as Gödel's higher doctorate). Student Karl Popper admired the quality of Hahn's mathematics lectures in the 1920s, endowed with historical information on occasion.[7]

Hahn also saw himself as a philosopher. Throughout his time as a student at Vienna University from the mid-1890s to his higher doctorate in 1905 he took part in some of the philosophical discussion groups that surrounded certain chairs in the university. After teaching elsewhere for several years, he returned to Vienna University as a full professor of mathematics in 1921. During 1922 he took the leading role in the appointment to the chair of natural philosophy of the German physicist and philosopher Moritz Schlick; after appointment in 1923 Schlick created what was to be the best-known discussion group, the so-called "Vienna Circle", in which Hahn was a leading member.[8] Further, while the Circle had no agreed philosophy among all its members, Schlick, Hahn and later Carnap strongly advocated positivism and empiricism, acknowledging major influences from Ernst Mach (who had held that chair in the 1890s) and Russell's philosophy.

3. NEWMAN'S VIENNESE PHILOSOPHY?

After his return Newman developed as a (pioneer) British topologist, with a serious interest in logic and logic education and a readiness to engage with Russell's philosophy; surely one sees heavy Viennese influences here, especially from Hahn. Soon after his return in 1923 he applied for a college fellowship, and submitted a paper on the avoidance of the axioms of choice in developing the theory of functions of a real variable,[9]

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[7] K. R. Popper, "Zum Gedenken an Hans Hahn", in Hahn, *Gesammelte Abhandlungen*, Vol. 1 (Vienna: Springer, 1995), pp. 1–20. In 1920 Hahn published with Meiner Verlag an annotated edition of Bernard Bolzano's *Paradoxien des Unendlichen* (1851).

[8] Our guide to the Vienna Circle is F. Stadler, *The Vienna Circle* (Vienna and New York: Springer, 2001). On Hahn see K. Sigmund, "Hans Hahn and the Foundational Debate", in W. DePauli-Schimanovich, ed., *The Foundational Debate* (Dordrecht: Kluwer, 1995), pp. 235–45; and "A Philosopher's Mathematician: Hans Hahn and the Vienna Circle", *The Mathematical Intelligencer* 17, no. 4 (1995): 16–19.

[9] M. H. A. Newman, "On Approximate Continuity", *Transactions of the Cambridge Philosophical Society* 23 (1923): 1–18. On the context see *passim* in P. Montel and A. Rosenthal, "Integration und Differentiation", in *Encyklopädie der mathematischen Wissenschaften* (Leipzig: Teubner), Vol. 2, pt. C (1923), pp. 1031–1135 [article 11C9b]); and F. A. Medvedev, *Scenes from the History of Real Functions*, trans. R. Cooke (Basel: Birkhäuser, 1991).

some unspecified item concerning solutions of Laplace's equation, and a long unpublished essay in the philosophy of science that was completed in August. Its title, "The Foundations of Mathematics from the Standpoint of Physics", named a topic that he could well have heard aired by several members of the future Vienna Circle. Maybe he wrote some of his essay in Vienna; unfortunately the leaves of the essay do not contain any watermarks.[10]

In it Newman contrasted the world of idealized objects and circumstances that was customarily adopted in applied mathematics (smooth bodies moving in vacua, and so on), "certain ideals, or abstractions [...] not applicable to those of real physical objects" with the world of real physical objects that one encounters and on which he wished to focus. He distinguished between these two kinds of philosophizing by *the different logics that they used*. The idealizing applied mathematicians would draw on the two-valued logic, for which he cited a recent metamathematical paper by Hilbert as a source;[11] but those interested in real life would go to constructive logic, on which he cited papers by Brouwer and Hermann Weyl.[12]

We see here not only a concern with philosophical questions but also a readiness to put logics at the centre of the answer—both most unusual for a mathematician, and invoking ways of thinking far more Viennese than Cantabrigian. His college referees, Ebenezer Cunningham and H. F. Baker, did not make much of the essay but agreed to the award of the fellowship. He neither revised it nor seemed to seek its publication, although occasionally he alluded to its concerns; and it must be a major

[10] Newman, "The Foundations of Mathematics from the Standpoint of Physics", 1923, manuscript, St. John's College Archives, item F33.1. It was given by J. F. Adams after he wrote his obituary of Newman, in which it is not mentioned ("Maxwell Herman Alexander Newman 7 February 1897–22 February 1984", *Biographical Memoirs of Fellows of the Royal Society* 31 [1985]: 436–52). It is also not noted in P. J. Hilton, "M. H. A. Newman", *Bulletin of the London Mathematical Society* 18 (1986): 67–72.

[11] D. Hilbert, "Die logischen Grundlagen der Mathematik", *Mathematische Annalen* 88 (1922): 151–65 (repr. in *Gesammelte Abhandlungen,* Vol. 3 [Berlin: Springer, 1935], pp. 178–91).

[12] L. E. J. Brouwer, "Begründung der Mengenlehre unabhängig vom logischen Satz von ausgeschlossenen Dritten", *Verhandlingen der Koninklijke Akademie van Wetenschappen te Amsterdam sect. 1, 12* (1918–19), no. 5 (43 pp.), no. 12 (33 pp.) (repr. in *Collected Works,* Vol. 1 [Amsterdam: North-Holland, 1975], pp. 150–221). C. H. H. Weyl, "Über die neue Grundlagenkrise der Mathematik", *Mathematische Zeitschrift* 10 (1921): 39–79 (repr. in *Gesammelte Abhandlungen,* Vol. 2 [Berlin: Springer, 1968], pp. 143–80).

source of his recognition of the importance of logic.

In these ways Newman built upon the awareness of logic that he must have gained at Cambridge from Penrose's interest in it. That contact will have continued, for after Vienna Penrose wrote several manuscripts on mathematical logic, especially psychological aspects, in which he was influenced by Russell and also by Ludwig Wittgenstein's notion of tautology as conveyed in the recently published *Tractatus Logico-Philosophicus* (1922). He even worked on a dissertation on the psychology of mathematics, but then abandoned it.[13] From 1925 he studied for a degree in medicine at Cambridge and London, and became a distinguished geneticist, psychiatrist and statistician, and also father of the mathematicians Oliver and Sir Roger Penrose.

4. Russell's logicized philosophy

Newman's new specialities were to lead him to criticize Russell's philosophy in 1927 and 1928. After getting *Principia Mathematica* off to Cambridge University Press in 1909, Russell had embarked on an ambitious philosophical programme in which he used or emulated techniques from logicism or mathematical logic as much as possible, especially the theory of types, the theory of definite descriptions, the logic of relations, and the importance of making inferences. In addition, he maintained in this philosophy the same empiricist approach that that he had adopted in 1899 before developing logicism. The first extended statement of this kind of philosophy was made in the book *Our Knowledge of the External World as a Field for Scientific Method in Philosophy* (1914), a widely influential text on the Vienna Circle among many others. Russell's empiricism focussed upon our percepts or sensations of the external world and reduced discussion of its unknowable entities that are causing the sensations in the first place. However, in later work he raised the status of entities by wondering if the structure of our percepts could allow or forbid us to infer similar structures about entities in the physical world.

Russell applied this alternative approach comprehensively in his book *The Analysis of Matter* (1927), in which he also allowed for modern physics by taking due notice of quantum mechanics and relativity theory.[14]

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[13] In the Penrose Papers, University College London Archives, see especially boxes 20–1 and 26–8.

[14] W. Demopoulos and M. Friedman, "Bertrand Russell's *The Analysis of Matter*: Its

It fell into three parts: "The Logical Analysis of Physics", an epistemological part on "Physics and Perception", and an ontological finale on "The Structure of the Physical World". Some chapters of the book were delivered as the Tarner lectures at Trinity College Cambridge in the autumn of 1926; Newman was in the audience, and helped Russell prepare for publication the two ontological chapters on the construction of points in space and on space-time order. When the book came out, he acutely criticized Russell's epistemological handling of structure-similarity in a lecture to the Cambridge Moral Science Club in December 1927 that appeared in the philosophical journal *Mind*.[15] It was this article to which Russell referred in his letter of April 1928 accepting Newman's criticisms.

5. ANALYSING RUSSELL'S ANALYSIS

The attempt to accommodate modern physics was one source of the importance of structure-similarity, since the candidate causal entities included, for example, electrons and quanta and not just sticks and stones. Russell took them to be "all logically complex structures composed of metaphysically more primitive" entities called "events".[16] Of particular significance was the inference from known perceived events to unperceived ones that are their causes, with structure-similarity as a desideratum though not as a necessity. However, Newman found this emphasis on structure to be excessive, and moreover questionably dependent upon properties such as continuity of our percepts and of the external world; structure alone could not deliver the information about the entities that Russell sought to obtain. He also queried Russell's ranking of structure-similarity into levels of "importance" according to the contexts in which they were used or not; for then it "would have to be reckoned among the

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Historical Context and Contemporary Interest", *Philosophy of Science* 52 (1985): 621–39; Demopoulos, "Russell's Structuralism and the Absolute Description of the World", in N. Griffin, ed., *The Cambridge Companion to Russell* (Cambridge: Cambridge U. P., 2003), pp. 392–419; and Hager (n. 3), Chap. 4.

[15] Newman, "Mr. Russell's 'Causal Theory of Perception'", *Mind* n.s. 37 (1928): 137–48. Newman helped Russell with Chaps. 28 and 29 of the *Analysis of Matter*, and criticized especially Chap. 20. Kurt Grelling quickly translated the book into German: *Philosophie der Materie* (Leipzig: Teubner, 1929). There is also a later French translation: *L'analyse de la matière*, trans. P. Devaux (Paris: Payot, 1965).

[16] The none too lucid *Analysis of Matter*, p. 9.

prime unanalysable qualities of the constituents of the world, which is, I think, absurd."[17]

Had Newman been familiar with the phenomenology of Edmund Husserl, he could have strengthened his criticism by pointing out that structures are "moments" of a manifold, unavoidably dependent upon it for their existence:[18] we do not have structures in isolation but always as a structure *of* some thing or things, like the price *of* the meal or *of* the shirts, or the colour *of* the diary or *of* the apple. In the same way Russell's structures are *between* things such as (un)perceived events and our percepts, and extra-structural information is needed before philosophy can be properly formed. In contrast to moments, parts of a manifold *can* be considered on their own; for example, the main course of the meal or the skin of the apple. While structure-(dis-)similarity is a relationship of major significance when developing a (new) theory under the influence of theories already available,[19] it cannot carry the same weight in epistemological contexts.

In his letter to Newman that he was to reprint in his autobiography and also appears below as letter **10.1**, Russell agreed with the criticisms and recognized their significance. Omitted categories included spatio-temporal continuity and co-punctuality, the latter arising in one of the chapters where Newman had helped him:[20] holding "between five events when there is a region common to all of them", it brought things more to the fore, but surely at the risk of either radically modifying his empiricism or else driving him back to phenomenalism.[21]

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[17] Newman (n. 15), p. 147, quoting *AMa*, p. 5. The passage quoted includes the sentence "Geometry is important, unlike arithmetic and analysis, because it can be interpreted so as to be part of applied mathematics", which surely contradicts the applicability of arithmetic and analysis.

[18] B. Smith, ed., *Parts and Moments: Studies in Logic and Formal Ontology* (Munich: Philosophia, 1982).

[19] See especially R. S. Kaushal, *Structural Analogies in Understanding Nature* (New Delhi: Anamaya, 2003); and also I. Grattan-Guinness, "Solving Wigner's Mystery: the Reasonable (Though Perhaps Limited) Effectiveness of Mathematics in the Natural Sciences", *The Mathematical Intelligencer* 30, no. 3 (2008): 7–17.

[20] *AMa,* p. 299. Newman did not comment on this bizarre structure-similarity. In a later presentation of his position Russell asserted that "Matter is what the physiologist sees, mind is what the patient is thinking" ("Mind and Matter", *PfM*, p. 151). Recently Hager (n. 3), p. 176, defended Russell's position, on grounds that surpass my understanding.

[21] Compare Demopoulos and Friedman (n. 14), pp. 630–2.

Russell did not publish a reply to Newman either then or later, and seems to have forgotten about it in later philosophical writings. When replying to criticisms from Ernest Nagel in the Schilpp volume of 1944 he confessed to "have been surprised to find the causal theory of perception treated as something that could be questioned", as if the events of 1928 had never happened![22] Perhaps for rendering brain surgery impossible, Nagel had also failed to warm to Russell's conclusion that since a percept *of* the world has to be also somewhere in the world, which must be in some nerve in the brain, *therefore* "what the physiologist sees when he looks at a brain is part of his own brain, not part of the brain he is examining."[23]

In *Human Knowledge* (1948) Russell gave perception some attention: in particular, in his discussion of "structure" he echoed Newman's position (but without naming him) that "In physics, assuming that our knowledge of the physical world is only as to the structure resulting from the empirically known relation of 'neighbourhood' in the topological sense, we have immense latitude in the interpretation of our symbols", such as waves or particles in quantum mechanics.[24] He attempted to reduce this latitude by invoking "compresence", "which holds between two or more qualities when one person experiences them simultaneously"; thus "Two events are 'compresent' when they are related in the way in which two simultaneous parts of one experience are related." He also appealed to "contiguity", "a property given in sight and touch", where "Two parts of my body are contiguous if the qualities by which I locate a touch in the two parts differ very little."[25] Thus, regarding space-time,

> Since physics is intended to give empirical truth, the ordering relation must not be a purely logical one, such as might be constructed in pure mathematics, but must be a relation defined in terms derived from experience. If the ordering relation is derived from experience, the statement that space-time has such-and-such a geometry is one having a substantial empirical content, but if not, not.

[22] Schilpp, pp. 335–6 (Nagel), p. 702 (Russell); see also p. 716. The reception of the *Analysis of Matter* among the Schilpp critics was largely critical, comprehensively so with W. T. Stace (p. 355).

[23] *AMa*, p. 383.

[24] *HK*, p. 273.

[25] *HK*, pp. 315, 347.

I suggest that the ordering relation is contiguity or compresence, in the sense in which we know these in sensible experience.[26]

These topics do not appear in *A History of Western Philosophy* (1945) (where Husserl is also omitted); they arise in *My Philosophical Development* (1959), but the *Analysis of Matter* is only mentioned once. Although the book was reprinted in 1954 (and also recently), these issues continue not to attract most of Russell's commentators, and Newman's essay has never been included in any of the book collections of reprints of articles on Russell's philosophy.

6. NEWMAN ON LOGIC

Newman's reply to Russell's letter, sent in May 1928, contained elegant reformulations of his reservations (letter **10.2**), but the bulk of it was devoted to logical knowledge. Logicians are often curiously coy about characterizing logic; for example, in *Principia Mathematica* logic is a theory of truth-functions of propositions, but the relationship to language remains mute; in particular, the status of propositional functions or relations is unclear beyond the overall commitment to empiricism. The Wittgensteinian characterization of logical propositions as always true or never true in the second edition of *Principia* only lightly lifts the veil. The choice of possibilities is increased by the interdefinability of connectives and quantifiers; the clarity of discussion in that pre-Gödelian age was hampered by the failure of most logicians explicitly to recognize metalogic as distinct from its host logic.

Maintaining that logic is primarily concerned with human meaning and beliefs, Newman stood further along the empiricist direction than did Russell, since in his letter "I should like to take as primitive" the notion of "*things happening*". His examples of such "processes" included terminating procedures, such as manipulations of mathematical symbols, clearly recollected from the unpublished essay of 1923. While he made pertinent contrasts between temporal and spatial knowledge, one senses a paucity of categories of knowledge that often affects reductionist philosophers, cutting their own throats with Ockham's razor. For example, Newman also characterized his position as "one thing happening after

[26] *HK*, pp. 346–7.

another". Granted that the failure to recognize the importance of temporal logic has been a major human failing for many centuries, do we have to go to this opposite extreme and require logic always to be time dependent? And is simultaneity between events impossible? The logical pluralism of the 1923 essay was much preferable. Things happening are not logical as such, but extra-logical perceptible occurrences that can be subject to theorizing within which (a) logic is one source of knowledge. Positivist Hahn held that logic was concerned exclusively with *means* of linguistic expression, not with the entities or events to which language may refer,[27] a line that Newman could have profitably followed.

In his reply of August 1928 from his summer home near Penzance, Russell expressed surprise at Newman's approach and invited him down for a chat (letter **10.3**). It seems not to have taken place at that time; however, in 1966 Newman wrote to Russell that "I remember talking to you about Gödel's proof soon after it appeared" in 1931 on some unspecified occasion.[28] The conversation would have been curious; for, while Russell advocated hierarchies of languages from 1921, he never distinguished metalogic from its host logic and so never grasped the significance of Gödel's theorems.[29]

7. TOPOLOGY (NOT) IN BRITISH MATHEMATICS

In a footnote to his article in *Mind* Newman gave as an example of partial structural similarity between perception and the external world a representation of a quartet of people in which acquaintances were neighbouring dots in a circle, and he noted that more complicated relationships required more dimensions in their representation: "Such properties of systems of relations have been the subject of mathematical researches for about twenty years, under the name of Combinatorial Topology."[30] He brought his principal mathematical interest to bear here, and also in his second letter. What did he have in mind?

Envisioned as a branch of mathematics by G. W. Leibniz under the

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[27] F. Ablondi, "A Note on Hahn's Philosophy of Logic", *History and Philosophy of Logic* 13 (2002)*:* 37–43.

[28] Newman, letter to Russell, 25 September 1966 (NA, 2–15–11; RAI 710).

[29] See Grattan-Guinness (n. 3), pp. 327–8, 388–91, 592–3.

[30] Newman (n. 15), pp. 139–40. The change of name to "algebraic topology" took place gradually from the 1940s.

name *analysis situs*, topology focuses upon place and position, properties of a domain such as inside or outside, neighbouring or distant, above or below, that remain unaltered under its continuous deformation. Among familiar topological situations is the impossibility of touring the seven bridges across the rivers of the city of Königsberg; a common kind of topological object is a map of a railway system when it represents the (lack of) connections between stations but not their distances apart. Another example is provided by the rectangle

Figure 1

Join *D* to *A* and *C* to *B*, and obtain a cylinder; then join end *AD* to end *BC* and get a doughnut. Alternatively, join *D* to *B* and *C* to *A* and obtain the Möbius-Listing band, which merges inside with outside and has only one side and one edge.

Topology had been developing especially at centres in German-speaking Europe (such as Vienna), Paris, the USA and the Soviet Union,[31] but not in the British Isles. This is surprising, for the tradition of Scottish "common-sense" philosophy had emphasized spatial-topological thinking, especially in mathematics: examples include vortex atoms with Lord Kelvin, fields of forces in the electromagnetism of Clerk Maxwell, and the theory of knots of P. G. Tait. The Englishman W. K. Clifford contributed a proposal to interpret matter and motion as electrical phenomena that manifested in the variable curvature of space.[32] Yet they had not played major roles in the rise of topology; for example, its only place in British mathematics lay in point-set topology. Thus when Newman returned from his Vienna experience he became to a notable extent a *pioneer* researcher in algebraic topology in Britain.

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[31] See *passim* in I. M. James, ed., *History of Topology* (Amsterdam: Elsevier, 1999).

[32] See R. Olson, *Scottish Philosophy and British Physics, 1750–1880: a Study in the Foundations of the Victorian Scientific Style* (Princeton: Princeton U. P, 1975); and *passim* in R. Flood, A. Rice and R. Wilson, eds., *Mathematics in Victorian Britain* (Oxford: Oxford U. P., 2011).

One of Newman's actions was to expound the "combinatory method" in August 1926 at the annual meeting of the British Association for the Advancement of Science.[33] Taking as the basic guide an article of 1907 on *analysis situs* in the German mathematical encyclopedia,[34] among the various methods that he described he preferred to represent the topological relationships of a "surface" by covering it by triangles with recti- or curvilinear sides. In two dimensions the basic "unit" was the "triangle", in three the "tetrahedron", and so on up (and also down to the "edge", and then to the "point"); for all dimensions the generic term was called "the simplex", and compounds of them "complexes".[35] The rectangle could be taken as a pair of triangles with a common edge. An important property of a simplex was convexity, which was featuring strongly at that time also in the geometry of numbers, mathematical economics and linear programming.

Figure 2, based upon Newman's diagram, shows a surface. Two "vertices" terminate each edge, and no edges intersect; the order of the letters on each edge is relevant but the straightness is not. A closed sequence of boundary edges defines an oriented "surface". For example, the ordered sequence of polygons *ECF*, *CDGF*, *GJHIEF* has as its boundary edges the sequence *EC*, *CD*, *DG*, *GJ*, *JH*, *HI*, *IE* (and also the reverse sequence), and all the internal edges are used in both directions and so cancel out; for example, edge *GF* in *CDGF* and edge *FG* in *GJHIEF*. In such cases the surface separates its interior from its exterior and so is two-sided; otherwise it is one-sided, like the band.

33 Newman, "The Combinatory Method in Analysis Situs", *Mathematical Gazette* 13 (1926): 222–7. At the same session Frank Ramsey talked about "mathematical logic" (*Report of the British Association for the Advancement of Science, Ninety-Fourth Meeting* [London: Office of the British Association, 1926], p. 342).

34 M. Dehn and P. Heegaard, "Analysis Situs", in *Encyklopädie der mathematischen Wissenschaften*, Vol. 3, pt. 1, pp. 153–220 (1907, article IIIAB3). An article updated progress in topology by the late 1920s: H. Tietze and L. Vietoris, "Beziehung zwischen der verschiedenen Zweige der Topologie", Vol. 3, pt. 2, pp. 144–227 (1929, article IIIAB13) (Newman on pp. 216–17).

35 The definability of dimension was itself a major research topic in topology since Georg Cantor had shown the isomorphism between the unit line and the unit square in 1877 (T. Crilly with D. Johnson, "The Emergence of Topological Dimension Theory", in James [n. 31], pp. 1–24).

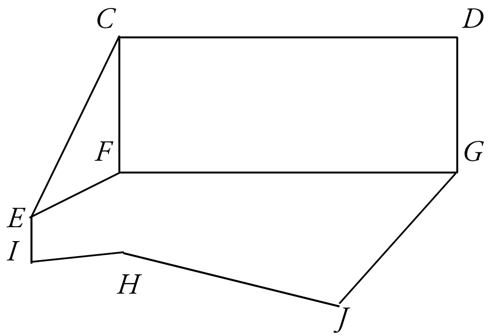

Figure 2

8. NEWMAN ON THE TOPOLOGY OF PHYSICS

This material lay behind the proposals of Newman's second letter (10.4) to Russell, which he sent from Italy in September 1928 soon after attending the International Congress of Mathematicians at Bologna (where Hilbert had made a strong presentation of metamathematics). His motivation was that he had recently finished reading the *Analysis of Matter* in detail, and saw scope for topology in some of Russell's treatment of both quantum mechanics and relativity theory.

One candidate for topological interpretation was the theory of emission and absorption in a "luminous event";[36] Newman saw it as expressible within space-time as a "mosaic" of "4-dimensional simplexes" *X*, and drew a tetrahedron containing seven vertices for the purpose. The periodicity of these phenomena was represented by corresponding repetitions of sequences of simplexes. But he was also understandably unsure about reconciling the topology with the space-time metric and the relation of the interval between two points[37] in relativity theory.

Newman envisioned "the final unified statement of physics" as an application of statistics to form "laws about small pieces of space time of atomic dimensions". These latter might be best expressed set-theoretically in terms of neighbourhoods, with relationships rendered in terms of edges of simplexes possessing common vertices, an approach that he had recently explored in combinatory topology in general.[38] He intro-

[36] *AMa*, esp. Chap. 33.
[37] *AMa*, esp. Chap. 35.
[38] Newman, "On the Foundations of Combinatory Analysis Situs", *Verhandlingen der Koninklijke Akademie van Wetenschappen te Amsterdam sect. I* 29 (1926): 611–41.

duced the notion of the "*n*-array" of vertices, specified by the collection of (n+1)-simplexes to which they belonged; for example, the septet belonging to *X* was "made into a 2-array" when the vertices of its triangles were appropriately labelled. In quantum mechanics talk of "electrons-, protons-, and quanta-at-an-instant" could be represented as "vertices of a '4-array' (groups of 5)" in the tetrahedron that represented (relativistic) space-time.

Newman concluded his letter with a return to "my chief grumble about your theory" of structure, namely the underrated status of the events on which structure is based and from which causality follows: for example, the need for continuity, upon which the topology will partly rest. He demanded, rather cryptically, that "the characterization of *points* should be logically simple", and suggested that co-punctual events be made primitive since there would then be no necessity to invoke convexity.

"Please excuse this enormous letter", ended Newman. While long it is also rather cryptic: in particular, beyond having elementary particles as vertices and relationships between them such as absorption as joining edges, it is not too clear what his topological representation of modern physics would look like. "Analysis situs" does feature in Russell's book, notably and not accidentally in the chapter on space-time order where Newman's assistance may have risen in places to ghost authorship; for example, "The 'lines' that we are defining are not to be supposed 'straight'; straightness is a notion wholly foreign to the geometry we are developing."[39] But no surface of a topological analysis of matter, such as Figure 2, emerges; the proposed revision of convexity would weaken connections with topology. Maybe some set-theoretic formulations would come back into play.[40]

Newman seems to have laid out his topological kitchen, but was not sure what to cook in it, either at that time or later. Indeed, "topology did not feature prominently in the physics of the story"[41] for some decades;

[39] *AMa,* p. 307.

[40] Alternatively, Newman might have had in mind the incidence matrix of a complex *U*, whose entries are defined in terms of the topological relationships between its constituent simplexes. However, he had not presented this approach in his lecture to the British Association.

[41] C. Nash, "Topology and Physics—a Historical Essay", in James (n. 31), pp. 359–416 (at 308–9).

the role of differential topology in relativity theory was the most prominent context, and it drew most on set-theoretic techniques.[42] Russell seems never to have replied to this letter; one may guess that its contents lay beyond his competence. However, topology features occasionally in later writings; in particular, in a popular article on "Physics and Metaphysics", published in May 1928, he stated, rather inaccurately, that

> To define a cage is a most complicated problem in a very modern branch of mathematics called topology, which is only properly understood in two universities, one that of Princeton, the other that of Moscow. If any of my readers wishes to know what a cage is, I advise him to write to Professor Veblen of the former university, but I cannot guarantee that the reply will be intelligible.[43]

Among his later philosophical books, *Human Knowledge* (1948) has little topological to offer.

9. NEWMAN'S LATER CAREER

Within weeks of his second letter to Russell, Newman was off to Princeton, where he spent the academic year 1928–29 with the topologist (and logician) Oswald Veblen.[44] His career in mathematical research was already dominated by topology, but his interest in logic was evident in a new course on the "Foundations of Mathematics" that he managed to insert into the Cambridge mathematical tripos; it covered not only mathematical logic and logicism but also metamathematics, Gödel's theorems, axiomatic theory and intuitionistic mathematics. Ready for the academic year 1933–34, he ran it only for the two succeeding years before it was closed down, perhaps because of disaffection among staff as well as among students. However, during the years 1938–41 he was somehow

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[42] See, for example, R. Penrose (son of Lionel and stepson of Newman), *Techniques of Differential Topology in Relativity* (Philadelphia: SIAM, 1972). J. Dieudonné, *A History of Algebraic and Differential Topology 1900–1960* (Basel: Birkhäuser, 1989), concentrates on the pure theory.

[43] Russell, "Physics and Metaphysics", *Saturday Review of Literature* 4 (1928): 210–11 (repr. *Papers* 10: 273). He will have had chances to discuss topology with Veblen in Sep tember 1927, when they (and the Russian topologist P. S. Alexandroff) travelled on the same ship to the USA.

[44] W. Aspray, "The Emergence of Princeton as a World Center for Mathematical Research, 1896–1939", in Aspray and P. Kitcher, eds., *History and Philosophy of Modern Mathematics* (Minneapolis: U. of Minnesota P., 1985), pp. 346–66.

able to continue placing questions on foundations in the examination papers.[45]

But Newman's secondary interest in foundations was to have major effects upon his later career. In 1931 Alan Turing arrived as an undergraduate, and after graduating three years later he sat in on Newman's foundations course in 1935 and learnt of metamathematics and recursion theory, of which Newman was the only British student. The effects on Turing's life were momentous: expertise in computability, code-breaker extraordinary at the government Code and Cypher School at Bletchley Park during the Second World War, and postwar investigator in computing at the National Physical Laboratory and then at Manchester University. Newman was alongside much of the time: colleague and even co-author on logic at the School, and then head of the Department of Mathematics at Manchester.[46]

While Russell and Newman never seemed to have held their discussion on logic for which Russell had hoped, contacts occurred from time to time later.[47] In 1936 Hardy had proposed Newman as Fellow of the Royal Society, seconded by J. E. Littlewood, and the election took place in 1939. As a Fellow Newman furthered the cause of logic there, with Russell's backing: he proposed and Russell seconded both the nomination of Turing as Fellow of the Royal Society in 1950 and of Gödel as Foreign Member in 1966; each nomination was successful. Newman sent a message for the celebrations of Russell's 90th birthday in 1962, and when Russell died in 1970 he agreed to be the chief obituarist for the Royal Society; but by then he was himself in his 70s and not able to fulfil the task. However, he contributed a pleasant survey of Russell's logic in a memorial meeting mounted by the Rationalist Association in June

[45] For details of this phase of Newman's career, including transcriptions of all his questions, see Grattan-Guinness (n. 6).

[46] A. Hodges, *Alan Turing: the Enigma* (London: Burnett Books and Hutchinson, 1983), esp. pp. 90–4. Among the large literature on the early history of computers see, for example, J. M. Copeland, ed., *Colossus: the First Electronic Computer* (Oxford: Oxford U. P., 2006). The details of Newman's activities at the School still seem to be somewhat lacking, but his place in computer science is nicely sketched in D. Anderson, "Max Newman: Topologist, Codebreaker, and Pioneer of Computing", *IEEE Annals of the History of Computing* 29 (2007): 76–81.

[47] Documents on both sides are gathered together in NA, folder 2–15.

1970.[48] He died in 1984, in his 88th year, and much of his remarkable and unusual career has stayed in the shadows.

10. THE FOUR LETTERS

10.1 *Russell, 24 April 1928, from Petersfield*[49]

Many thanks for sending me the off-print of your article about me in "Mind".

I read it with great interest and some dismay. You make it entirely obvious that my statements to the effect that nothing is known about the physical world except its structure are either false or trivial, and I am somewhat ashamed at not having noticed the point for myself.

It is of course obvious, as you point out, that the only effective assertion about the physical world involved in saying that it is susceptible to such and such a structure is an assertion about its cardinal number. (This by the way is not quite so trivial an assertion as it would seem to be, if, as is not improbable, the cardinal number involved is finite. This, however, is not a point upon which I wish to lay stress). It was quite clear to me, as I read your article, that I had not really intended to say what in fact I did say, that *nothing* is known about the physical world except its structure. I had always assumed spatio-temporal continuity with the world of percepts, that is to say, I had assumed that there might be co-punctuality between percepts and non-percepts, and even that one could pass by a finite number of steps (from one event to another compresent with it) from one end of the universe to the other. And co-punctuality I regarded as a relation which might exist among percepts and is itself perceptible.

I have not yet had time to think out how far the admission of co-punctuality alone in addition to structure would protect me from your criticisms, nor yet how far it would weaken the plausibility of my metaphysic. What I did realise was that spatio-temporal continuity of percepts and non-percepts was so axiomatic in my

[48] Newman's piece appeared untitled in *New Humanist* 1 (Dec. 1972): 321–2; materials in NA, 2–15–20, 22 and 32.

[49] Published by Russell in *Auto.* 2: 276–7.

thoughts that I failed to notice that my statements appeared to deny it.

I am at the moment much too busy to give the matter proper thought, but I should be grateful if you could find time to let me know whether you have any ideas on the matter which are not merely negative, since it does not appear from your article what your own position is. I gathered in talking with you that you favoured phenomenalism, but I do not quite know how definitely you do so.

10.2 *Newman, 29 May 1928, from St. John's College*

I hope you will excuse my long delay in answering your letter—making up examination questions leaves little time for more abstract speculations.

I made no positive suggestions in my article in "Mind" because I thought it would be foolish to do so without having a complete system at least roughly outlined in my head, and I cannot pretend I have. But the general position on the foundations of logic that I have held for some years—and hope some time or other to work out—has some bearing on the problem.

In the first place you will probably agree that the problem is essentially one of *meaning*—we have certain beliefs that are undoubtedly connected with unperceived parts of the world—e.g. about the origin of the solar system—and the problem is to analyze them in terms of concepts of whose significance we feel as sure as possible. That the beliefs refer in some way to structure[,] your analysis I think places beyond doubt, but I confess that "causal proximity" between unperceived "events" does not seem to me to have the necessary assured significance, at any rate without further examination.

The concepts ordinarily regarded as fundamental in logic or science have won their place rather on account of their convenience than because they are truly primitive. The adoption of $\exists x \,.\, \phi x$ as an undefined symbol allows formal logic to be developed with beautiful economy, but the statement, *e.g.*, that if I "count" through the letters on the page before me the process will terminate, belongs I think to an altogether more rudimentary set of ideas from which "$\exists$" should be constructed when *meaning* is the quarry. The notion

I sh[oul]d like to take as primitive is that of *things happening*. In logic we are concerned with perceived happenings, which can be individualized to "one thing happening after another". This leads to the notion of "going through a row of things", and the most primitive type of proposition is that a certain process of this kind will come to an end. The operations of mathematics consist in going through and manipulating rows of symbols, and the statement that there exists a solution of a certain equation means that a certain given process terminates with a number that satisfies the equation. I have hitherto pursued this idea chiefly in connection with the foundations of mathematics, but I incline to believe that it is this notion of "things happening" that is the primitive one underlying our beliefs about the external world.

It is possibly just this that you mean by saying that "the world consists of events", but I conceive of your events as in a way more static entities. For example, on my view an atom is to be abstracted from the set of "happenings"—quantum changes—associated with it; the question whether it "exists" between the changes is meaningless.

We may, I think, go further and admit as significant the notion of "things happening one after another" (in the same locality). It is easy to stress too much the assimilation of space and time required by the General Th[eor]y of Relativity. Not only does the th[eor]y itself distinguish time-like from space-like vectors, but there are aspects of time which it does not touch at all. E.g. the important fact that we are *now* in the year 1928, and no other, in the month of May, has no exact spatial analogue, and does not appear as a fact in the Th[eor]y of Relativity, which does not distinguish events which are happening from those that are not. There is something absolute about past events—they are past, not "relative" to the present, but just past, over, gone. The relation of London to my present location is quite different in this respect from the relation of the past to my present. And it seems to me that I have a direct apprehension of the meaning of (unperceived) things happening in succession; whereas the notion of spatial separation is directly traceable to qualities of my percepts and cannot be directly applied to unperceived events.

Whether this notional theme is happening in succession ("at the same place") is sufficient to make a world I don't know. In any caseI am afraid you will not make much of this rather confused account. It would be a great pleasure to me to discuss these ideas with you if ever you have the time to spare.

10.3 *Russell, 10 August 1928, from Carn Voel*

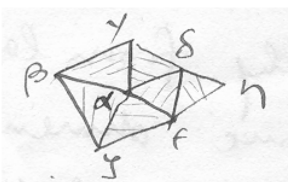
It was only for lack of time that I didn't answer your letter of 29 May, which interested me very much. The point of view you suggest on logic is very different from what I have been accustomed to, but I feel that something of the sort may well be right. I should dearly love to discuss the matter—I suppose you are not by any chance near here for the vacation?

Any time during the next fortnight I could put you up if you cared to come.

10.4 *Newman, 2 September [1928], from Lugano, Italy. His archive contains one page of his own copy of the letter.*

It was very kind of you to send me your new book—I have only just had time to read it through. I am very pleased if you found the list of papers I sent you useful.[50]

Two things I found particularly interesting were the general analysis of periodicity and the idea of the single "luminous event" connecting emission and absorption. On the de Broglie–Schrödinger Theory elect[ron]s and quanta are themselves simply centres of spherical disturbances, and so your analysis gives a method of identifying matter from considerations of structure alone. I am not quite sure that "quality" is really necessary for the recognition of periodicity. Suppose space-time were a sort of "mosaic" of 4-dimensional simplexes (a 4-dim-simplex is the analogue of *triangle* (2-dim.) and *tetrahedron* (3-dim.))



(2-dimensional "mosaic")

[50] [Presumably Newman was referring to Russell's citations of Vietoris, Felix Hausdorff, P. S. Urysohn and Menger in *AMa,* Chaps. 28 and 29.]

The microscopic character of sp[ace] time at a point α might depend on the way the simplexes fitted together there—i.e. on the coincidences between their other vertices. (In 2 dimensions all that can vary is the number of simplexes containing α, but in 3 or more dimensions a real variety of arrangements is possible). These properties are independent of any choice of co-ordinates, or indeed of any "continuous structure" of the interior of the simplexes, but yet are structural, not qualitative. Now might not periodicity mean that along a certain chain of simplexes the same arrangement recurs at intervals of p simplexes? The tendency of physics does seem to be towards explaining everything in terms of the *arrangement* of objects of at most 3 different sorts (electrons, protons, quanta).

With regards to "luminous events", it would be a very great advance to define interval length by counting, but is the actual counting of domains from a set covering the space-time continuum consis[t]ent with a (3: 1)-quadratic form for ds^2? It seems a complicated question.

Now that I have a clearer view of the whole theory than was possible listening to lectures, it is hard to doubt that the final unified statement of physics, when it comes, must be on these lines, namely, that the field laws and the quantities involved in them must all be derived statistically from laws about small pieces of space time of atomic dimensions. There has really been no serious attack before on the problem of the relation between quantum and field physics so far as I know. Some half-hearted attempts have been made by Einstein and others to get tensor forms of the quantum equations, but this hardly touches the deeper problems to which your book is devoted. My feeling is, however, that it will be necessary to commit oneself either to finite neighbourhood[s] or to infinite "converging" sequences; and if, as one hopes, the former should prove adequate, to some more definite assumptions about their *arrangement.* I have for some time thought that space time might be a 4-dimensional pattern of simplexes of the kind mentioned above, and have wondered if everything might come out of its structure. You will notice that all statements about such a pattern are expressible as relation[s] between *vertices*, e.g., that $\alpha\beta\gamma$ and $\alpha\gamma\delta$ have a common edge means that they have two common vertices. Any collection of things E becomes a 2-dimensional pattern or *array*, if a list is given of the trios selected from E which are to be regarded as "triangles" or units. Thus the collection α, β, γ, δ, ϵ, ζ, η is made into a *2-array* ("pictured" in p. 1 [above]) by specifying that its *units* are

$$\alpha\beta\gamma,\ \alpha\gamma\delta,\ \alpha\delta\epsilon,\ \alpha\epsilon\zeta,\ \alpha\zeta\beta,\ \epsilon\delta\eta$$

I have suggested, in my Amsterdam paper, how *manifolds or spaces* could be distinguished from other arrays of this kind.[51] All properties of such "manifolds" are really properties of the grouping of certain things—the *vertices.* It is tempting to suppose that electrons-, protons-, and quanta-at-an-instant may be "vertices" of a "4-array" (groups of 5). This hypothesis has the advantage of making logically simple object[s] which are unanalysed in physical theory; the "vertices" are not, of course, "infinitesimal"—the notion of size is hardly applicable to them. (In this theory there w[oul]d be some physical property correspond[ing] to "belonging to the same simplex".)

This brings me to my chief grumble about your theory. I find it hard to see what reason there is for believing in the existence of the *events* on which it is based, and especially to understand what is the difference (on which so much depends) between events and other domains in sp[ace]-time. I don't mean this to be understood in the rock-bottom solipsist way with which there is no arguing; but is it credible even in the "third degree", according to your "trichotomy" of p. 388[52]—as credible as causality? The argument for causality is still some sort of generalization of experience—at least I think that to be plausible it must be capable of statement as such—an inference from happenings we perceive to happenings we don't. But what does it mean to say that emitting and absorbing electrons are connected by a single event, while some other pairs of p[oin]ts are not? For the statement to contain the material for an *explanation* of the null interval[,] should there not be some property distinguishing *events* from other portions of space-time, other than their defined relation to the phenomena they explain? I cannot find anything resembling these favoured traits in physics; nor does the example of percepts,—events in one "specious present"—seem to help. For one thing that does seem clear is the *continuity* of our

[51] [Newman (n. 38).]

[52] [Namely, "the three grades of certainty": "the highest, my own percepts", then "the percepts of other people", and then "events which are not percepts of anybody".]

stream of percepts as it is originally "received"; any separation into parts ("specious presents") is an act of analysis, and to some extent arbitrary. The "vertex" theory I mentioned above is perhaps a slight improvement, for though it has little contact with immediate experience, electrons, etc., at-an-instant, do appear as entities in physical theory, in no uncertain way. But should not all such schemes be kept in their place as apparatus for enunciating compactly the laws governing complicated phenomena, as opposed to theories, like the causal theory of perception, which claim to deal with real properties of the world?

With regard to *topology:* whatever the metaphysical status of events, it seems important that the characterisation of *points* should be logically simple. This can hardly be said of the convexity condition which underlies your definition, when it is written out in terms of your fundamental concepts. Could you not avoid the whole difficulty, without committing yourself to any disagreeable hypothesis, by taking as primitive "copunctuality" between any finite number of events? Of course without assuming that, given N, there actually exists a set of N co-punctual events. Then a point would be [the] set of events, α, of which every finite subset is co-punctual, but no subset is co-punctual with an event not belonging to α. If co-punctuality is "pictured" as having a common domain, this def[inition] w[oul]d, I think, be satisfactory without any private reservation about convexity. There remain the definitions of "between" and "line" but I don't see why these need be brought in so early: I don't think you use them afterwards.

Please excuse this enormous letter[.]

ACKNOWLEDGMENTS

For permission to reproduce these letters I thank William Newman and the Master and Fellows of St. John's College Cambridge, and the Bertrand Russell Archives at McMaster University in Canada. I am indebted also to University College London Archives. For advice I am much indebted to Roger and Oliver Penrose, Joseph Kouneiher, and the referees.