

MODES AND LEVELS OF PERPLEXITY

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John Ongley and Rosalind Carey. *Russell: a Guide for the Perplexed*. London: Bloomsbury, 2013. Pp. ix, 212. ISBN: 978-0-8264-9753-6. £45 (hb), £14.99 (pb); US\$80 (hb), US\$24.95 (pb).

I

This book is the 41st in a series of guides to the work of various philosophers and philosopher groups, written at a level of readability for students. The authors survey the course of Bertrand Russell's philosophical career, starting out with his work on logic and mathematics. The main philosophical tools are presented: in logic, they are term, proposition, proposition function, relation, paradox, and first-*vis-à-vis* higher-order logic. Then, moving on to epistemology, the principal philosophical notions are presented, such as sense-data, facts related to language, Russell's overriding preference for positivistic philosophies and suspicions of metaphysics, knowledge by acquaintance and by description, theories of truth, logical atomism, percept, events, meaning, and inference in the context of scientific theories. It is nice to have many of the principal components of Russell's philosophy exhibited together, including the later works *An Inquiry into Meaning and Truth* and *Human Knowledge*.

The perplexed reader would like to be given *precise* references for Russell's assertions on various points, especially when he was so prolific. A cited short item does not need more precision; but the two main chapters that cover the logic and epistemology story often seem to invoke *The Principles of Mathematics* or *Principia Mathematica* and yet contain only one precise citation of a longer text in their 64 pages (p. 85). The other chapters of the book are somewhat better in this respect.

The index is excellent, though W. V. Quine was missed at pp. 29–30. The table of contents would have benefited from including the titles of the sections.

II

Much ground is covered; however, the account is mitigated by the failure to provide a biographical factual survey that should have constituted the opening chapter. Little or no description or explanation is given of, among others, Cambridge University, its mathematical tripos, Russell's research fellowship in 1895, the philosophical journals *Mind* and *The Monist*, Russell's conversion from neo-Hegelian to positivistic philosophy following G. E. Moore around 1899, the start of personal contacts with Giuseppe Peano in 1900, the gradual involvement of A. N. Whitehead from around 1902, the financial rescue of *Principia Mathematica* by the grant from the Royal Society in 1909, the serious mistake in Volume 2 detected by Whitehead in 1911 while it was in press, the initial contact with Ludwig Wittgenstein from 1911, Russell's switch from logic to epistemology in the early 1910s, Whitehead's intentions for the fourth volume of *Principia* that was abandoned in 1918, the circumstances of the second edition published in the mid-1920s, Russell's partial return to serious philosophical writings from the mid-1930s onwards, or the Trinity fellowship from 1944. Presumably the authors have made most of these omissions intentionally; but the target audience is the loser.

The opening "introduction" does mention some of these events and publications; but its purpose is to provide a preliminary look at some of the main features of Russell's logic, including the comprehension axiom "that every predicate defines a class" (p. 3), type theory, definite descriptions, "the no-class theory of classes" (p. 10), and the status and role of "analysis". Why do these features need prologues when the others do not? When they are re-discussed in the later chapters on logic and epistemology, several near-repetitions occur. Furthermore, quantification does not receive sufficient consideration in the book; and the logical connectives and rules of inference none at all! The silence over his incoherent all-purpose "implication"¹ is especially regretted.²

III

The writing of the two main books in logic, the *Principles* and *Principia*, is rather lightly handled. In particular, the authors note some of the complications of the composition of the *Principles*,³ but they leave the reader to draw the surprising conclusion that logicism was conceived during its preparation and not before.

The treatment of the theory of definite descriptions comes with the usual

¹ RUSSELL, "The Theory of Implication", 1906.

² CORCORAN, "Meanings of Implication".

³ GRATTAN-GUINNESS, *The Search for Mathematical Roots, 1870-1940*, Ch. 6.

omission of its essential role in logicism of supplying a means of defining *single-valued* mathematical functions. Moreover, the immediately ensuing discussion of the “no-class theory” of sets would have benefited from a note on Russell’s substitution theory of sets and relations, which drew on descriptions for its basic notion of the matrix; it enjoyed high status in Russell’s logic for at least a year before failing in various aspects in 1907. Descriptions would also have provided a good situation to explain contextual definition, which is usually ignored by historical commentators.

Very welcome is the attention paid in several places to *The Analysis of Matter* (1927), Russell’s brave philosophical foray into science and especially in modern physics that has never been given the consideration that it merits. It also excited a challenge from the mathematician Max Newman that neither Russell then nor later, nor philosophers following in his wake, have fully addressed.⁴

IV

The account of “number”, meaning mostly positive integers, is quite extensive, with Peano’s axioms contrasted with Russell’s definitions. For some reason the account of irrational numbers is related more to Richard Dedekind’s theory than to Russell’s own. The theory of real numbers excludes Whitehead’s alternative theory given in Volume 3 of *Principia*; also omitted is Russell’s generalization in Volume 2 of ordinal numbers to ordinal similarity, his chief contribution to the mathematics of logicism.

Russell’s paradox is duly delivered, although the authors do not say that it is a double contradiction, nor that it is the result of adapting Cantor’s power-set axiom (pp. 57–9). The unsuccessful type theory of the *Principles* is not analysed, but that of *Principia* is discussed at some length.

The final chapter of the book deals with “the infinite”. Its location as an appendix in all but name is not very satisfying, but it begins well with some consideration of the algebraic logic tradition of C. S. Peirce and Ernst Schröder. Their use of the “earlier set theory of Boolean algebra” (p. 176: “part-whole theory” is perhaps a more precise characterization) contrasts fundamentally with Russell’s reliance upon Cantor’s set theory. The almost total lack of bonhomie between these two traditions of symbolic logic is exceptionally perplexing,⁵ and would have been worth some discussion.

The chapter continues with a clear summary of aspects of Cantor’s transfinite arithmetic and the theory of continuity, important *parts* of logicism but understood by the algebraists to be mathematical topics. It ends with a very welcome discussion of two axioms of special importance to Russell that often

⁴ GRATTAN-GUINNESS, “Logic, Topology and Physics”.

⁵ GRATTAN-GUINNESS, *Search for Mathematical Roots*, Chs. 4, 6–8.

are ignored by historical commentators: the axioms of infinity and of choice. His concern with them overlapped in 1904: around the summer, after a dispute with C. J. Keyser, he acknowledged that assuming an infinitude of individuals needed axiom support; and he also spotted the need for choice, slightly *earlier* than its popularizer, Ernst Zermelo. Each axiom also involves an important philosophical issue: for infinity, the positivistic requirement that individuals be physical objects (minimizing this need had led Whitehead to his mistake in 1911); for choice, how can the infinitude of independent sections in creating the choice class be expressed in Russell's *finitary* logic? His little-known paper⁶ on these two axioms deserves attention.

v

Another aspect on which the perplexed reader will seek guidance is the influence and impact of Russell on others, whether positive or negative. His reception history is very rich: international, various in its philosophical approaches, and sometimes relating to the logic and sometimes to logicism. There are a few pages on Wittgenstein's *Tractatus*; but nothing is said about, for example, Dorothy Wrinch, Frank Ramsey, Jean Nicod, Rudolf Carnap (the coiner of "logicism" in 1929), Kurt Gödel, Quine, Alfred Tarski or Karl Popper.

The case of Gödel is especially intriguing: in his famous paper of 1931 on the undecidability of first-order arithmetic he not only refuted logicism but also asserted the central importance of distinguishing a logic from its "metalogue" (Carnap's word in 1931, because of Gödel). Now Russell had made a move similar to the latter in 1921 when he distinguished a language from its metalanguage (to coin Tarski's later word), in preference to Wittgenstein's doctrine of showing and saying. Nevertheless, always a logical monist, Russell could not grasp metalogue, and so never understood the significance of Gödel's theorem for logicism, despite trying into his 90s. The quality of perplexity is high here!⁷

vi

The bibliography of the book consists of around 100 items that have been cited in the text; about 60 are Russell items, though the *Principles* and *Principia* are excluded. We are also given the bibliographical details of volumes in Russell's *Collected Papers* (which itself is not discussed), but only from the fourth volume onwards. Furthermore, the only use that is made of this edition is to confine to it citation details of papers.

⁶ RUSSELL, "Sur les axiomes de l'infini et du transfini" (1911).

⁷ GRATTAN-GUINNESS, *ibid.*, pp. 327–8, 388–91, 592–3.

Of the writings on Russell's logic and philosophy only Ayer⁸ and Hylton⁹ are cited. Thus the score or more of us who have been pestering publishers with our book manuscripts on Russell, and the editors of *Russell* and other journals with our papers — all of us are cast aside. The older literature is no luckier; for example among quite a few, the Schilpp volume¹⁰ on the philosophy.

VII

The authors correctly begin with three chapters on logicism and in emphasizing the place of logic in his main philosophical positions. However, the elucidation of their interpretation is compromised by the lack of attention to biography and to influence, the low scale of referencing, and lapses on technical issues and definitions (not all of which were noted above). So, unfortunately this book does not meet the aspirations of the series to provide “clear, concise, and accessible introductions to thinkers, writers, and subjects that students and readers can find especially challenging ... guiding the reader towards a thorough understanding of demanding material” (p. [i]). No fully satisfactory presentation of Russell's philosophy at this level exists; his own *My Philosophical Development* is still the leader.

⁸ AYER, *Bertrand Russell*.

⁹ HYLTON, *Russell, Idealism, and the Emergence of Analytic Philosophy*.

¹⁰ GRATTAN-GUINNESS, *ibid.*, pp. 327–8, 388–91, 592–3.

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