UNCOVERING THE MATHEMATICS BEHIND RUSSELL'S PHILOSOPHY OF MATHEMATICS

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In *My Philosophical Development*, Russell recorded the disappointment that both he and Whitehead felt about the reception of *Principia Mathematica* by the mathematical community. While the philosophical parts of the book, including those parts dealing with philosophical logic, were of course widely discussed and of tremendous importance in the subsequent development of philosophical logic, the purely mathematical aspects of the work went largely ignored. Russell was perhaps exaggerating when he claimed he used to know of "only six people who had read the later parts of the book" (*MPD*, p. 86) but it was certainly true that the impact of the book on mathematicians working in areas outside of the philosophical foundations of their subject was minimal. The situation has hardly changed since the publication of those remarks in 1959. *Principia Mathematica* remains the target of philosophical, not mathematical, attention. In this outstanding new book on Russell's logicism, however, Sébastien Gandon offers a welcome exception to the rule. While the book is still very much a book on Russell's philosophy, its central claim is that new light can be shed on that philosophy by examining the hitherto neglected mathematical parts of both *Principia Mathematica* and *The Principles of Mathematics*.

What parts of Russell's philosophy are to be better understood in light of these forays into the "terra incognita" (p. 2) of Russell's treatment of advanced mathematics in *Principia* and the *Principles*? Gandon's insightful suggestion is that the very notion of analysis at the heart of Russell's logicist project can be grasped in a new and more complete way by reflection on Russell's development of areas of mathematics such as his theories of geometry and quantity. The suggestion is a compelling one, not least because Gandon does a superb job of arguing the case through his own painstaking analysis of Russell's mathematics in the book.

To understand why these seemingly remote parts of Russell's philosophical writings carry such significance, Gandon argues that Russell's logicist project must be understood both in terms of its contribution to philosophy and to

mathematics. Logicism makes a bold assertion about mathematics—that mathematical truths (or, perhaps, some subset of them—e.g. those that exclude geometry in the case of Frege's version of logicism) are nothing more than logical truths. As we know, the logicist project does not rest content with bold assertion, it seeks to prove the assertion. The proof will be a demonstration that every mathematical truth can be translated into a logical truth. Frege's attempt at that demonstration famously failed because it overlooked Russell's paradox. Russell and Whitehead's attempted demonstration may have avoided that pitfall, but it had problems of its own (the axioms of reducibility and infinity) that left many unconvinced of its success. Gandon's project here is not to provide a fresh argument for accepting Russell and Whitehead's demonstration, nor even for accepting the truth of logicism. Rather, it is to examine more closely what the claim made by the logicist is and to subsequently draw a subtle but important distinction between the projects of Frege and Russell, as well as the project which Gandon, more tentatively, ascribes to Wittgenstein.

To illustrate the differences between the three approaches, Gandon invites us to reflect on what kind of analysis of mathematical reasoning is being proposed by the logicist, and suggests that the three positions mentioned above present themselves as three possible outcomes of that reflection. The logicist is faced with an *analysandum*—pre-logicized mathematics—to which the *analysans* must be related in a certain way. One option is to take the *analysans* as effectively replacing the *analysandum* and thus taking the logicist as licensed in revising mathematical theory and practice as she sees fit. This is the position Gandon ascribes to Frege. At the other extreme, the view he ascribes to Wittgenstein, is that *any* revision of mathematical practice would simply count as a failure to capture the *analysandum*. Logicism looks destined for failure on this model (as indeed Wittgenstein seemed to think it was), as it simply proposes too many obstacles to efficient mathematical practice: if the mathematician is reserved the ultimate judgment as to the viability of the analysis, it seems highly unlikely that any analysis on which mathematical practice changes at all will meet her approval. So where does this leave Russell? This, according to Gandon, is where the neglected parts of *Principia* and the *Principles* show their true importance. What they reveal is the extent to which Russell, unlike either Frege or Wittgenstein, was prepared to go repeatedly back and forth between the mathematical content and the philosophical analysis of it, seeing his project as constrained in both directions:

> The Fregean considers that the perspective that one gets *ex post*, once the logical construction is completed, is the only valuable one. On the contrary, the Wittgensteinian claims that the only important standpoint is the one one has *ex ante*, before the process of logicization takes place. For Russell, neither of them is right:

> the two perspectives, *ex ante* and *ex post*, are legitimate. What appears, at one stage of the system, as something which is given from outside … becomes, at a later stage, a part of the logical machinery.… (Pp. 197–8)

It is precisely this feature of Russell's analysis of mathematics that Gandon thinks will remain concealed as long as we continue to ignore the mathematical parts of Russell's work.

Gandon's claim is an important one. No philosopher prized analysis more highly than Russell, and the analysis of mathematics was the most central part of his work. What Gandon has done in this book is point out that a century after Russellian analysis first instigated analytical philosophy as we now know it, there is still much to investigate about what exactly analysis meant to Russell. Everyone interested in Russellian analysis ought to read this book.

WORKS CITED

RUSSELL, BERTRAND. *PoM*, *MPD*.

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