

## REFLECTIONS FROM *PRINCIPIA MATHEMATICA*

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Alexandre Guay, ed. *Autour des Principia Mathematica de Russell et Whitehead*. (Collection Histoire et Philosophie des Sciences.) Dijon: Editions Universitaires de Dijon, 2012. Pp. 168. ISBN 978-2-36441-001-4. €20.00 (pb).

*T*his collection, by several distinguished French philosophers, is intended to be a work on problems which are suggested by or perhaps stem from Whitehead and Russell's *Principia Mathematica*. It turns out to be a somewhat eclectic collection. Some of the papers explicate points in *Principia*, some delve into issues from *Principia* or perhaps suggested by something in it, and

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some into issues suggested by later works of Russell's. The collection was put together for the centenary of *Principia*, with the idea of focusing on "its posterity, the doors it has opened, the questions it has brought up, and the techniques it has initiated" (p. 5).

The essays are all interesting, but in some of them the main focuses are only introduced and are not developed very much, appearing to explicate points already made in detail elsewhere and not adding a great deal that is new. The first essay, for example, gives an overview of the development of Russell's logic, the theory of classes and the account of relations, the antinomies, the theory of descriptions and even the ramified theory of types. While the review is useful, there doesn't seem to be here anything more than what would be found in the previous works of Denis Vernant, Jules Vuillemin, or Philippe de Rouilhan, on which the author draws. Hints of further issues are to be found in the discussion, but they are not developed because of the sheer number of points being reviewed.

Denis Vernant, in the fifth essay, gives a thorough review of Russell's account of similarity of relations and his use of this in relation-arithmetic, mostly from Chapter 6 of *Introduction to Mathematical Philosophy*. Vernant points out that Russell uses the terms "analogy" and "exactly analogous" in several places and thinks of these in terms of his definition of structure. At the end of the essay Vernant begins with Russell's formal account of structure and seeks to give an account of analogy, and specifically metaphor, using the tools of similarity of structure. Again, the discussion of similarity relations is good, but does not really go beyond previous work, and the application to metaphor, while interesting, is very brief.

The sixth essay, by Nadine Gessler and Denis Miéville, discusses Lesniewski's criticisms of *Principia* and gives some hint of Lesniewski's own positions. Lesniewski's work is very interesting and has probably not been given its due. These authors briefly mention Lesniewski's concerns about classes, the assertion sign and definitions, and then give a short presentation of Lesniewski's own views. This last part is again just sketched out.

The authors do point out difficulties Lesniewski had with parts of *Principia*, including the sloppiness about use and mention, the requirement that definitions be non-creative and the theory of types. They also point out that Lesniewski had a view of logic (reminiscent of Arnauld and Nicod) as "the art of thinking", and they discuss his suspicion of sets as abstract entities rather than mereological wholes of the kind which can be grounded in experience. Lesniewski apparently thought of mathematical logic as limiting and not encompassing the whole of thinking. This is curious, because often Russell and Whitehead are accused of expanding logic in order to capture all mathematics as part of logic. Gessler and Miéville accent the differences between Lesniewski's position and *Principia*, but in many ways Lesniewski's views are actually

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closer to those of Whitehead and Russell than to, e.g., the views of Zermelo or of present-day critics of logicism. Lesniewski thought that the theory of types was an artificial way of resolving the paradoxes, but in fact the whole no-classes theory, criticized by Gessler and Miéville, was designed not to postulate typed entities as a way around the paradoxes. From the perspective of *Principia*, it is the axiom system of Zermelo that will seem to be an artificial way of solving the paradoxes, and with this, Lesniewski agreed, although perhaps also because he rejected the whole concept of set as understood by Zermelo. Lesniewski also adopted a theory of types (of properties) in his ontology. Even if the ramified theory is taken to be a theory of symbolism and not of the symbolized, a simple typing of properties or universals is not at all incompatible with *Principia*.<sup>1</sup>

Some of the essays take a topic found in *Principia* or in other works of Russell's and discuss more recent work in the area. In the third essay, for example, François Clementz takes as his starting point the discussion in *Theory of Knowledge* on the question of the direction of a relation and the ontological status of converse relations. Clementz proceeds to draw an interesting connection between Kit Fine's work on neutral relations and Russell's concerns in *Theory of Knowledge*. Clementz in the end rejects Fine's "anti-positionalism" and makes a distinction between two senses of order (p. 77) which allows us to say that in one sense the order of a relation is the same as the order in its converse, although the direction will be opposite.

François Rivenc takes as his starting point the remarks in Appendix C of the second edition of *Principia*. Russell had developed a new foundation for his mathematical logic in the second edition, one based on extensionality, which held that "functions of propositions are always truth-functions and a function can only occur in a proposition through its values" (*PM*, 2nd edn., I: iv). Appendix C addresses the apparent counter-examples to this principle of extensionality in such propositions as "*A* believes *p*" and "*p* is about *A*". Rivenc gives a fairly thorough discussion of the issues and Russell's Appendix C method of solving the problem, namely his distinction between the "factual" and the "assertive" propositions (*PM* I: 666). While Russell stood by this distinction as a key to the solution to the difficulty, he gave the details of his analysis as a tentative proposal, simply showing that one could give an analysis where "*A* believes *p*" and "*p* is about *A*" are not functions of *p*.<sup>2</sup> After

<sup>1</sup> Gregory Landini has argued that Russell has what he calls type\* distinctions. See LANDINI, *Wittgenstein's Apprenticeship with Russell* (2007), p. 57. Bernard Linsky also argued that Russell held that universals were typed. See LINSKY, *Russell's Metaphysical Logic* (1999), p. 33. Both hold that these types are simple.

<sup>2</sup> See *PM* I: 662: "It is not necessary to lay any stress upon the above analysis of belief, which may be completely mistaken. All that is intended is to show that "*A* believes *p*" may very well not be a function of *p*, in the sense in which *p* occurs in truth-

his careful exposition of Russell's remarks, Rivenc moves on to later treatments of this issue, particularly the discussion of opacity in the works of Carnap and Quine and the various issues involved with quantifying into opaque contexts. He even brings in John Perry's concerns with indexical beliefs. The discussion is very thoughtful and ends with a persuasive critique, here sympathetic to Dennett, of "beliefs" as replacements for "judgments" in the earlier epistemology of Russell (and Husserl).

The final article, by Joseph Vidal-Rosset, begins from Russell's remarks in *An Inquiry into Meaning and Truth* on the Law of Excluded Middle.<sup>3</sup> What Russell said over the course of that work was that a strict empiricism, i.e. one which held the verification theory of meaning, was incompatible with the law of excluded middle. Vidal-Rosset sees this remark as in accord with the views of Neil Tennant that the Law of Excluded Middle (if true) is synthetic *a priori*. Vidal-Rosset labels as the "Russell-Tennant Thesis" the thesis that the validity of the law of excluded middle is incompatible with empiricism and consequently a coherent philosophy of knowledge must either renounce classical logic or renounce empiricism. Vidal-Rosset is aware of the radical difference in viewpoint between Russell and Tennant in that Russell, holding something like this thesis (understanding empiricism to include the principle of verification), chose to renounce empiricism while Tennant, also holding a thesis like this, chose to renounce classical logic. While Vidal-Rosset sees Russell's difficulties as an antecedent, his primary concern is a defence of Tennant's position against general attacks on intuitionistic logic. Some of these concerns are mentioned by Russell, but also by Hilbert and Quine. Vidal-Rosset focuses on the criticism that disallowing the law of excluded middle for the mathematician would be (in Hilbert's words) not allowing a boxer to use his fists, and his defence consists in reviewing the arguments which show that whenever a formula is provable in classical logic, a conditional with that formula as a consequent and with the conjunction of all the  $p_i \vee \sim p_i$  for each  $p_i$  that is an atomic formula in that formula as antecedent is provable in intuitionistic logic.

While Vidal-Rosset mentions Quine's concerns about the analytic/synthetic distinction, he doesn't go into Quine's indeterminacy thesis, nor does he discuss Tennant's attacks on this thesis. He doesn't fully explain Tennant's own take on the analytic/synthetic distinction, which includes Tennant's controversial claim that some existence claims are analytic. Russell's own views

functions."

<sup>3</sup> Vidal-Rosset calls this the principle of the excluded third most of the time, but also calls it the Law of Excluded Middle, understanding by this  $p \vee \sim p$ . He sees this as following from bivalence (p. 142). It is important to note that this principle can still be held even if bivalence is denied. ARISTOTLE holds the law of excluded middle in *On Interpretation*, pp. 18b–19a, even though he suggests the future contingent propositions are no more true or no more false than their denials.

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about the analytic/synthetic distinction also notoriously changed, but it isn't clear that simply because Russell held in *Inquiry into Meaning and Truth* that the law of excluded middle conflicted with empiricism that he would have held that it was synthetic *a priori*.

I have a concern about the long passage from Russell which Vidal-Rosset cites in addressing an argument that Russell gave in *Inquiry into Meaning and Truth*.<sup>4</sup> In the passage in question Russell is trying to show that it may be possible for a verificationist not to give up the law of excluded middle. He first allows inductive generalizations from particulars to count as empirical verification, and then argues that a verificationist might thereby extend the law of excluded middle to such statements as "it snowed on Manhattan Island on the first of January 1 A.D." But he then turns to such sentences as "there is a cosmos which has no spatio-temporal relation to the one in which we live." This sentence, Russell said, the verificationist will have to hold as meaningless. Russell himself doesn't hold this, but he says that in this case the verificationist would still be able to hold to the law of excluded middle because the sentence would not be a proposition. Vidal-Rosset seems to mistake the purpose of Russell's argument and seems to attribute to Russell the position that he was attributing to the strict verificationists. Now Vidal-Rosset does think that it is incorrect to attribute the extreme verification theory to intuitionism, and Tennant has certainly not aligned his view with this one, but those philosophers in the heyday of verificationism did hold such a position, and Russell's remarks were directed toward them.

Of all the articles in this collection, the one most closely working with *Principia* and the one most developed in terms of its argument, is the contribution from Sébastien Gandon. This piece is essentially a French version of Chapter 5 of his book *Russell's Unknown Logicism*. In that work Gandon's argument played an important role in his overall position that Russell and Whitehead developed a topic-specific logicism which was sensitive to the concerns and reasoning which occurred in the pre-logicized mathematics. Here Gandon confines himself to showing the virtues of the *Principia* account of quantity over the alternatives. In a somewhat demanding discussion, Gandon outlines the accounts of rational and real numbers in Dedekind, Burali-Forti and Frege (supplemented by the contemporary work of Bob Hale). Gandon then discusses the account in *Principia* and argues that while in one sense the Frege–Hale definitions of the rationals and reals lead more quickly to certain results, the definitions from *Principia* are "more natural and direct" (p. 52). Gandon's work gives readers a rare look at the theory of quantity developed in Volume III of *Principia*.

<sup>4</sup> The argument occurs on p. 278 of *IMT*<sub>2</sub> (p. 347 of the Norton *IMT*). Vidal-Rosset discusses it on pp. 164–5.

In sum, the essays in this collection contain a wide variety of topics, all of them interesting, and they show that Russell studies, as well as contemporary philosophy of logic, are alive and well in France today.

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