# LOGICISM BEYOND *PRINCIPIA MATHEMATICA*

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William Demopoulos. *Logicism and Its Philosophical Legacy*. Cambridge: Cambridge U. P., 2013. Pp. xii, 272. ISBN: 9781107029804. £60.00; US$104.99 (hb).

This book brings together eight previously published essays along with three new essays and a brief introduction. In one way or another, each essay pursues either logicism or some broader implication of logicism that a major figure like Russell or Carnap explored. In a narrow sense, logicism is just the position that grounds the concepts and claims of arithmetic (or all of mathematics) in logic. However, in Demopoulos' hands, logicism becomes a project of much greater significance. A note added to Chapter 5, the classic 1985 article "Bertrand Russell's *The Analysis of Matter*" (co-authored with Michael Friedman), indicates this broader scope and Demopoulos' own take on logicism. Originally Demopoulos and Friedman had concluded their article with both the observation that there are serious "intellectual tensions produced by logicism's attempt to account for both pure mathematics and applied mathematics (mathematical physics)" and the pessimistic conclusion that "it appears that we can account for the distinctive character of the one only at the expense of the other" (p. 107). Now Demopoulos indicates that even though "something close to [this] is true of Carnap's Ramsey-sentence reconstruction of the language of science", "the four previous chapters are an extended argument against" the earlier conjecture. For Demopoulos, the main problem is to make sense of both pure and applied mathematics in something like the original logicist form found in Frege, Russell and Carnap. The solution to this problem arises only when we have pinpointed exactly where these logicist projects failed and how their failures can be overcome.

Chapter 1 focuses on Frege and his logicism for arithmetic. Demopoulos' main contention is that Frege aims to reduce arithmetic to logic in order to demonstrate the autonomy of arithmetic from geometric intuition and empirical experience. Frege's worry about intuition is not motivated by scepticism or by a concern about the cogency of our knowledge of arithmetic (p. 11). The point of logicism is to free arithmetic from the obscurity associated with intuition and to get a clearer grasp of the generality that is characteristic of arithmetic. (Chapters 8 and 9 develop this point in more detail and use it to motivate a brand of logicism for arithmetic that is taken to improve on current neo-Fregean approaches.) For this reason Hume's Principle takes a central role in Frege's reconstruction of arithmetic for it not only establishes the autonomy of arithmetic, but also secures the core use of arithmetic in application, i.e. in counting (p. 19). Unlike Dummett and other commentators Demopoulos does not think that this aspect of Frege's logicism is flawed. Instead, flaws arise only with the accounts of applied mathematics offered by later figures like Russell and Carnap. The root of these problems is that they extend a particular logicist strategy to other areas of mathematics, such as geometry, and to our scientific knowledge more generally.

Chapter 2 considers "Carnap's Thesis", which Demopoulos summarizes as "the assertion that certain applied mathematical theories are not *factual*" (p. 28). Although the thesis is clearly central to Carnap throughout his career, Demopoulos ultimately argues for a non-Carnapian basis for the factual/non-factual distinction. The root of the difference is the kind of criteria of identity that are appropriate for the objects at the heart of the theory:

> … the factuality of an applied mathematical theory will be shown to turn on the recognition that the criteria of identity appropriate to its objects are *empirically constrained* in a way that the criteria of identity appropriate to the objects of a non-factual theory are not. (P. 28)

For arithmetic, the natural numbers associated with two sortal concepts are identical just in case the objects that the concepts apply to can be completely paired up in a one–one fashion. That is Hume's Principle. It says not only what the numbers are, but also establishes their autonomy from empirical contingencies. In applications of geometry, however, the situation is quite different. Demopoulos presents what he calls Einstein's "analysis of time" as a kind of analogue to Frege's analysis of natural number (p. 42). The former analysis can be seen as involving a criterion of identity for the times of occurrence of events. Here there is also an equivalence relation, namely the relation of simultaneity. Following Einstein, Demopoulos emphasizes the empirical constraints that are involved in this criterion of identity. The nature of light and signalling processes turn out to be central to Einstein's analysis. This shows that the applied geometry of special relativity is factual in a way that applied arithmetic is not.

This is a striking proposal that has immediate implications for a number of debates about scientific knowledge. Demopoulos' main argument for this emphasis on criteria of identity is that many of the alternatives explored by others are unable to recover a sufficiently robust factual/non-factual distinction. It appears that the prime example of this problem is Russell's 1927 *Analysis of Matter* and the problem raised by Newman in his 1928 *Mind* article. Russell's epistemic structural realism maintains that "of 'percepts' we know *both* their quality and structure (where Russell's use of the term 'quality' includes relations), while of external events we know only their structure" (p. 93). This structure is limited to "what can be expressed by mathematical logic" (quoted at p. 92). Newman pointed out that this undermines the substantial character of our knowledge of the physical world. For, subject to some cardinality constraints, any structural relation ascribed to the physical world can be shown to be satisfied (pp. 96–7). Newman's construction assumes that the experienced relations of percepts play no role in constraining the admissible relations that are taken to obtain in the physical world. As a result, Russell's analysis is deemed a failure as it eliminates any substantial knowledge of the physical world (beyond some restrictions on its cardinality). Demopoulos (writing with Friedman) take these limitations to exhibit one way that an overly logicist, structural approach to applied mathematics can fail: "despite its intention, Russell's structuralism collapses into phenomenalism" (p. 100).

It is not immediately clear how Russell's structuralism relates to his logicism or how the failures of Russell's structuralism illuminate the general problem of applied mathematics that motivates Demopoulos. Russell, at least, saw Newman's problem as the result of a mistake in how he had presented his views. Writing to Newman in 1928, Russell says: "I had always assumed spatio-temporal continuity with the world of percepts" (quoted at p. 101). This would block Newman's construction if Russell required that some experienced relation between percepts was used to define the known relations between other events. This appears to be Russell's later practice in places like *Human Knowledge*.[1] Demopoulos may think that this sort of move would sacrifice the features of structuralism that make it promising. In particular, it is

[1] This issue is pursued in PINCOCK, "Carnap, Russell and the External World" (2008),

difficult to see how one could know that any "continuity" obtained between what we experience and what goes on in the physical world. Any account of scientific knowledge that simply assumed this would be illegitimate.

Carnap's philosophy of science offers other examples of the pitfalls of an overly logicist approach to applied mathematics. In Chapter 7 Demopoulos develops an interesting reconstruction of Russell, Ramsey and Carnap that concludes that Carnap's mature philosophy of science resolves an often missed opposition between Russell and Ramsey (p. 142). If we divide the vocabulary of a language into its observational and theoretical parts, then we can isolate the consequences of a theory T that involve only the observational terms. This set of "$L_O$-consequences" captures the observational content of T. Demopoulos points out a potential gap that can arise for some theories between the models that satisfy its $L_O$-consequences and the models that satisfy the Ramsey sentence for T.[2] It may be the case that the Ramsey sentence for T has models only when we extend or add to the domain of the models of the $L_O$-consequences. These theories then involve an illegitimate extrapolation from the observable. By contrast, a theory that satisfies what Demopoulos calls "Ramsey's principle" would have models of its Ramsey sentence that require only models with the same domain as the models of its $L_O$-consequences (p. 155).

On this reconstruction, Ramsey treats theoretical claims as "merely effecting a more tractable representation of our observations" (p. 160). Carnap allows theories that violate Ramsey's restrictions, and so in this respect he returns to the full-blown structural realism that Russell intended. But unlike Russell, Carnap handles any extensions of the domain by using "the mathematical background of our linguistic framework" (p. 161). This is a "deflationist interpretation" (p. 160) of the role of theoretical terms that sacrifices the substantial knowledge that would distinguish a realist interpretation from a merely instrumentalist attitude. The viability of this attitude towards scientific knowledge is considered in more detail in Chapter 4 of this volume. Demopoulos argues that Carnap's mature approach is not able to handle cases like the atomic hypothesis, which turned out to be "susceptible to definitive resolution by empirical and analytic methods" (p. 83). The upshot is that all three versions of structuralism found in Russell, Ramsey and Carnap sacrifice too many of our commitments about scientific knowledge. The structuralist thesis

pp. 121–2. In particular, I note that Russell's account of the space-time order in *Human Knowledge* uses the relation of compresence that is found in experience (*HK*, pp. 329–30).

2 The Ramsey sentence for T results from taking a finite axiomatization of T, replacing each theoretical predicate with a second-order variable, and prefixing an existential quantifier for each such variable. So, if "$T(F, G)$" is a finite axiomatization of theory T with theoretical predicates $F$ and $G$, then T's Ramsey sentence is "$\exists X \exists Y\ T(X, Y)$".

that "The theoretical component of what our theories express is wholly captured by statements which depend only on the logical category of their constituent concepts" (p. 161) is thus rejected.

Throughout Demopoulos is less interested in exploring the textual details of a philosopher's positions, and more focused on the conceptual links and insights that these positions lead to. Clearly, the failings of the implementations of the structuralist thesis that Demopoulos considers do not show that Demopoulos' alternative is correct. It remains to be seen if a more direct argument is available that would justify the focus on different sorts of criteria of identity. There is only the beginning of this sort of argument in this volume, with brief considerations of simultaneity (pp. 36–9, 137–9) and Gupta's version of empiricism (pp. 164–8). One hopes that the essays collected here will soon be supplemented with a new volume that would deepen and extend Demopoulos' approach to applied mathematics.

Many readers of this journal will wish to pay special attention to Chapter 10, which reprints the 2007 article "The 1910 *Principia*'s Theory of Functions and Classes" with a new appendix on a paradox for propositions and its importance. Demopoulos provides a helpful reconstruction of *Principia*'s theory of propositional functions and classes that emphasizes its epistemic aims, as opposed to the metaphysical goal of eliminating classes: "Except for finite classes, our knowledge of a class cannot consist in knowledge of its members but must appeal to a propositional function which the members of the class all satisfy" (p. 218). This leads Demopoulos to present a number of principles concerning propositional functions that are jointly sufficient to derive versions of Russell's vicious circle principle that can motivate the full, ramified theory of types.

As a philosophy major at the University of Western Ontario in 1995 I was fortunate enough to enroll in Demopoulos' history of analytic philosophy class, where we considered Frege's *Foundations of Arithmetic*, Russell's *Logical Atomism* lectures and Wittgenstein's *Tractatus*. The essays in this volume preserve the intensity and commitment to rigorous argumentation that I first encountered in that class twenty years ago. Much of my own philosophical work since then has struggled with the relationship between pure and applied mathematics. Like Demopoulos, and Russell before him, I remain convinced that this question has significant implications for scientific knowledge more generally. It is a great pleasure to have the opportunity to note these intellectual debts, and I look forward to Demopoulos' next contribution to these important debates.

## WORKS CITED

NEWMAN, M.H.A. "Mr. Russell's Causal Theory of Perception". *Mind* 37 (1928):

137–48.

Pincock, Chris. "Carnap, Russell and the External World". In M. Friedman and R. Creath, eds. *The Cambridge Companion to Carnap*. Cambridge: Cambridge U. P., 2008. Pp. 106–28.

Russell, Bertrand. *AMa*. *HK*.

Whitehead, A.N., and B. Russell, *PM*.