

# RUSSELL AND WITTGENSTEIN ON INCONGRUENT COUNTERPARTS AND INCOMPATIBLE COLOURS

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Russell (in *Principles of Mathematics*) and Wittgenstein (in *Tractatus Logico-Philosophicus*) largely agree on the twin questions of why pairs of congruent objects cannot always be made to coincide and why surfaces can never be uniformly two colours at once. Both philosophers take space and colour to be mathematically representable, construe the relevant impossibilities as mathematical and hold that mathematical impossibility is at root logical. It is not by chance that Russell says nothing about the phenomena in his Introduction to the *Tractatus* or surprising that Wittgenstein was unmoved by the objection that his account of colour incompatibility puts paid to his early philosophy.

**P**hilosophers have long been puzzled by the impossibility of making all similarly shaped objects (for instance right-hand and left-hand gloves) coincide and the impossibility of surfaces of one colour all over being a second colour all over at the same time (green as well as red, for instance). Not many have felt comfortable treating such “incongruent counterparts” and “incompatible colours” as excluded merely as a matter of fact, as empirically impossible in the sense that human beings over twenty metres tall are empirically impossible. And few have managed to regard the phenomena as excluded as a matter of pure reason, as logically impossible in the sense that people who are both tall and short are logically impossible. Nor has it been generally accepted that incongruence of (some) counterparts and incompatibility of colours are synthetic a priori, the idea of knowledge about the world obtained prior to examining what’s there being hard to swallow. Here I consider Bertrand Russell and Ludwig Wittgenstein’s discussions of the two impossibilities, discussions the

importance and interest of which are mostly unnoticed or misunderstood.

It is underappreciated, in fact hardly appreciated at all, that Russell and Wittgenstein offer remarkably similar explanations of why the thumbs of a pair of gloves placed down on a table invariably point in different directions and why surfaces that are uniformly red cannot at the same time be uniformly green, blue or any other colour. There is more than a hint of Russell's treatment of the incongruence of counterparts and the incompatibility of colours in *The Principles of Mathematics* in Wittgenstein's treatment of the phenomena in the *Tractatus Logico-Philosophicus* (*PoM*, pp. 417 and 467, *TLP* 6.36111 and 6.3751).<sup>1</sup> While disagreeing on many points and expressing themselves differently, Russell and Wittgenstein agree that the phenomena are real and require explanation. Moreover and more strikingly, when set side by side, their explanations turn out to run, a few minor differences aside, along the same lines and stand and fall pretty much together. Indeed it is difficult to shake the impression that Wittgenstein, a notably careful reader of the *Principles*, restates in the *Tractatus*, even appropriates, what Russell had earlier written about the phenomena.<sup>2</sup>

Russell and Wittgenstein both hold that the incongruence of counterparts and the incompatibility of colours pose no special problem. In the case of incongruent counterparts, Russell writes:

In three dimensions, a curious fact has to be taken account of, namely, the disjunction of right and left-handedness, or of clockwise and counter-clockwise.... In this fact, however, there seems, to my mind, to be nothing mysterious, but merely a result of confining ourselves to three dimensions. In one dimension, the same would hold of distances with opposite senses; in two dimensions, of areas. (*PoM*, pp. 417–18)

And Wittgenstein in a similar vein writes:

The Kantian problem of the right and left hand which cannot be made to cover one another already exists in the plane, and even in one-dimen-

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<sup>1</sup> RUSSELL, *The Principles of Mathematics*, 2nd edn. (1937) and L. WITTGENSTEIN, *Tractatus Logico-Philosophicus* (1932).

<sup>2</sup> Compare G. LANDINI, *Wittgenstein's Apprenticeship with Russell* (2007), pp. 86–8

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sional space.... A right-hand glove could be put on a left hand if it could be turned around in four-dimensional space. (TLP 6.36111)

(In this connection Wittgenstein points out that it is impossible to make “o—x” and “x—o” in “---o—x--x—o----” cover one another without moving into two dimensions.) What both philosophers are underlining is that incongruent counterparts cannot be made to coincide because their congruence is linked to the dimensions of the space in which they are located.

As for the incompatibility of colours, Russell and Wittgenstein again see less of a problem than other philosophers. Russell writes:

The most fundamental characteristic of matter lies in the nature of its connection with space and time. Two pieces of matter cannot occupy two places at the same moment, and the same piece cannot occupy two places at the same moment.... By these properties, matter is distinguished from whatever else is in space. Consider colours for example: these possess impenetrability, so that no two colours can be in the same place at the same time. (PoM, p. 467)

And Wittgenstein for his part writes:

For two colours ... to be at one place in the visual field, is impossible, logically impossible, for it is excluded by the logical structure of colour. Let us consider how this contradiction presents itself in physics. Somewhat as follows: That a particle cannot at the same time have two velocities, *i.e.* that at the same time it cannot be in two places, *i.e.* that particles in different places at the same time cannot be identical. (TLP 6.3751)<sup>3</sup>

While Russell refers to properties that colour shares with matter and Wittgenstein speaks of “the logical structure of colour”, both take what Russell calls a “fundamental characteristic” to exclude the possibility of more than one colour at a place at a time.

Russell’s discussion of incongruent counterparts and incompatible colours seems to have been given a pass, but not Wittgenstein’s. His

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<sup>3</sup> The reference to the visual field is inessential. In the material that served as the source for the remarks in the *Tractatus*, WITTGENSTEIN speaks of the impossibility of a point being simultaneously red and green. See *Notebooks 1914–1916*, 2nd edn. (1979), p. 81. No doubt the revision was made to focus the discussion and preempt petty criticism.

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discussion of the phenomena has been severely panned. For instance, in a major study of Wittgenstein's philosophy, Robert Fogelin complains that the material on right- and left-handed gloves in the *Tractatus* is "just awful" and the material on the simultaneous occurrence of colours involves a "remarkable" lapse, explicable "only with reference to Wittgenstein's vaunting confidence that the truth of his thoughts was 'unassailable and definitive'".<sup>4</sup> As Fogelin sees it, Wittgenstein misses the boat since he "solves the problem of incongruent counterparts by denying that the counterparts are congruent" and introduces incompatible colours, "[t]he most famous counter-example to his thesis that the only necessity is logical necessity", as "an *illustration* of the thesis that the only necessity is a logical necessity".<sup>5</sup> How reasonable is this? Are Wittgenstein's remarks as ham-fisted as Fogelin suggests and does Russell—insofar as he anticipates Wittgenstein—mishandle the business just as badly?

Wittgenstein does, it is true, say two incongruent lines in one-dimensional space are "congruent figures" and state that "[t]he right and the left hand are in fact completely congruent" (*TLP* 6.36111). This does not, however, by itself show he begs the question and assumes the problem can be skirted by offering "an alternative definition of congruency".<sup>6</sup> More charitably interpreted, he is referring to "geometrically similar figures" when he speaks of "congruent figures [*kongruenten Figuren*]" and is taking "completely congruent [*vollkommen kongruent*]" to mean "absolutely similar" (*TLP* 6.36111). The reason he thinks the impossibility of a left and a right hand covering one another "has nothing to do with it" is that such similar figures "cannot be made to cover one another without moving them out of [the space they are in]" and "[a] right-handed glove could be put on a left hand if it could be turned around in four-dimensional space". While sketchy and in need of elaboration, this line of argument cannot be reasonably rejected out of hand. It is scarcely a "just awful" idea that the "Kantian problem" can be solved by noting that a right-hand glove could be put on a left hand in four-dimensional space.

When it comes to incompatible colours, Wittgenstein is again on firmer ground than Fogelin—along with many others before and

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<sup>4</sup> R. FOGELIN, *Wittgenstein* (1987), pp. 90, 92.

<sup>5</sup> *Ibid.*, pp. 90–1.

<sup>6</sup> *Ibid.*, p. 90.

since—believes. The problem is not introduced as an illustration of logical necessity but held to be solved by noticing that “two colours ... to be at one place in the visual field ... is excluded by the logical structure of colour”. Nor does Wittgenstein suggest, as Fogelin also claims, that “the necessity of colour incompatibility as a *logical* necessity” can be shown by providing a physical analysis of colour.<sup>7</sup> Had Wittgenstein believed this, he would have left himself open to the charge of putting off the evil day, spatial incompatibility and temporal incompatibility being no less problematic than colour incompatibility. But all he actually says is that similar contradictions occur “in physics” (*TLP* 6.3751). To observe, as he does, that colour incompatibility is comparable to spatial incompatibility and temporal incompatibility is not to say that colour incompatibility is analysable in terms of velocities and positions.<sup>8</sup> As in the case of his examination of the incongruence of counterparts, more discussion would be welcome, but what he says is not terribly and obviously inadequate.

Instead of jumping to the conclusion that Wittgenstein’s explanations of incongruent counterparts and incompatible colours fail miserably, we would do better, surely, to consider why he devotes so few words to them and comments on them at the end of the 6.361s and 6.37s in remarks of low “logical importance” (*TLP*, p. 31). The most plausible answer is that he did not see himself as having much to add to what Russell had already said in the *Principles*. Given the similarity of their views—and the point, stressed by Gregory Landini, that “[i]n reading the *Tractatus* it is essential to keep in mind that it assumes the viability of many of Russell’s analyses—if only in broad outline”<sup>9</sup>—it is a good bet that Wittgenstein included the passages because he thought he could account for the phenomena more or less as Russell had accounted for them. Nor should it be overlooked that Russell, a philosopher with a nose for logical error second to none, passes over Wittgenstein’s treatment of the two impossibilities in silence in the Introduction that he wrote for the *Tractatus*. Since he did not hesitate to criticize Wittgenstein on many points in the book (some quite small), how likely is it, I wonder, that he would have refrained from

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<sup>7</sup> *Ibid.*, p. 91.

<sup>8</sup> Compare M. and J. HINTIKKA, *Investigating Wittgenstein* (1986), pp. 125–6.

<sup>9</sup> LANDINI, p. 86 (mentioned in connection with Wittgenstein’s remarks about incongruent counterparts and incompatible colours).

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commenting on the discussion of incongruent counterparts and incompatible colours were it as dreadful as Fogelin alleges?

Mindful of how closely Wittgenstein's account of the phenomena in the *Tractatus* resembles Russell's account of them in the *Principles*, Gregory Landini is less quick to criticize what Wittgenstein says. As he reads the relevant remarks, Russell and Wittgenstein were pursuing "an eliminativistic program of conceptual analysis", one that promotes "elimination" over "reductive identity" (Landini, pp. 14, 86). In his view Russell reconceptualizes congruence "within [his] new research program of logical analysis" and "Wittgenstein's discussion of Kant on incongruous counterparts speaks loudly as an endorsement of Russell's eliminativism, whose intent is precisely to offer a reconstruction of the notion of congruence" (pp. 86–7). In addition, Landini maintains that "[a] similar, though less compelling case can be made with respect to Wittgenstein's discussion of colour incompatibility", the *Tractatus* being "committed" to the "formidable task" of finding "an eliminativistic analysis of the ordinary notion of 'colour' ... if only at the limit of scientific inquiry" (p. 87). Part of this seems right, part questionable. There is much to be said for the suggestion that "Wittgenstein's position in the *Tractatus* [regarding the Kantian problem] is simply borrowed from Russell" and "an examination of Russell's *Principles* [regarding colour incompatibility] again reveals Wittgenstein's debt" (pp. 86–7). Much less certain is whether Russell and Wittgenstein were in the present instance engaged an eliminativistic project.

While saving Russell and Wittgenstein from the charge of failing to notice the obvious, Landini's interpretation of their treatment of incongruent counterparts and incompatible colours labours under the difficulty that they are more aptly described as analysing notions than as doing away with them, as clarifying rather than eliminating. Regarding the problem of incongruent counterparts, it is questionable whether Russell thinks "the ordinary conception of congruence used by Euclid, Kant, and others" should be eschewed and a "reconceptualization" provided within a new eliminativistic programme (*ibid.*). He is more straightforwardly read as aiming to dispel confusion about congruence rather than as offering a "new definition" (p. 87). Far from suggesting that "Kant's discussion of incongruent counterparts relies upon an erroneous importation of ordinary language notions of superposition and motion into the properly *metrical* notion of

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congruence” (p. 86), Russell confines himself to noting that metrical equality is, while necessary, insufficient for congruency (*PoM*, p. 418). The gist of his analysis is that congruence is not metrical equality since it requires the possibility of moving one figure onto the other, i.e. it goes beyond having the same shape since it involves “motion and superposition” (*ibid.*).

And it is questionable, too, whether Russell meant to eliminate, as opposed to analyse, the concept of colour. To observe that “[t]he most fundamental characteristic of matter lies in the nature of its connection to space and time” and that colour shares one of the properties “commonly attributed to matter” (*PoM*, p. 467), is to clarify, not to remove (or reduce), and likewise for his observation that two colours cannot be in the same place at the same time because “colours ... possess impenetrability”. Nor does Wittgenstein assume it possible to find “an eliminativistic analysis of the ordinary notion of ‘colour’ which separates out the logical components from the material” (Landini, p. 87). Leaving aside the awkward fact that Wittgenstein asserts that “[a]ll propositions of our colloquial language are ... logically completely in order” (*TLP* 5.5563), he nowhere maintains that “the problem of colour incompatibility [should be tackled] as a problem of the incompatibility of surface reflectance potentials”, still less does he equate the incompatibility with “the impossibility of a particle (photon) having different velocities at the same time” (Landini, p. 87). To the contrary, he works with the concept of a “space of colours [*Farbenraum*]” (2.0131) and opts for the non-eliminativistic (and non-reductive) view that colour incompatibility is traceable to “the logical structure of colour”.<sup>10</sup>

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<sup>10</sup> I would also question whether Russell and Wittgenstein are in the present context following in the footsteps of Heinrich Hertz, a scientist who “offers an example of a reconstruction of an approach that is eliminativistic” (LANDINI, p. 89). Hertz’s dynamics is discussed in the final chapter of the *Principles* and Newtonian mechanics is treated in the *Tractatus* along Hertzian lines (with Hertz explicitly mentioned more than once). But neither Russell nor Wittgenstein takes congruence and colour to be eliminable on the model of Hertz’s “systematic reconceptualization of Newtonian dynamics” (*ibid.*). To take “the impenetrability of colours”, as Russell does, to be a feature of “the logical structure of matter” is not to take it as “a logical feature of the analytic reconceptualization of space, time and matter” (*ibid.*, p. 88). And even allowing that Hertz employs the “‘picture theory’ ... as an aid in achieving [the] reconceptualization of ‘force’”, how clear is it that Wittgenstein’s treatment of incongruent counterparts and incompatible colours is “an example of a reconstruction which employs picturing as an eliminativistic tool” (pp. 88–9)?

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By way of reinforcing the point that Russell means to clarify rather than eliminate the notions of incongruent counterparts and incompatible colours, it helps to consider how he accounts for them in the *Principles*. When explaining the incongruence of counterparts, he first points out that the fact of incongruence is “descriptive in nature” in the sense that “[b]etween two non-coplanar rays, or between four non-coplanar points taken in an assigned order, there is always one of two opposite relations, which may be called right and left” (p. 417). From this, he then notes, it follows—as a “geometrical consequenc[e]”—that “volumes ... become magnitudes with sign, in exactly the same way in which distances on a straight line have sign when compounded with their sense”. Whence, he concludes, there is “a distinction between two figures whose metrical properties are all identical”, a distinction that is “no more puzzling than the distinction between the [oppositely-directed] stretches  $AB$  and  $BA$ , which are metrically indistinguishable” (p. 418). His thought is that not “all geometrical facts [are] metrical”, and in the case of incongruent counterparts, a necessary condition for congruence—that “there must be a continuous series of equal figures leading from one to the other”—fails to be satisfied. Since “there is no gradual transition” between the counterparts, he would have us notice, “[n]o motion will transform [one into the other]”, and “at some point in the series a sudden jump would be necessary.”

Russell’s discussion of colour incompatibility in the *Principles* is subsidiary to the discussion of matter and correspondingly less straightforward than his discussion of incongruent counterparts. But it is equally clarificatory (rather than “eliminativistic”). What primarily needs appreciating, Russell argues, is that “division of space always implies division of any matter occupying the space, but division of time has no corresponding implication”, these properties being sufficient to distinguish matter “from whatever else exists in space” and following on from this colour as well (*PoM*, p. 467). Given these properties, it is, he notes, impossible for two pieces of matter and two colours to occupy the same place simultaneously, to penetrate one another. In arguing this point, it is important to notice, Russell is “concerned merely with the analysis of rational Dynamics considered as a branch of pure mathematics”, not with “the nature of matter that actually exists” (p. 465). Just as the properties of space, time and matter “involve none of the so-called laws of motion, but only the nature

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of motion itself”, so too the properties of colour are “purely kinematical” (p. 468). “To obtain a material universe, so far as kinematical considerations go, we have only to consider a class of [relations doing service for material points] subject to the condition that the logical product of any two relations is to be null”, a condition that “insures impenetrability”.

There is no mention of any of this in the *Tractatus*, but it is reasonably assumed, as Landini notes, that Russell’s discussion informs Wittgenstein’s thinking. Charitably read, Wittgenstein is restating, albeit in a somewhat garbled form, Russell’s point that congruence requires more than metrical equality and it is “only to those who regard motion as essential to the notion of metrical equality that right and left-handedness form a difficulty” (*PoM*, p. 418). Moreover he is best understood as adopting Russell’s views about colour incompatibility with the proviso that the phenomenon is due to the nature of colour. As he says in an ancestor of the remark in the *Tractatus*: “[T]he very language of physics reduces [the logical impossibility of a point being red and green at the same time] to a kinetic impossibility”.<sup>11</sup> While conceding that “the fact a particle cannot be in two places at the same time does look more like a logical impossibility [than the fact that a point cannot be red and green at the same time]”, he insists that the sole difference is that the one follows from “the structure of space and of particles”, the other from the “difference of structure of red and green”.<sup>12</sup>

Pivotal to how incongruent counterparts and incompatible colours are discussed in the *Principles* and the *Tractatus* is the idea that the required explanations are mathematical in nature. Russell and Wittgenstein hold that the possibility of incongruent shapes coinciding and surfaces having two colours all over at the same time is ruled out by principles of mathematics in much the same way that Euclidean triangles with interior angles adding up to  $170^\circ$  are ruled out by principles of geometry. They are at one in viewing the problems as problems of mathematical physics, and the full force of their remarks emerges only when considered in the context in which they state their

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<sup>11</sup> *Notebooks 1914–1916*, p. 81. There is no suggestion here that colour is subject to a reductive analysis in terms of position, velocity and the like. WITTGENSTEIN is not stating or presupposing the possibility of a “reductive identity”. Also compare the original German: “[S]chon die physikalische Ausdrucksweise reduziert sie zu einer kinetischen Unmöglichkeit.”

<sup>12</sup> *Ibid.*

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solutions. Each broaches the problems in the course of discussing mathematical physics, the strategy of the one no less than the other being to show that the impossibilities follow as “geometrical consequences” when the phenomena are construed in the way that mathematical physicists construe space, time, position and velocity. It cannot be accidental that Russell explains the phenomena in Parts VI and VII of the *Principles* (which are devoted to “Space” and “Matter and Motion”) and Wittgenstein explains them in the part of the *Tractatus* devoted to physical theory (and Newtonian mechanics).

To state the point another way, Russell and Wittgenstein proceed on the assumption that mechanics (and other physical theories) “determine a form of description” (*TLP* 6.341). They regard shape and colour in the time-honoured manner favoured by mathematical physicists as mathematically representable and argue that incongruent counterparts and incompatible colours are excluded by the relevant mathematical representations. In this regard they are in agreement with Hertz, for whom “[t]he subject matter [of physics]” in Book I of *Principles of Mechanics* is “completely independent of experience”.<sup>13</sup> They treat matter, space, motion and colour as independent of how the world happens to be, think of the formal as circumscribing the actual and take the impossibilities of pure mathematics to reappear as impossibilities about how things are when “we pass from pure to applied mathematics” (*PoM*, p. 8). It is, they hold, the underlying mathematics that precludes the possibility of right-hand gloves covering left-hand gloves and the possibility of red points being green at the same time. The incongruity of counterparts (whether one-, two- or three-dimensional) and the incompatibility of colours (as well as the incompatibility of positions and velocities) are, they jointly maintain, traceable to what Hertz variously refers to as a “mode of expression” or “mode of representation”.<sup>14</sup>

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<sup>13</sup> HERTZ, *The Principles of Mechanics Presented in a New Form* (2007), p. 45. Compare Isaac Newton’s concern with “principles not philosophical but mathematical” in Books I and II of *Principia* (1934), p. 397. None of this gainsays the point in n.10 that Russell and Wittgenstein treat incongruent counterparts and incompatible colours differently from how Hertz treats force. In this connection it should be recalled that in the *Tractatus* Wittgenstein takes qualities and quantities to be representable by points in abstract mathematical spaces. Compare: “A speck in the visual field ... has, so to speak, a colour space around it” (*TLP* 2.0131).

<sup>14</sup> HERTZ, pp. 9, 24.

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It should now be clear that Russell and Wittgenstein had good reason to observe that incongruence arises in one (and two) dimensions as well as three and that colour incompatibility is comparable to position and velocity incompatibility. They were not benightedly imagining that their mentioning these other cases solved the problems. Rather they alluded to them, I take it, because they believed that anyone who reflects on the simpler cases, cases that are more obviously mathematical in nature, will take the more complicated cases in stride (and see that there is “nothing mysterious” about them). What a consideration of the simpler cases reveals, they presumably thought, is that the impossibilities are no stranger—and no more worrisome—than mathematical impossibilities, in fact are to all intents and purposes mathematical impossibilities. Both note that the concepts of incongruent counterparts and incompatible colours are no more problematic than the concepts of oppositely signed lines in one dimension and particles located at two points, concepts that are in turn no more problematic than clear-cut mathematical impossibilities—for instance, the concept of an even prime number greater than two or the concept of a regular plane figure with just two interior angles.

The burden of the discussion up to now has been that Russell and Wittgenstein treat incongruent counterparts and incompatible colours as mathematical impossibilities, not as impossibilities originating in how things happen to be or how the mind happens to operate. Even granting all this, however (and conceding that much of the criticism that has been or might be levelled against them misfires), there is an important hurdle still to be overcome—the claim being that the impossibilities are logical, not mathematical. Russell and Wittgenstein both take the occurrence of incongruent counterparts and incompatible colours to be, as Wittgenstein says about colours at the same place in the visual field simultaneously, “impossible, logically impossible” (*TLP* 6.3751). (Also compare Wittgenstein’s further claim in the same remark that “[t]he assertion that a point in the visual field has two different colours at the same time, is a contradiction”.) It thus remains to be explained how the impossibilities can be regarded, as Russell and Wittgenstein regarded them, as logical and why it is a mistake to complain, as Fogelin complains, that Wittgenstein slyly treats non-tautologous necessary truths as tautologous.<sup>15</sup> Otherwise put, how

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<sup>15</sup> FOGELIN, p. 90.

could Russell and Wittgenstein think of themselves as justified in taking mathematical impossibility to be reducible to logical impossibility and continue to construe representation in terms of logical principles as well as mathematical spaces?

Russell's thinking is easily appreciated, a central theme of the *Principles* being the reducibility of mathematics to logic. He worked long and hard to demonstrate that mathematical formulae can be expressed as logical formulae and mathematical theorems proved from principles of logic. It is only worth adding that he claims to establish the reducibility of rational dynamics along with the reducibility of mathematics itself. Indeed, in the part of the *Principles* in which he discusses incongruent counterparts, he observes that "the definition of a kind of space is always possible in purely logical terms [without] new indefinables" (p. 436). For him "the abstract logical method ... [enables] us to define all the classes of entities which mathematicians call spaces, and to deduce from the definitions all the propositions of the corresponding Geometries" (p. 461). Moreover when summarizing his argument at the end of the book, he claims to have shown that "pure mathematics (including Geometry and Rational Dynamics) can be derived wholly from the indefinables and indemonstrables [referred to at the beginning of the book]" (p. 497) and that "the purely logical nature of mathematics is established throughout" (p. 498).

How Wittgenstein understands the relationship of mathematics to logic is less clear. He is commonly read as holding logic is categorically different from mathematics since logic comprises tautologies and mathematics equations (*TLP* 6.1 and 6.2). There is, however, reason to think he agreed with Russell and regarded mathematical impossibilities as logical impossibilities. In 1916, when he drafted the remarks about colour incompatibility (the remarks on incongruent counterparts came later), he had not drafted the remarks on mathematics and by all appearances embraced Russell's argument that mathematics is reducible to logic. Only in 1917, when he composed the remarks in the *Tractatus* on mathematics, does he appear to think that the logicist programme requires substantial modification, if not replacement. But even then he is most reasonably understood as assuming that his recently acquired conception of a number as an "exponent of an operation" (6.021) can be smoothly grafted onto what he had earlier written about logic. Rather than shift his ground, he apparently believed it possible to preserve the spirit, if not the letter, of Russell's discussion.

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Though hardly a ringing endorsement of “logicism”, his claim that mathematics is “a logical method” (6.2), “a method of logic” (6.234), suggests he remained convinced that his (Hertzian) mathematical conception of representation goes hand in hand with his (Russellian) logical conception and was still of the view that incongruent counterparts and incompatible colours are logically—as well as—mathematically, impossible.<sup>16</sup>

As I understand the history, then, Russell and Wittgenstein construed shape and colour mathematically, argued that incongruent counterparts and incompatible colours are mathematically impossible, and concluded—given how they viewed the relationship of mathematics to logic—that the two impossibilities are also logical. For Russell the final step in the argument was an immediate consequence of his logicism, for Wittgenstein a less direct but still arguable consequence of his treatment of logic and mathematics. (For present purposes it is neither here nor there whether the notion of an operation in the *Tractatus*, essential to Wittgenstein’s account of mathematics and logic, should be regarded as falling in the province of mathematics rather than logic.) Whatever the merits of their arguments, Russell and Wittgenstein plainly regarded incongruent counterparts and incompatible colours as logically—because mathematically—excluded. Small wonder that they proceeded as though they were saying nothing controversial, that Russell lets Wittgenstein’s treatment of the phenomena pass without comment in his Introduction and that Wittgenstein was oblivious to the objection that his discussion of colour incompatibility falls badly short.<sup>17</sup>

None of this is to suggest that Russell and Wittgenstein are in complete agreement regarding incongruent counterparts and incompatible colours. They were temperamentally very different kinds of thinker and their ways of working at philosophical problems poles apart. For

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<sup>16</sup> For further discussion, see my “Wittgenstein on Colour Exclusion”, forthcoming.

<sup>17</sup> The last point is defended in my “Wittgenstein on Colour Exclusion”. In this paper I criticize the usual view that Wittgenstein’s discussion of colour incompatibility was the Achilles’ heel of the *Tractatus*. Wittgenstein’s second thoughts concerning the phenomenon were, I argue, a consequence, not the cause, of the turn his philosophy took in 1929. In opposition to conventional wisdom, I maintain that “Some Remarks on Logical Form” post-dated rather than pre-dated his repudiation of what he had written on the topic in the *Tractatus*. Incidentally Wittgenstein seems never to have returned to the question of the source of the incongruence of pairs of gloves and other such counterparts.

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one thing Russell attended to the technical niceties and discussed the nature of spatial incongruence and colour incompatibility in some depth while Wittgenstein was content here, as elsewhere in his finished writings, to state his conclusions and leave the job of figuring out the details to the reader. In the case of colour incompatibility, for example, Russell specifies features of matter he thinks are generally counted as essential to it and notes that colour shares some of its properties, whereas Wittgenstein simply declares that the simultaneous occurrence of colours is precluded by “the logical structure of colour”. And in contrast to Russell, who is at pains to establish that mathematics and mathematical physics are reducible to logic, Wittgenstein assumes that they are—as he construes them—all of a piece.

Russell and Wittgenstein differed, too, regarding the kind of logical status the incongruence of counterparts and incompatibility of colours enjoy. They agreed that the impossibilities are logical but disagreed on what this amounts to. In the *Principles* Russell states that “logic is just as synthetic as all other kinds of truth” (p. 457) and for many years afterwards took logical propositions to say something, to have assertible content, while in the *Tractatus* Wittgenstein takes logical propositions to “say nothing” and avers that “[t]heories which make a proposition of logic appear substantial are always false” (*TLP* 6.1–6.11). He appears to align himself with Russell when he announces that “[t]he logical propositions describe the scaffolding of the world” (6.124). But he no sooner says this than he backtracks and adds: “or rather they present it” inasmuch as “[t]hey ‘treat’ of nothing”. (Also compare 6.3211: “[T]he a priori certain [always] proves to be something purely logical”.) However much Wittgenstein appropriated from Russell, he considered propositions expressing impossibilities to be *Scheinsätze* (pseudo-propositions), not—as Russell would have it—genuine *Sätze*. In this respect he was further than Russell from Kant, for whom, at one time at least, the existence of incongruent counterparts affords (a priori) evidence for the reality of absolute space.

Allied to this last difference between Russell and Wittgenstein, there is a yet deeper difference. While offering similar accounts of incongruent counterparts and incompatible colours, they do not understand what they have done in the same way (and do not agree on what philosophers can claim to achieve). They take themselves to be engaged in different sorts of task and to have accomplished different

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things. They were fundamentally at odds over the nature of the phenomena themselves and why, contrary to conventional wisdom, they are philosophically inconsequential. For Russell, explaining incongruence and incompatibility is a matter of getting clear about how things are, for Wittgenstein a matter of getting clear about how we speak and think. Whereas Russell takes the mystery of why some counterparts are incongruent and colours incompatible to be dispelled by providing explanations that are as substantive as the natural scientist's explanations, just more abstract, Wittgenstein takes the mystery to be as inconsequential as a linguistic or logical riddle. One could say Russell means to contribute to the acquisition of a philosophical picture of the fundamental features of the world, Wittgenstein to expose a couple of philosophical questions as pseudo-questions.

So I see an examination of Russell and Wittgenstein on incongruent counterparts and incompatible colours as having the twofold benefit of shedding light on Russell's project in the *Principles* and clarifying Wittgenstein's thinking in the *Tractatus*. When their remarks are considered together, one sees more clearly not only what Russell believed he could establish and how he hoped to establish it, but also what Wittgenstein took him to have established and how far he was willing to go along with him. It is not for nothing that Russell seems to have accepted Wittgenstein's remarks about incongruent counterparts and incompatible colours, and that in the Preface of the *Tractatus* Wittgenstein registered his debt to "the writings [*Arbeiten*] of [his] friend Bertrand Russell" and disclaimed "novelty in points of detail". For all their differences, Russell and Wittgenstein had more than a little in common, and it does neither of them any favour to overlook where they come together, to say nothing of encouraging what I take to be the baneful view of Wittgenstein's discussion of colour incompatibility as the point at which the *Tractatus* begins to unravel.<sup>18</sup> Philosophical agreement can be as revealing as philosophical disagreement, and doubly so when top-flight philosophers with profoundly different philosophical outlooks come down on the same side of the fence.<sup>19</sup>

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<sup>18</sup> Again see my "Wittgenstein on Colour Exclusion" for more discussion.

<sup>19</sup> I am grateful to Paul Forster and Marcos Silva for comments on an earlier draft of this paper and to two anonymous reviewers for suggesting improvements.

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