

WELL-ORDERING IN THE RUSSELL–NEWMAN CONTROVERSY

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There is a curious letter of 24 April 1928, reproduced in Russell’s *Autobiography*. It is from Russell to Max Newman. It is my thesis that there is a crucial “not” missing from the text and interpretations of the letter. This small point, if it is correct, has a very large impact for clarifying how Russell saw Newman’s challenge to his *structural realism* according to which all of our empirical knowledge in physics concerns structure alone.

I. INTRODUCTION

Russell’s *structural realism* is the thesis that physics must confine itself to knowledge about the structural features of the world. He articulated this thesis in his book *An Outline of Philosophy* (1927). In *The Analysis of Matter* (1927), he writes:

[W]herever we infer from perceptions, it is only structure that we can validly infer; and structure is what can be expressed by mathematical logic....
(*AMa*, p. 254)

Russell continues:

The only legitimate attitude about the physical world seems to be one of complete agnosticism as regards all but its mathematical properties.
(*AMa*, pp. 270–1)

One of the most neglected features of Russell’s structural realism is that it is part of his four-dimensionalist philosophy of time. It is in this setting that we best understand the origins of his thesis that events are fundamental and “things” are series of events through time.

On Russell’s view, a bit of matter is a series of transient particular

events obeying the laws of the new physics of the 1920s. In *Analysis of Matter*, he puts it as follows:

We shall find, if I am not mistaken, that the objects which are mathematically primitive in physics, such as electrons, protons, and points in space-time, are all logically complex structures composed of entities which are metaphysically more primitive, which may conveniently be called “events”. It is a matter for mathematical logic to show how to construct, out of these, the objects required by the mathematical physicist. It belongs also to this part of our subject to inquire whether there is anything in the known world that is not part of this metaphysically primitive material of physics.... Physics, in itself, is exceedingly abstract, and reveals only certain mathematical characteristics of the material with which it deals. It does not tell us anything as to the intrinsic character of this material. (AMa, pp. 9–10)

The issues driving this conclusion are not those of the ivory tower epistemologist, the Humean empiricist or sceptic; they concern the very nature of the *empirical* science of mathematical physics of the 1920s.

Russell explains that common sense imagines “things” in the physical world, but empirical atomic physics has led to resolving “things” into groups of electrons and protons and the like.¹ He then asks (AMa, p. 244): Are electrons and protons part of the ultimate furniture of the world, or are they groups of events, or causal laws of events? Now Russell points out that one cannot settle this issue by appealing to perceptions of “things” since a group of events arranged about a centre with no substance at the centre “will produce exactly the same percepts; therefore the substance at the centre, if there is one, is irrelevant to science ...” (*ibid.*). Russell goes on to explain that the recent changes in physics, initiated by Einstein, further buttress this conclusion. Let me quote at length:

The substitution of space-time for space and time has made it much more natural than formerly to conceive of a piece of matter as a group of events. Physics starts, nowadays, from a four-dimensional manifold of events, not, as formerly, from a temporal series of three-dimensional

¹ Modern particle physics, with its bosons, quarks and the like, has come a long way from the views of 1927 where the proton and the electron seemed to be fundamental.

manifolds, connected with each other by the conception of matter in motion. Instead of a permanent piece of matter, we have now the conception of a "world line", which is a series of events connected with each other in a certain way. The parts of one light-ray are connected with each other in a manner which enables us to consider them as forming, together, one light-ray; but we do not conceive a light-ray as a substance moving with the velocity of light. Just the same kind of consideration may be held to constitute the unity of an electron. We have a series of events connected together by causal laws; these may be taken to *be* the electron.... (Ibid.)

Russell is clearly a four-dimensionalist and an eternalist in the philosophy of time.² This is playing a central role in his advocacy of structuralism. It is the relations of events that matter to physics, and relations of events are just another way of talking about kinds of structures. Different systems of relations of events are mathematically conceivable, and mathematical physics becomes empirical only when empirical studies are employed to select among viable structures each adequate to the data.

It is important to emphasize that these philosophical points about the nature of physics are not only to be found with Russell. They were corroborated by Einstein as well. Consider the following:

I am convinced that we can discover by means of purely mathematical constructions the concepts and the laws connecting them with each other, which furnish the key to understanding of natural phenomena. Experience may suggest the appropriate mathematical concepts, but they most certainly cannot be deduced from it. Experience remains, of course, the sole criterion of the physical utility of a mathematical construction. But the creative principle resides in mathematics.

... I am compelled to make use of a mathematical conception. The physical world is represented as a four-dimensional continuum. If I assume a Riemannian metric in it and ask what are the simplest laws which such a metric system can satisfy, I arrive at the relativist theory of gravitation in empty space....

At this point, we still lack a theory for those parts of space in which electrical density does not disappear. De Broglie conjectured the existence of a wave field, which served to explain certain quantum properties

² This interpretation of Russell is easily corroborated in his 1924 essay "Logical Atomism" (1924) and in *An Outline of Philosophy*.

of matter. Dirac found in the spinors field-magnitudes of a new sort, whose simplest equations enable one to a large extent to deduce the properties of the electron. Subsequently I discovered, in conjunction with my colleague, that these spinors form a special case of a new sort of field, mathematically connected with the four-dimensional system, which we called “semi-vectors”. The simplest equations which such semivectors can be reduced furnish a key to the understanding of the existence of two sorts of elementary particles, ... and it looks as if they described, in an easy manner, certain essential properties of electrical particles.³

Russell was influenced by the revolutionary ideas in the physics of fields. They led him to be brave enough to offer his *constructive* thesis that the elementary particles themselves may be identified with (eliminated in favour of) processes in the fields. Thus the fundamental idea is that of an event field, and this falls within the general mathematical logic of *kinds* of structures made possible by the way relations order their fields. Which event field best describes the physical world, of course, remains an empirical question. But the point is that a *structural* orientation to physics is fully present in the new relativistic and quantum physics.

In an article published in *Mind* in 1928, M. H. A. Newman posed a challenge to the thesis of structural realism that Russell had defended in *Analysis of Matter*.⁴ There is a letter of 24 April 1928 from him responding to Newman. Russell reprinted it without comment in his *Autobiography*. He explains what he takes to be Newman’s point, namely, that structural realism is either false or trivial since it would make physical knowledge consist only of knowledge of the cardinality of the physical world.

Newman’s paper has been discussed by many authors, and his argument is often likened to Putnam’s “Model Theoretic Argument”, which attempts to challenge Metaphysical Realism (the thesis that even the most ideal and empirically well-tested physical theory can be false).⁵ The model-theoretic argument uses the result that any consistent *first-order* theory has a countable model. Putnam’s idea is then to point out that whatever objects there are in the world, if their

³ EINSTEIN, “On the Method of Theoretical Physics” (1934), p. 19.

⁴ NEWMAN, “Mr. Russell’s Causal Theory of Perception”.

⁵ See PUTNAM, “Realism and Reason” (1977).

cardinality is the same as that of the model of a first-order consistent theory that is ideal, then there is no way that world's objects could fail to be regarded in a mathematical sense as a model of the theory. One cannot justify a protest on grounds that this contrived model in the world's objects is *unintended* since the theory has already exhausted all empirical tests even at the limit of inquiry.

As H. G. Merrill pointed out, Putnam's argument doesn't work—given that we are prepared to accept a conception of *physical* properties and relations.⁶ Not every contrived set of the world's objects (contrived, because it is induced by the one-to-one correspondence between the world's objects and the objects of the model of our consistent and ideal first-order theory of physics) will be acceptable as an extension for the predicates of the language of the theory. After all, not every such set would form the extension of a *physical* property or relation. Of course, Merrill's criticism requires the notion of a physical universal (property or relation), and Putnam finds this notion problematic. What properties and relations are physical, he thinks, can only be a question of what predicates and proper axioms governing them are adopted by our formal first-order physical theory. So we have something of a stalemate.⁷

In any event, the debate between Putnam and Merrill seems orthogonal to issues animating Russell and Newman concerning structural realism. In *Analysis of Matter*, Russell explains that there is no such thing as matter (in the traditional sense). Not even electrons survive in the traditional physical atomistic sense. It is difficult to see how to apply Putnam's idea that the world's "objects" be assigned to numbers which are serving in the domain of a model of our ideal consistent first-order theory of physics. Moreover, Russell didn't demand that physical theories are to be first-order theories. He uses the full apparatus of *Principia's* predicate variables to express his mathematical constructions and to express mathematical physics. Putnam's argument uses a semantic result considering consistent first-order theories which does not apply to theories that are not first-order, and thus does not apply to the simple type theory as found in *Principia*. (Putnam's model-theoretic result that every consistent first-order theory has a

⁶ MERRILL, "Putnam's Model Theoretic Argument against Realism" (1980).

⁷ For a nice discussion see DEMOPOULOS AND FRIEDMAN, "The Concept of Structure in *The Analysis of Matter*" (1977).

countable model does not even apply to a consistent standard second-order theory, which demands that predicates must occur in predicate positions only.) It is far from clear, therefore, that the Putnam argument has any bearing at all on Russell's conception of Newman's objection to his structural realism. In what follows, we shall argue that he did not see Newman's concern as arising from issues pertaining to model theory. We shall find that Russell has a fully adequate response to Newman's concern—as he understood it.⁸

2. RUSSELL'S LETTER TO NEWMAN: A MISSING "NOT"?

The best way to enter into the debate between Russell and Newman is to look very carefully at the letter of reaction that Russell wrote Newman on 24 April 1928. What is curious about the reaction is that Russell seems to agree, at least in spirit, with Newman's concern and expresses his being "ashamed" for not having noticed it for himself. He writes:

Many thanks for sending me the off-print of your article about me in *Mind*. I read it with great interest and some dismay. You make it entirely obvious that my statements to the effect that nothing is known about the physical world except its structure are either false or trivial, and I am somewhat ashamed at not having noticed the point for myself.

It is of course obvious, as you point out, that the only effective assertion about the physical world involved in saying that it is susceptible to such and such a structure is an assertion about its cardinal number. (This by the way is not quite so trivial an assertion as it would seem to be, if, as is not improbable, the cardinal number involved is finite. This, however, is not a point upon which I wish to lay stress.) (*Auto*. 2: 176)

There is more. But we shall return to the remainder of the letter later. Let us first stop here to ponder the meaning of Russell's parenthetical clause and his admission of being "ashamed".

Looking at Russell's parenthetical clause, I want to argue, suggests that there is a missing "not" (or, alternatively, that the proper interpretation of the letter has Russell intending a negative). As he pointed

⁸ It should be noted that Grattan-Guinness edited the complete sequence of four Newman–Russell letters. See GRATTAN-GUINNESS, "Logic, Topology and Physics: Points of Contact between Bertrand Russell and Max Newman" (2012).

out in *Principia Mathematica* (2: 311, 3: 131; see *262.23) and in *Introduction to Mathematical Philosophy* (1919, p. 57), the mathematical theory of kinds of orderings is a matter of relation numbers. This point, mind you, is discussed in some detail in Chapter 24 of *Analysis of Matter* itself. In Russell's view, order is due to relations which organize their fields. Now kinds of structure (kinds of ordering) are given by *similarity* between relations that produce the orderings. The finite cardinality of the field of a relation does not determine the ordering. But when it comes to a *well-ordering* relation and *order types*, the situation is different. Order types are given by similarity between well-ordering relations. A relation well-orders its field when it linearly orders its field in such a way that every non-empty subset of its field has a least element under the ordering. This is of central importance to Russell's letter to Newman. The reason is that when it comes to well-ordering relations, *finite* cardinality of the field *does* determine its order type. In the case where the field of the well-ordering relation has an *infinite* cardinality, the cardinality does not determine the order type.

Now let us assume that in Russell's view, a well-ordering of physical events is required. It follows that if the cardinality of events is finite then the structure (the order type) is given by the cardinality alone. If, on the other hand, the cardinality is infinite, then the structure (the order type) is *not* given by the cardinality alone. With this in mind, Russell's parenthetical clause must be interpreted to have intended a negative. It might be better read as follows:

“This by the way is not quite so trivial an assertion as it would seem to be, if, as is not improbable, the cardinal number involved is *not* finite.”

Admittedly, in Russell's original typed letter, which he seems to have looked over carefully before signing, we don't find any sign of a missing “not”.⁹ But we can, in fact, interpret Russell's intent as negative

⁹ This was pointed out to me in conversation by Kenneth Blackwell. The letter may be found at <http://www.cdpa.co.uk/Newman/MHAN/exhibition-panel.php?Title=Philosophy&Picture=Philosophy.jpg>. Russell corrected the ribbon copy twice for the spelling of “spacio-” (which indicates he dictated the letter to someone unfamiliar with the common spelling of “spatio-”) and drew parentheses around “from one event to another compresent with it”. The *Autobiography* follows the uncorrected carbon copy, which I have emended in light of the ribbon copy.

even without adding the word “not” for clarification. It all turns on the proper reading of his parenthetical remark. Given a well-ordering relation, structure is trivial if its field is finite, and it is not trivial if its field is not finite. The key lies in reading the import of Russell’s counterfactual clause “as it would seem to be”. He is expressing the negative, but expressing it in the *counterfactual* which asserts that structure would be trivial if the cardinality were finite. Either way, it seems clear that what he has in mind is the issue of the cardinality of the field of the relation in question.

Now we can find very strong support for the interpretation that he had this cardinality issue in mind by establishing that in *Analysis of Matter* his structural realism was applied in a way that required a *well-ordering* relation of events. What is of importance for our “missing-not” interpretation of the letter to Newman is whether, in *Analysis of Mind*, Russell holds: (1) There are at least \aleph_0 many events involved, and (2) the relevant events are well-ordered. Admittedly, it is doubtful that Russell held that the universe, *sub specie aeternitatis*, consists of a well-ordered series of events. Perhaps the universe is linearly ordered but not well-ordered. In what way, then, do well-ordering relations become involved?

The answer comes by looking at Russell’s discussion of the construction of points and in particular space-time points. In offering a construction, he notes the following:

The whole of this theory, however, aims only at constructing such properties of space-time as belong to *analysis situs*; everything appertaining to intervals and metrics is omitted at this stage, since causal considerations are required for the theory of intervals. (AMa, p. 401)

Realizing that Russell is a four-dimensionalist helps us to understand his comment that his construction belongs to *analysis situ*. He seems to be considering slices of the universe at a time. He is constructing a grid of points for the configuration of the universe at a given slice of its four-dimensional tube. This is where a well-ordering relation comes into play. Russell needs it for his construction of points. He writes:

A “point” is a co-punctual group which cannot be enlarged with ceasing to be co-punctual.

In order to demonstrate the existence of points so defined, it is sufficient to assume that all events (or at least all events co-punctual with a given co-punctual quintet) can be well ordered. If Zermelo's axiom is true, this must be the case; if not, it may involve some limitation as to the number of events. I have been led by the arguments, first of Dr. H. M. Sheffer, and then of Mr. F. P. Ramsey, to the view that Zermelo's axiom is true; I am therefore less reluctant than I should have been formerly to assume that events can be well ordered. (*AMa*, pp. 299–300)

Contrasting his method of construction with the “enclosures series” of Whitehead's method of extensive abstraction, Russell observes that Whitehead assumes that every event encloses and is enclosed by other events. He rejects Whitehead's construction. In its effort to construct a *continuum* of points, Whitehead's construction *requires* that there be no minimum duration of an event and no maximum duration. Russell held that these are ultimately empirical questions, and he finds that empirical studies suggest that both are false (*AMa*, p. 292). Russell thinks Whitehead's approach is ingenious and logically viable, but it is unnecessary—given we have at least \aleph_0 many events, each of finite duration, and given that we can well-order them. Having set out his construction, he writes:

It follows from this that a *compact* series of points contained within a stretch of collinear points is continuous. It does not follow that there are compact series of points; this would require existence-axioms which there is no object in introducing, since we do not know whether space-time is continuous or not. It is, however, interesting to observe that an initial apparatus of \aleph_0 events suffices to generate a continuous space-time of points, by means of the relations of co-punctuality and logical inclusion. (my italics; *AMa*, p. 311)

Russell adds that the extension of his construction to surfaces, volumes and four-dimensional regions presents no difficulties in principle.

With well-ordering relations squarely in view, we get a clear understanding of how Russell understood the Newman problem—at least at the time of writing his letter. If the physical world (at a time) has a *finite* cardinal number of events, and we apply Russell's *analysis situ* construction of points (which require well-ordering), then its cardinality alone will determine the structure (i.e., the order type). All our

knowledge of its structure would be captured by the cardinality alone.¹⁰ We can now clearly see why Russell would take Newman to be making the point that it may happen that the structure (order type) belonging to *analysis situ* of the universe at a given time is determined by cardinality alone. Moreover, we can fully explain Russell’s parenthetical comment— that if there are infinitely many events in any given state of the universe at a time, then the cardinality does *not* determine the order type (the structure). Indeed, since Russell had assumed \aleph_0 events for his construction of points, his structural realism had already *avoided* the Newman problem (so understood). But this was a point upon which Russell did not want to lay emphasis. This, it seems to me, best explains the content of his letter to Newman.

3. RUSSELL’S RESPONSE TO NEWMAN

As we noted, Russell admits being “ashamed” at having not already seen Newman’s point. It is natural enough for Russell to admit this sense of shame since the relationship between cardinality and order type is one he knew so well. He was, quite naturally, as he puts it, “dismayed” at his not having noticed its relevance to his construction of points within his structural realism—once he has assumed a well-ordering of events for the construction. Russell’s admission of shame is thus fully explained by our thesis of the missing “not”.

In sum, if we fix the missing “not” in Russell’s letter to Newman, we get a better understanding of how Russell interpreted Newman’s objection. We see that the issue of well-ordering was central to Russell’s interpretation of Newman’s objection that his structural realism is a trivial matter of cardinality. There is more in Russell’s letter, however. Our orientation to his conception of Newman’s concern sheds new light on it. The letter goes on to admit that the assumption of infinity is not the only (or most important) response that can be made to Newman’s concern. In the remainder of the letter, Russell writes:

It was quite clear to me, as I read your article, that I had not really intended to say what in fact I did say, that *nothing* is known about the

¹⁰ Amusingly, it is worth noting that it is far from *trivial* to imagine empirical tests which would determine what the cardinality of the universe, or even a region of the universe, is at a given time slice.

physical world except its structure. I had always assumed spatio-temporal continuity with the world of percepts, that is to say, I had assumed that there might be co-punctuality between percepts and non-percepts, and even that one could pass by a finite number of steps (from one event to another compresent with it) from one end of the universe to the other. And co-punctuality I regarded as a relation which might exist among percepts and is itself perceptible.

I have not yet had time to think out how far the admission of co-punctuality alone in addition to structure would protect me from your criticisms, nor yet how far it would weaken the plausibility of my metaphysic. What I did realize was that spatio-temporal continuity of percepts and non-percepts was so axiomatic in my thoughts that I failed to notice that my statements appeared to deny it.

I am at the moment much too busy to give the matter proper thought, but I should be grateful if you could find time to let me know whether you have any ideas on the matter which are not merely negative, since it does not appear from your article what your own position is. I gathered in talking with you that you favoured phenomenalism, but I do not quite know how definitely you do so. (Auto. 2: 176-7)

There are many points here to consider. We shall have to take them in turn, rearranging some of the passages for continuity of content.

We do well to first note that transient particular events, series of which compose states of perceiving minds, are called “percepts” by Russell. Transient particulars that are not in any series that compose perceiving minds are “non-percepts”. Phenomenalism, as Russell defines it, is the thesis that all transient events are percepts. Secondly, our new orientation to the Newman issue which emphasizes well-ordering relations may help to explain what Russell meant when, in his letter to Newman, he goes on to say: “... I had assumed that there might be co-punctuality between percepts and non-percepts, and even that one could pass by a finite number of steps (from one event to another compresent with it) from one end of the universe to the other.” This difficult passage is illuminated by *Analysis of Matter* in Russell’s construction of space-time points. He writes: “If there is a minimum to the size of events, we may assume that it is always possible to pass from one event to another by a finite number of ‘steps’” (p. 304). We find Russell offering a conditional, but in the letter he is confessing that he had assumed its hypothesis. The universe of Einstein’s space-time metric is finite in volume but unbounded, and

thus Russell may have imagined the events in any given space-time slice of the four-dimensional manifold that is the universe to be well-ordered. If this is correct, he is alluding to his assumption of well-ordering in the letter to Newman.

In any case, it is clear that certain passages of the letter belong together because they concern the same subject matter—namely, Russell’s assumption that percept/non-percept connections were, as he put it, “axiomatic in my thought”. Consider again his remark: “It was quite clear to me, as I read your article, that I had not really intended to say what in fact I did say, that *nothing* is known about the physical world except its structure. I had always assumed spatio-temporal continuity with the world of percepts, that is to say, I had assumed that there might be co-punctuality between percepts and non-percepts....” This passage belongs with the following: “I have not yet had time to think out how far the admission of co-punctuality alone in addition to structure would protect me from your criticisms, nor yet how far it would weaken the plausibility of my metaphysic. What I did realize was that spatio-temporal continuity of percepts and non-percepts was so axiomatic in my thoughts that I failed to notice that my statements appeared to deny it.” A look at *Analysis of Matter* corroborates Russell’s comments to Newman. Russell makes what we shall call an *assumption of correlation*, and he is admitting to Newman in his letter that he should have made it more apparent in *Analysis of Matter* that this is itself a bit of empirical knowledge.

It is very important to realize, however, that while Russell accepts that his assumption of correlation is physical knowledge, he holds that it *not* of the sort of knowledge that would significantly undermine his thesis that physical knowledge as to the nature of events must be limited to structural knowledge. The assumption of correlation is neither physical knowledge as to the nature of events nor physical knowledge of the relations in which they stand. Russell admits that he has not had time to think fully about the extent to which this point avoids all concerns Newman might raise. But Russell’s conciliatory comments should not be taken to imply that he was at all worried that they do not *fully* resolve Newman’s objection.

The assumption of correlation in Russell’s *Analysis of Matter* maintains that we know that there are many-one (functional) correlations between event causes (transient event particulars) and percepts. We have only to look at his Chapter 24 on structure in scientific

inference. Russell explains that when two relations have the same relation number, all their mathematical (logical) properties are identical. He notes that “the inferences from perceptions to their causes, assuming such inference to be valid, are concerned mainly, if not exclusively, with logical properties” (*AMa*, p. 151). This assumption of correlation, however, does not give rise to a kind of knowing of the physical world’s objects that is out of sorts with Russell’s structural realism. The important point for our purposes is how inference about the nature (not the existence) of the causes of percepts is limited to structural knowledge alone. This is the case even though the inference is only possible because of an assumed correlation (if only many-one, and not one-one) between percepts and the causes of those percepts.

The relation S of event(s) causing a given percept is known to be many-one (and not one-one) since several non-percepts may cause a given percept, but no non-percept causes two distinct percepts. Even a rudimentary understanding of the physical world, Russell explains, gives rise to the recognition that the same (qualitatively identical) percepts may have different events causing them. (Differing percepts, however, must have differing events causing them.) Thus, the *projective* (if you will) relation S between causing events and the percept caused is only many-one (i.e., it is functional). But its converse cannot reasonably be assumed to also be many-one. (For this reason, one cannot be justified in holding that S is a one-one relation.) One of Russell’s examples concerns the fact that the many distinct events happening on the sun—even seen with a powerful telescope—do not each produce distinct percepts. Several, even infinitely many, distinct events might project onto just one percept. But when two percepts are caused, we may safely assume that each has a separate cause. In consideration of this issue, it is important to point out that in *Analysis of Matter*, there is an odd slip. In a passage in which Russell discusses the relation S , he writes:

If we have xSx' and ySy' where S is many-one, and if y and y' differ, we can infer that x and x' differ....
(*AMa*, p. 255)

This is false. Consider the many-one relation “ x has y as closest friend”. Trump has Trump as closest friend. Obama has Michelle as closest friend. Obama is not Michelle, but obviously it doesn’t follow that Trump is not Trump. What Russell meant to say was this:

If we have xSy and $x'Sy'$, where S is many-one, and if y and y' differ, we can infer that x and x' differ.

This, of course, follows immediately by logic from the fact that S is many-one. The slip, as we are calling it, seems never to have been caught in subsequent editions of the book. This slip cannot be attributed to a printer's error.¹¹ It is in his original manuscript. We find:

It is obvious as a matter of logic that, if our correlating relation S is many-one, not one-one, logical inference in the sense in which S goes is just as possible as before, but logical inference in the opposite sense is more difficult. That is why we assume that differing percepts have differing stimuli, but indistinguishable percepts need not have exactly similar stimuli. If we have $xSx' + ySy'$, where S is many-one, + if $y + y'$ differ, we can infer that $x + x'$ differ; but if $y + y'$ do not differ, we cannot infer that $x + x'$ do not differ. We find often that indistinguishable percepts are followed by different effects - e.g. one class of water causes typhoid + another does not. In such cases

Of course, the slip is wholly innocuous since we can clearly discern what Russell intended to say. The import for the debate with Newman, however, is significant. It makes it clear that S is a relation *between* causing events (non-percepts) and percepts caused. It is not a relation between percepts, nor is it a relation between non-percepts.

Russell admits that we know there is such a relation S and we can know that S is functional, but we cannot assume that the converse of S is functional. This feature of S does not, however, prevent useful inferences in empirical science based on the relation S . Russell admits only that because the converse of S is not functional, it makes inference in physical science *precarious* about causes of percepts. He writes: "It is obvious as a matter of logic that, if our correlating relation S is many-one, not one-one, logical inference in the sense in which S goes [i.e., from events causing percepts in its domain to its range of

¹¹ Special thanks to the Editor for the facsimile of the original handwritten page (fol. 279) of *Analysis of Matter* containing the typo from RA REC. ACQ. I.

percepts caused] is just as feasible as before, but logical inference in the opposite sense [i.e., from percepts in the range to the event causes in the domain] is more difficult" (*AMa*, p. 255). This is the empirical knowledge about (co-punctual) structural nodes that Russell admits to Newman that he had always (tacitly) assumed as part of his structural realism.

It is important to realize that Russell's admission to Newman that knowledge about *S* is empirical knowledge is *not* the admission that the structural relation between percepts is the same as the structural relation holding between the events that are the causes of the percepts at the nodes. That would be the sort of non-structural knowledge that Russell *denies* that physicists can have. Russell explicitly denies that the relation between percepts can be known to be the same as that between the non-percepts correlated with each at the nodes:

... I am concerned to point out that we can only infer the logical (or mathematical) properties of physical space, and must not suppose that it is identical with the space of our perceptions. Indeed, as I shall try to prove later, the whole of a man's visual space is, for physics, inside his head; this will follow from causal considerations.

The same sort of considerations apply to colours and sounds. Colours and sounds can be arranged in an order with respect to several characteristics; we have a right to assume that their stimuli can be arranged in

an order with respect to corresponding characteristics, but this, by itself, determines only certain logical properties of the stimuli. (*AMa*, p. 252)

As we can see, it would be quite mistaken to imagine that Russell was conceding that his structural realist position must be modified to allow empirical knowledge that the relationship between percepts is known to mirror (or be the *same* as) the relation between the non-percepts causing them. This would be to go well beyond the *assumption of correlation*. For example, the relation between the great many non-percepts collectively causing a given node of the visual space of percepts in the brain may be vastly different from the relation between the nodes of the non-percepts that cause it.

In corroboration of this interpretation, it is of some interest to note that Russell goes on to say that there is but one exception: temporal relations among percepts are the same as those between non-percepts

(*AMa*, p. 253). Russell maintains that knowledge of this temporal sort is *not* empirical knowledge. This is due to his thesis that temporal relations between percepts are themselves inferred and thus are on a par with temporal relations between non-percepts. He writes:

... all that we perceive is in the present, and the time order of the original events is inferred from relations among the simultaneous events which constitute our present recollection. Thus the conclusion seems to be: Psychological time may be identified with physical time, because neither is a datum, but each is derived from data by inferences of the sort we have found elsewhere, namely, inferences which allow us to know only the logical or mathematical properties of what we infer. (*AMa*, p. 254)

Russell later explains in *Analysis of Matter* that he takes a five-term relation of “co-punctuality” which holds between five events when there is a region common to all of them. A group of five or more events is then called “co-punctual” when every quintet chosen out of the group has the relation of co-punctuality (p. 299). In contrast, the relation of “comprentence” holds of two events which overlap in space-time (p. 294).

As we see, there is no need at all to try to go beyond the assumption of correlation that he admitted in his letter to Newman. In Russell’s mind, at least, the objection Newman raised was fully resolved by his assumption of correlation *alone*. In short, we have been able to explain Russell’s sense of being “ashamed” and also to explain why his assumption of correlation does not undermine his position that physical knowledge is not trivial and yet it cannot go significantly beyond structural knowledge. It remains to consider what it was, if anything, concerning Newman’s *phenomenalism* that Russell found important enough to address in his letter.

4. PHENOMENALISM

The remaining part of Russell’s letter we find Russell asking whether Newman’s own work accepts “phenomenalism”, and whether phenomenalism might be something that Newman himself hoped would address concerns about structural realism being trivial. Obviously, Russell takes Newman to be an ally who might have ideas about

how to address the concerns he raised. What did Russell mean by *phenomenalism*? The connotations of “phenomenalism” that we have today largely originate with C. I. Lewis and are thus quite different from those that Russell intended to associate with the word. In the hands of Lewis, *phenomenalism* is a thesis that assumes methodological solipsism, attempting to paraphrase all statements about physical objects in terms of statements about actual and probable *mental* (phenomenally qualitative) states of sense experience of a given person.¹² Russell did not accept methodological solipsism in the 1920s, and his theory of physical knowledge embraces the existence of transient event particulars which may occur in several minds or in no minds at all. *None* of the transient event particulars, whether in minds or otherwise, are assumed by Russell to have a qualitative character.

What Russell meant by “phenomenalism” in the context of his neutral monism is quite different from what Lewis meant. Happily, Russell tells us precisely what he means by “phenomenalism”. He defines phenomenalism as the thesis that there are only percepts (*AMa*, p. 209). Put in terms of Russell’s neutral monism, phenomenalism demands that *all* transient particulars be those that occur in some series or another that constitutes one or another mind. This is certainly not a methodological solipsism, since the percepts of several minds may be taken into consideration. Russell presents his notion of phenomenalism as a serious rival to what he calls “the causal theory of perception”. He regards it as an “intermediate” (a “half-way house”) between the causal theory and solipsism, which maintains that *my* percepts are the only percepts and that there is no matter at all (*AMa*, p. 398). He explicitly points out (*AMa*, p. 399) that (in his definition of the notion) phenomenalism is not to be confused with the thesis that there are only *my* percepts (which is a form of solipsism). He rejects this sort of phenomenalism in *Analysis of Matter*:

Phenomenalists appear to take testimony for granted, *i.e.* to assume that the words which they see and hear express what they themselves would express if they used them. But this involves causality, and involves it in the form in which the cause is in one person and the effect in another.

¹² Of course, any methodological solipsist imagines their own states to be representative of those of any other mind. Thus, the methodological solipsistic phenomenalist construction of physical objects for one person is supposed to be the same as the construction for any other.

There does not seem, therefore, to be any substantial justification for this half-way house.

We therefore assume, though with less than demonstrative certainty, that percepts have causes which may not be percepts, and, in particular, that when a number of people have similar percepts simultaneously, there is what may be called a “field” of causally connected events, which, it is found, have relations that often enable us to arrange them in a spherical order about a centre. Thus we arrive at a space-time order of events, which is found to be the same whichever of many possible methods of arriving at it we adopt; in this order, a percept is located in the head of the perceiving.

(*AMa*, p. 399)

Russell goes on explain how accepting the “field” of non-percepts (naturally resulting from a causal theory of perception that is a rival to phenomenalism) accords with his structural realism. His explanation is worth quoting at length since this nicely recaps his position:

In drawing inferences from percepts to their causes, we assume that the stimulus must possess whatever structure is possessed by the percept, though it may also have structural properties not possessed by the percept. The assumption that the structural properties of the percept must exist in the stimulus follows from the maxim “same cause, same effect” in the inverted form “different effects, different causes”, from which it follows that if, *e.g.*, we see red and green side by side, there is some difference between the stimulus to the red percept and the stimulus to the green percept. The structural features possessed by the stimulus but not by the percept, when they can be inferred, are inferred by means of general laws—*e.g.* when two objects look similar to the naked eye but dissimilar under the microscope, we assume that there are differences in the stimuli to the naked-eye percepts which produce either no differences, or no perceptible differences, in the corresponding percepts.

These principles enable us to infer a great deal as to the structure of the physical world, but not as to its intrinsic character. (*AMa*, p. 400)

Here again we see that Russell’s structural realism adopts an assumption of correlation—i.e., “principles” or “general laws”—and this, as he admits in his letter to Newman, is physical knowledge. But we also see that these principles or general laws, in Russell’s view, do not jeopardize the significance of his thesis of structural realism.

Russell’s letter is asking Newman whether he holds the phenomenalist’s thesis that there are no non-percepts—i.e., no transient

particulars that are not in some series constituting one or another mind—and whether such a thesis plays any role in Newman’s own views about the nature of scientific inference. Why does Russell ask Newman this question? The answer is that Russell wonders whether Newman imagines that phenomenalism, so conceived, might altogether avoid his assumption of correlation. Russell’s letter indicates that in conversation Newman had showed some sympathy for phenomenalism. The point is that, in examining Russell’s comments in his letter, we see nothing whatsoever to indicate that he thought Newman had found an insuperable criticism of structural realism. The assumption of phenomenalism is itself a way out.

In every case, we see that Russell held steadfastly to his structural realism in his correspondence with Newman. Moreover, Russell never changed his mind about structural realism. The fact is that the Newman criticism (as Russell understood it) had very little, if any, long-term impact on him. And this is as it should be.

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