IS MOTION "CONTRADICTION'S IMMEDIATE EXISTENCE"?

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A driving concern of Russell's rejection of Idealism was his conviction that reality is free of contradictions. However, echoing the neo-Hegelians that Russell is usually taken successfully to have refuted, Graham Priest has argued that the analysis of motion provides a motivation to adopt dialetheism (the thesis that some contradictions may be true). Furthermore, Priest argues that the Russellian account of motion as given in *The Principles of Mathematics* fails accurately to capture the phenomenon. In this paper we argue that Priest's objections to Russell are neither new nor decisive. We show that even if one shares Priest's concerns about the Russellian model there are alternatives inspired by Russell's own contemporaries that do not entail dialetheism. We conclude that not only are Priest's objections to Russell unconvincing, but even one who shares Priest's intuitions has no reason to resurrect the Hegelian account of motion:

> [M]otion itself is contradiction's immediate existence. Something moves, not because at one moment it is here and at another there, but because at one and the same moment it is here and not here, because in this "here" it at once is and is not. The ancient dialecticians must be granted the contradictions that they pointed out in motion; but it does not follow that therefore there is no motion, but on the contrary, that motion is existent contradiction itself.   (Hegel, *Science of Logic*)

I. INTRODUCTION

C entral to Russell's rejection of Idealism was his discovery that, contrary to the impression he had gained as a student of mathematics at Cambridge, mathematical analysis had the resources to solve the contradictions that Hegelians took to be evidence of the unreality of the physical world of appearance:

> Hegelians had all kinds of arguments to prove this or that not "real". Number, space, time, matter were all professedly convicted of being self-contradictory. Nothing was real, so we were assured, except the Absolute, which could think only of itself since there was nothing else for it to think of and which thought eternally the sort of things that idealist philosophers thought in their books. (*MPD*, p. 62)

In rejecting Hegelian idealism, Russell rejected the view of mathematics as contradictory: "above all, I no longer had to think that mathematics is not quite true" (*ibid.*, p. 49). Russell saw mathematics as providing solutions to philosophical problems that the Hegelians had taken to be insoluble:

> In the whole philosophy of mathematics, which used to be at least as full of doubt as any other part of philosophy, order and certainty have replaced the confusion and hesitation which formerly reigned. Philosophers, of course, have not yet discovered this fact, and continue to write on such subjects in the old way. But mathematicians [...] have now the power of treating the principles of mathematics in an exact and masterly manner, by means of which the certainty of mathematics extends also to mathematical philosophy. Hence many of the topics which used to be placed among the great mysteries—for example, the natures of infinity, of continuity, of space, time and motion—are now no longer in any degree open to doubt or discussion. Those who wish to know the nature of these things need only read the works of such men as Peano or Georg Cantor; they will there find exact and indubitable expositions of all these quondam mysteries.[1]

Russell's view that mathematics offered solutions to the Hegelians'

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[1]   RUSSELL, "Mathematics and the Metaphysicians" (1901), *Papers* 3: 369.

deepest metaphysical problems quickly became orthodoxy.[2]

A classic example is the phenomenon of motion, which Hegel had called "contradiction's immediate existence" (Hegel, p. 440). Russell's theory of motion, underpinned as it is by the mathematics of infinite series,[3] has been widely endorsed as the solution to apparent contradictions in motion such as that exemplified by Zeno's arrow paradox.

Despite Russell's best efforts, some philosophers maintain the view that reality can be contradictory. Graham Priest[4] explicitly defends Hegel's analysis of motion in support of his wider defence of metaphysical dialetheism.[5] *Metaphysical* dialetheism—as opposed, for example, to semantic dialetheism[6]—is the philosophical thesis that reality may contain contradictions. If the optimum theory of motion can be shown to require appeal to contradictions then not only does Priest have grounds for defending a dialetheic analysis of the physical world, but a central component of Russell's rejection of idealism is undermined.

Priest has argued that a sympathetic reconstruction of Hegel's argument that motion is "contradiction's immediate existence" (Hegel, p. 440, cited in Priest, p. 341) reveals the argument to be "rigorous and precise" (*ibid.*, p. 343). Priest's reconstruction of Hegel interprets the argument as recommending dialetheism. Motion, Priest has Hegel argue, "realises a contradiction" (*ibid.*, p. 341). Furthermore, the subject matter makes the alleged dialetheia a particularly interesting one. As Priest says, "according to [the formal reconstruction of Hegel's] view, to be in motion is to occupy more than one place (in fact a continuum of places) at the same time, and hence to be and not to be in some places" (*ibid.*, p. 343) Priest therefore attributes to Hegel:

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[2]   For extensive discussion of Russell's break from the Hegelian tradition, see S. CANDLISH, *The Russell/Bradley Dispute* (2007); P. HYLTON, *Russell, Idealism and the Emergence of Analytic Philosophy* (1990); N. GRIFFIN, *Russell's Idealist Apprenticeship* (1991).

[3]   See RUSSELL, *PoM*, esp. Chs. XLII, LIV.

[4]   "Inconsistencies in Motion" (1985).

[5]   Not all dialetheists share Priest's convictions regarding motion. See J. C. BEALL, *Spandrels of Truth* (2009), pp. 127–9, for compelling arguments against Priest from a dialetheic standpoint.

[6]   See E. D. MARES, "Semantic Dialetheism" (2004) for detailed discussion of the differences between these two variants of dialetheism.

(i)   A moving object o is located at position x at time t and o is not located at x at t.

If (i) is true, the physical world is inconsistent. The issue is therefore of interest not only for addressing the question of whether dialetheism in general is a plausible doctrine—repeated examples contribute to a cumulative case—but also for the debate internal to dialetheism concerning the applicability of the thesis to various domains.[7]

   Of course, even if Priest is reading Hegel accurately, we have a good argument for a dialetheic theory of motion only if that theory is superior to alternative consistent theories of motion. The orthodox position, and the only alternative that Priest considers, is Russell's famous "at-at" theory of motion:

> Motion consists in the fact that, by the occupation of a place at a time, a correlation is established between places and times; when different times, throughout any period however short, are correlated with different places, there is motion; when different times, throughout some period however short, are all correlated with the same place, there is rest.
>
> (*PoM*, §446)

Priest offers two objections to Russell's theory that he takes to be decisive, and which he argues Hegel's account avoids. We consider both objections in §2 below.

   In what follows we will argue that Priest's objections to Russell's account of motion are neither new (they were well rehearsed by Russell's own contemporaries almost a century ago[8]) nor decisive. Moreover, we will show that even if one were to be persuaded by these original objections to Russell they do not in any way compel us to endorse the reality of contradictions. We conclude that Priest's objections provide no reason to think that the metaphysics of motion will be a source of compelling evidence in favour of metaphysical dialetheism.

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[7] For discussion of the relative merits and plausibility of different varieties of dialetheism (e.g. metaphysical, semantic, legal, epistemic) see, among others, PRIEST, "Truth and Contradiction" (2000), and MARES (2004).

[8] Most notably BERGSON. See, for example, his *Creative Evolution* (1911).

2. PRIEST'S OBJECTIONS TO RUSSELL

Priest raises two objections to Russell's account of motion. First, that Russell's view entails that "there is no such thing as an intrinsic state of change. (The cinematographic objection.)" (Priest, p. 343). Second, that Russell fails adequately to address Zeno's arrow paradox.

Consider the problem of intrinsic change. Priest says:

> … it follows from Russell's definition that there is no such thing as an *intrinsic* state of change. If one had a body in motion and took […] a logical "picture" of it at a certain instant, the "picture" one would obtain would be no different to one of a similar body in the same place but at rest.   ("Inconsistencies in Motion", p. 339)

Russell would very likely have granted this and agreed that there is no feature distinctive of change that could be observed by looking at an object at an instant. He says a similar thing in response to Zeno, as we shall see. Once we adopt the mathematical insights of the nineteenth century and allow that a body can have instantaneous velocity, we have a measurable quantity the value of which for the stationary body differs from its value for the body in motion. True, this difference in velocity could not be discerned by looking at a "logical picture" (by which we assume Priest means an idealized picture of a body at an instant). A body at rest and a body in motion might appear identical in such a picture, but observed over the course of several instants, the difference in velocity between the two objects becomes evident.

Anything in motion would of course appear static if it were artificially imagined static. Further, as Priest himself allows, motion—whether intrinsic or not—takes time. What we can say, therefore, is that a continuously moving body and a stationary body would look different if watched for a non-instantaneous interval of time. One would be seen to move. If we take an ordinary photograph of a person, we cannot observe their breathing. A logical picture of a breathing person is, presumably, identical to a logical picture taken at the same instant of a person holding their breath. Whatever that shows us, it doesn't show us that our account of breathing needs revision, only that some things don't show up on logical pictures of instants. Motion takes time, and it is therefore unsurprising that it cannot be identified without reference to an interval of time.

Turning to Zeno's paradox of the arrow, Priest says:

> Consider an object in uniform motion, say, the tip of an arrow, travelling from A to B, and take an instant, t_0, of its motion. At t_0 the arrow advances not on its journey towards B. (If it did make some headway on its journey, this would take time. The temporal stretch involved would not therefore be an instant.) Thus at $t = t_0$, total progress made equals zero. But a temporal interval $a \leq t \leq b$ is made up of such points. It would seem therefore that since no progress is made in any basic part of the interval $[a, b]$, no progress can be made in the whole, i.e. the arrow never makes any progress on its journey at all. This is absurd.
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> ("Inconsistencies in Motion", p. 340)

Is this absurd? Suppose we reconstruct Zeno's argument like this:

     Consider an arrow in flight.

(1)   At an instant the arrow is in a space exactly matching its dimensions.

(2)   Anything in a space exactly matching its dimensions is not moving.

(3)   Anything not moving at each instant during a period of time is not moving at all during that period of time.

(4)   So, the arrow in flight is motionless.[9]

Russell, as we have seen, rejects (3), claiming instead that not moving at each instant during a period of time is compatible with being in a different place at the end of that same period of time. As he puts it,

> Weierstrass, by strictly banishing from mathematics the use of infinitesimals, has at last shown that we live in an unchanging world, and that the arrow in its flight is truly at rest. Zeno's only error lay in inferring (if he did infer) that, because there is no such thing as a state of change,

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[9] "In a space exactly matching its dimensions" is a rendering of Aristotle's "opposite to something equal to itself" at *Physics* 239ᵇ6. Edward Hussey has the arrow occupying "'a space equal to itself' (i.e. of the same shape and size)" ("Pythagoreans and Eleatics" [2003], p. 157), and Kirk and Raven "confidently [reconstruct the argument] as follows: "An object is at rest when it occupies a space equal to its own dimensions. An arrow in flight occupies, at any given moment, a space equal to its own dimensions. Therefore an arrow in flight is at rest'" (*The Presocratic Philosophers* [1966], pp. 294–5).

therefore the world is in the same state at any one time as at any other.
                    ("Mathematics and the Metaphysicians", *Papers* 3: 370)

The Russellian account gives a perfectly coherent and mathematically respectable response to Zeno, and Priest has done nothing more than say that Zeno's argument demonstrates that being in different places at the beginning and the end of a period of time is incompatible with being at rest at each instant during that period. But this is hardly a criticism of Russell, who has given a perfectly clear explanation of how the two things can be made compatible. Unless Priest has an argument to show that Russell has not avoided contradiction, it is hard to see any substance in his objection.

Priest accuses Russell of failing both to account for intrinsic change and to respond to Zeno's arrow paradox, but this is to misconstrue the dialectic of Russell's argument. Russell's response to Zeno is precisely to argue that there is no intrinsic state of change. Priest's two objections, it seems, reduce to one that Russell has already answered.

3. AN ALTERNATIVE CONCEPTION OF MOTION

Priest objects to Russell's analysis of motion into instants and the cinematographic view of motion it entails. Russell's mathematical response to Zeno embraces this consequence. This debate has been run many times before, and Priest's insistence on the "no number of not-movings make a moving" response is just dogmatism. Furthermore, as we shall now show, even those in the grip of this dogma should not concede that motion provides evidence of a true metaphysical contradiction. If the analysis of motion into instants, central to the arrow paradox and accepted by both Russell and Priest, is the root of the difficulty, any account of motion eschewing this analysis may escape the paradox and undermine Priest's conclusion. In fact accounts meeting this criterion were available in Russell's time. Consider for example the following simplified account inspired by Russell's contemporaries.

Rather than analyzing motion into instants, take as primitive temporal intervals corresponding to those given in perception as the so-called "specious present". Henri Bergson introduced his concept of *durée* to capture this very notion, talking of "succession without distinction […] an interconnexion and organization of elements, each

one of which represents the whole, and cannot be distinguished or isolated from it except by abstract thought."[10] He was apparently motivated by doubts that anticipate Priest's by almost three quarters of a century:

> every attempt to reconstitute change out of states implies the absurd proposition, that movement is made of immobilities.
> …
> Philosophy perceived this as soon as it opened its eyes. The arguments of Zeno of Elea, although formulated with a very different intention, have no other meaning.   (Bergson, p. 325)

Bergson was not alone among Russell's contemporaries in voicing Priest's concern. Samuel Alexander explicitly agrees with Bergson, saying "[m]otion is not a succession of point-instants, but rather a point-instant is the limiting case of a motion. So far we have seen Mr. Bergson to be right in his protest."[11] Whitehead's method of extensive abstraction, later extensive connection, allows him to develop a point-free geometry[12] and to derive the points and instants amenable to scientific theorizing from temporally extended "regions" or "events". He defines points, lines and areas "in terms of abstractive sets," and so defines "abstractive sets without reference to the notions, point, line, area."[13] Without a more thorough explanation of Whitehead's vocabulary it is impossible to do his position justice, but the central idea is that the starting point is "regions" and the different modes of connection between them. This theory models, without reifying, points and instants by deriving them by means of a series of definitions from this starting point. It answers all of Priest's concerns without sacrifice either of mathematical rigour or of the law of non-contradiction.

David Bostock examines both Whitehead's and Russell's efforts to construct points out of regions and concludes that, though their actual

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[10] BERGSON, *Time and Free Will* (1913), p. 101.

[11] ALEXANDER, *Space, Time and Deity* (1927), p. 321.

[12] See WHITEHEAD, *An Enquiry Concerning the Principles of Natural Knowledge* (1919), *The Concept of Nature* (1964), *Process and Reality* (1985) and, e.g., GERLA AND MIRANDA, "Graded Inclusion and Point-Free Geometry" (2004), "Mathematical Features of Whitehead's Point-free Geometry" (2008). Whitehead develops the basis of the approach in (1919) and (1964) and adopts some modifications in (1985) inspired by DE LAGUNA, "Point, Line and Surface as Sets of Solids" (1922).

[13] WHITEHEAD, *Process and Reality* (1985), p. 298.

methods need modification, the construction can in fact "be done either way round".[14] That is, we can construct points from regions, or regions from points, taking either as our primitive notion. Bostock observes that "in these days our mathematicians and our physicists are much more familiar with the other direction [i.e. constructing regions from points] and there is surely no reason for them to drop this approach in favour of [constructing points from regions]" (*ibid.*, p. 47). But for our purposes here, "just as good" is good enough. We might, if we felt the force of Priest's concerns, develop a geometry that could form the basis for theorizing about motion without starting by analyzing the world into points and instants.

4. CONCLUSION

Priest's objections to Russell collapse into one. The same objection was voiced by Russell's contemporaries. It didn't trouble Russell then; if it troubles us now, we should reject the analysis of motion into instants, not the law of non-contradiction.

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[14] BOSTOCK, "Whitehead and Russell on Points" (2010), p. 46.

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