A SEXIST JOKE IN *PRINCIPIA MATHEMATICA*

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Attempts have recently been made (Blackwell, 2011; Berumen, 2014) to catalogue the humour in *Principia Mathematica*, but so far overlooked has been a joke in the symbolism of *PM* itself.

In "The Wit and Humour of *Principia Mathematica*"[1] Kenneth Blackwell presented what many of us took to be a comprehensive survey of the jokes in *Principia Mathematica*, though Michael Berumen quickly discovered another one in the distant reaches of volume 3. But it turns out that there is yet another joke in *Principia*—a sexist one, too—hiding in plain sight in the symbolism itself. That it was missed is hardly surprising, since the symbol in question is not extensively used in *Principia* and, so far as I am aware, not at all outside it. It is a special kind of variable and so is given no formal definition in *PM,* although notations derived from it are (∗38·01·02·03). Indeed, it is a kind of variable one doesn't expect to find in *PM.*

It is, in fact, an operator variable. Now, operator variables have not found much serious use in logic,[2] for the logical operators are taken to be improper symbols, which lack independent semantic significance. In model-theoretic terms, they are not assigned values in the domain and thus are not susceptible to variation. Russell himself takes this for granted in the opening sentence of *The Principles of Mathematics* where the propositions of pure mathematics are said to be of the form "*p* implies *q*" where "neither *p* nor *q* contains any constants *except logical constants*" (emphasis added). It is true that his second sentence

[1] Blackwell, "The Wit and Humour of *Principia Mathematica*" (2011).
[2] The notable exception is Leśniewski's protothetic, where they play an important role.

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immediately invites one to ask why the logical constants are permitted to remain while all others must be absent, for he gives no account of the special nature of the logical constants but merely lists them.[3] Later, in "The Philosophical Importance of Mathematical Logic" (1911), he identifies the logical constants as those which resist replacement by variables (*Papers* 6: 35–6). So it is a bit surprising to find that the first volume of *PM,* which had been published just the previous year, contains an operator variable.

The operator variable in *PM*, however, has nothing to do with these issues. Nor does the fact that operators are improper symbols preclude altogether the possibility of replacing them by variables. True, such variables cannot be objectually interpreted on the domain, but they can be interpreted substitutionally as taking the various constant operator symbols as their substitution values. In this way they do occasionally appear in a metalanguage for the practical purpose of stating general principles of operator behaviour concisely. For example, one might state a formation rule thus: "If p and q are wffs and O is a binary truth-functional operator, then p O q is a wff."

Unfortunately, neither model-theoretic semantics nor the modern object/meta- distinction were available to Whitehead and Russell, so they were not able to make the case for their operator variable in quite the way we would do today. But they did quite clearly give it a substitutional interpretation. The operators over which it ranges are not truth-functional propositional operators or quantifiers: they are binary operators on classes and relations, such as $\cap$, $\cup$, $\dot{\cap}$, $\dot{\cup}$, |, ↿, ↾ and ⥏. The operators in question form what Whitehead and Russell call "double descriptive functions" (*PM*, ∗38). They are term-forming operators on pairs of terms. Or to put it in Whitehead and Russell's idiom: they are functions which form definite descriptions from pairs of arguments: either pairs of classes—thus, $\alpha \cap \beta$ and $\alpha \cup \beta$ (the intersection and union of α and β); pairs of relations—thus, $R \mathbin{\dot{\cap}} S$, $R \mathbin{\dot{\cup}} S$, and $R \mid S$ (the conjunction, disjunction and relative product of R and S); or a pair consisting of a class and a relation—thus, α ↿ R, R ↾ α and R ⥏ α (the relation R with, respectively, its domain, converse domain, or both restricted to α). The operator variable is interpreted

[3] Or rather some examples of them. Part I of *Principles* is devoted to "The Indefinables of Mathematics" and it is only at the end that he puts forward his final list (p. 106). In both cases, the lists are closed under definition.

substitutionally: it can be replaced by any of the eight operator symbols just mentioned. Its substitutional range, however, is not limited to these eight symbols, and additional symbols are explicitly mentioned later (*PM* 2: 301), e.g. $\uparrow$, $\downarrow$, though the list is not exhaustive. It takes as substitution instances any binary function on terms such that the resulting term exists (*PM* 1: 296).

The reason Whitehead and Russell want a single piece of notation which will encompass all such operators is that they are trying to capture the general mathematical notion of an arithmetical operation. The ideas of ∗38 come into their own only much later in the work, especially in relation arithmetic (see, e.g., ∗150 and ∗182). Though the basic form of the notation seems to be rarely used in *PM*, derived forms do appear occasionally. So what symbol do Whitehead and Russell use for this quite rare and rather special variable? They choose

x ♀ y

because woman is not constant!

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