Documents

RUSSELL'S CORRECTED PAGE PROOFS OF *PRINCIPIA MATHEMATICA*

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We report here on the set of complete proofs of Volumes I and II of Whitehead and Russell's *Principia Mathematica* newly acquired by the Bertrand Russell Archives. These proof sheets, marked with a number of corrections, were likely bound for Russell by Cambridge University Press, though not exactly the same as the first edition. We assess the information to be gained from the texts and the corrections, most significantly around ∗110 in Vol. II and the lost dot of the empty relation $\dot{\Lambda}$ in Vol. I. All are in Russell's hand and described in an appendix. We also note several revisions in the first edition that were made after these proofs. We discuss the provenance of the volumes, and Russell's correspondence about proofs of *PM* with M. H. Dziewicki, but we find that there is insufficient evidence to determine the chain of possession from Russell to their discovery for sale in Australia in recent years.

I. PROOFREADING "PM"

Despite accounts in his letters at the time of having written several thousand pages of *Principia Mathematica* and conveying them to Cambridge University Press on 19 October 1909,[1] almost nothing is known to survive. There are rejected drafts and an

[1] Grattan-Guinness, "The Royal Society's … Support … of *PM*" (1975), p. 91.

russell: the Journal of Bertrand Russell Studies n.s. 39 (winter 2019–20): 141–66
The Bertrand Russell Research Centre, McMaster U. ISSN 0036–01631; online 1913–8032

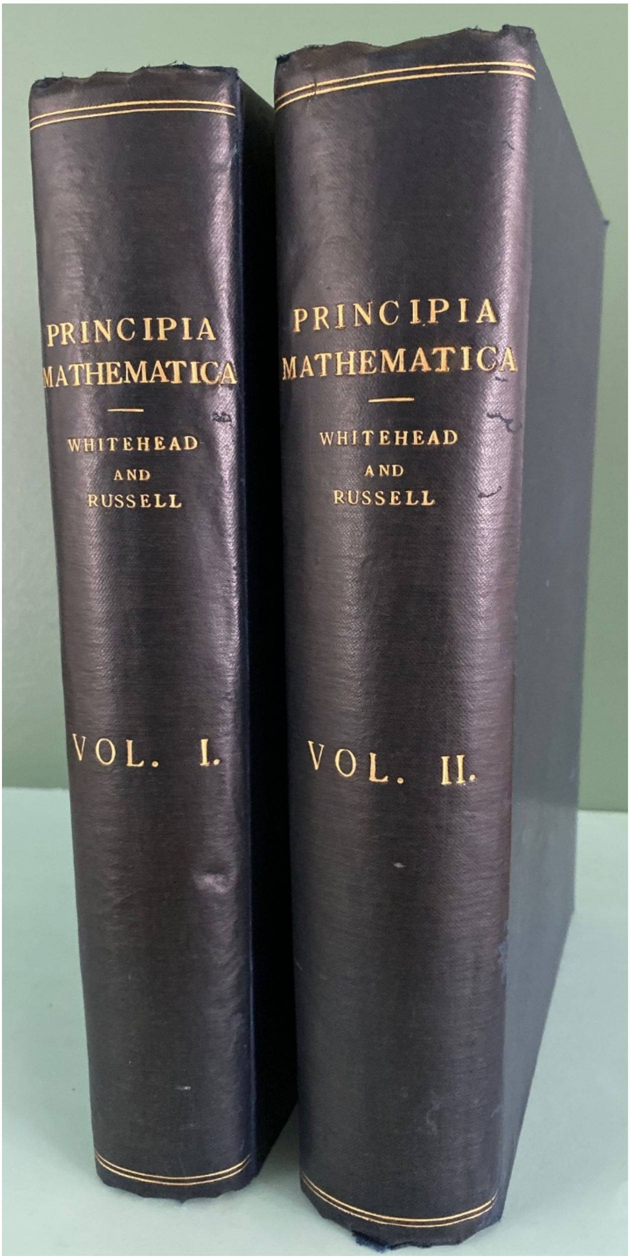


Illustration 1. The bound proof volumes.

immense index of where propositions were used, but only two and one-half leaves of the manuscript from which type was set. One leaf is with a letter to Lady Ottoline Morrell at the University of Texas,[2] she having ignored Russell's instruction to destroy the leaf. The other one and one-half leaves are in the Bertrand Russell Archives, where Russell reused the versos for another purpose. They were found among other documents inserted in his copy of the first edition of *PM*.[3] They are of interest, not for revisions on the leaves or between them and the text of the first edition, but for the printer's markings of the signature line and the name of a particular compositor. Of proofs at any stage, only a single leaf was known, a proof of the "Additional Errata to Volume I" that appeared in Volume II, page [viii].[4] The proof is marked "1st", dated 20 November 1911 and initialled "H.S.D." There had been no hint of the survival of further proofs. Now bound proof sheets of the first two volumes have come to light, providing, among much else, a second stage of the "Additional Errata to Volume I".[5] The Appendix records and describes all the proof markings and whether the corrections appear in the Errata, the first edition, or the second.

The acquisition appears to be Russell's reference copy of the final or (in the case of some sheets) near-final page proofs of the first two volumes.[6] Consider not only the size of the original manuscript but also the multiple stages of proofs: his desk would soon have been awash in manuscript and proofs as the proofreading of the volumes proceeded over a three-year period. The proof sheets were large: 22 × 30 inches[7] for these Imperial Crown Octavo volumes. The paper in the proof volumes and the first edition is identical. The signatures are signed numerically.[8] They were probably proofread unfolded, since

2 It is reproduced in Anon., "Illustrations" (2011), p. 81.

3 The leaves are reproduced in Linsky and Blackwell, "New Manuscript Leaves and the Printing of the First Edition of *PM*" (2005), pp. 144–5, 148.

4 The proof is reproduced in Blackwell, "Russell's Mathematical Proofreading" (1983), p. 158, and in Anon., "Illustrations", p. 83.

5 Also extant are five leaves in Whitehead's hand of proposed errata to the first edition of *PM*. Some were corrected in Volume III, some listed as errata there (and made in the second edition), and some never corrected. See RA2 710.111549a–e.

6 Scans of the two volumes are available in McMaster Library's digital archive. Visit digitalarchive.mcmaster.ca/islandora/object/macrepo%3A90174.

7 See Collins, *Authors' & Printers' Dictionary* (1938), pp. 174, 277.

8 Being preliminary to the text, the first sheet in Volume I is unsigned. In Volume II the preliminary sheets, which include Whitehead's late insertion of a "Prefatory Statement of Symbolic Conventions", are signed a (on leaf 5 only) and b (on leaves

reading a folded sheet would require opening the top and fore-edge of all the leaves with little paper remaining to hold the sheet together.[9] We know the sheets started to arrive regularly in spring 1910[10] until Whitehead stopped the printing a year later, in January 1911. They resumed three to four months later and until Volume II was finished. The proofreading of Volume III was completed in February 1913.[11] Russell evidently discarded the manuscript as it was returned by the Press, and also the intervening stages of proofs. However, rather than keep a perhaps unstable pile of folded sheets by him as he worked on new proofs, he had his set bound. To judge by the cloth and the binding's quality, the binding could have been done by Cambridge University Press, but the typeface used on the spines, the unbevelled boards and lack of blind-stamped rules, the binding of the endpapers, and red speckling of the edges differentiate the proof bindings from the first edition. The proof volumes are bound identically, but it would make sense for Russell to have had the first set bound in the interval between finishing the proofreading of Volume I and starting that of Volume II. After Volume I's appearance from the Press a short time before publication in December 1910,[12] he would no longer have needed a handy copy of its proofs on which to record new errata and for referring to what the authors had passed for the press as he worked on Volume II. The same consideration applies to the binding of the page proofs of Volume II as he worked on Volume III before Volume II was published in April 1912. There was no practical need to bind the proofs of Volume III. The accumulated proofs of that volume were probably discarded when it was published in April 1913.

Despite the unexpected survival of the bound proofs, the number of stages of proofs is unknown, and would have varied according to whether the revisions required checking. There were always at least two stages. Consider the subject phrases in the right running heads, e.g. "THE LOGICAL CALCULUS" (p. 93 of the first edition, p. 89 of the second). Although *PM*'s chapter headings are strong clues to what

1 and 2).

9 Some sheets in the bound volumes show they were once folded vertically, as if for carrying in one's coat pocket or mailing.

10 Russell to Philip Jourdain, 10 April 1910, RA3 Rec. Acq. 33b, is the first reference.

11 Russell to Lady Ottoline Morrell, letters of February 1913, RA3 Rec. Acq. 69.

12 Dates of publication are taken from *PM*'s bibliographical entry in *B&R* I: 19–22.

should go in the running heads, this one doesn't mirror any such heading. The compositors[13] would not have decided on the wording, and the authors would have filled in the topic words on the first proofs.[14] If there were lengthy insertions or deletions in the text, they might have had to readjust the running heads. The bound proofs have a fair number of corrections, and we know that as "final revises" the corrections are incomplete (see secs. II–IV below). Whether the authors were sent new proofs again—until everything was confirmed to be correct—is unknown. What *is* known is that the sheets for each sixteen-page section (or signature or gathering) were printed in their 750 copies (500 for Volumes II and III[15]) so that the limited supply of cold metal type (especially for unusual "sorts") could be distributed and reused for a new set of sixteen pages.[16] As an example of this approach at the Press, Whitehead in August 1911 pointed out an error in Volume II, ∗74·12, commenting: "If the sheet is printed off, keep this as an erratum." We do not know how many sheets were on the go (in varying stages of composition and revision), but it could hardly be less than three. Both men saw proofs. At one point Whitehead wanted Russell "to return my marked proof in time for me to compare them with the revises".[17] It was Russell's job to collate the corrections and deal with the Press. Distribution of the type is why *PM* had to be reset in the early 1920s when demand justified reprinting the work.[18]

[13] *Cf.* Russell, *The Conquest of Happiness* (1930), on trades and professions whose practitioners derive pleasure from their work: "I have known also compositors who were experts in setting up mathematical type, or Nestorian script, or cuneiform, or anything else that was out of the way and difficult" (p. 150).

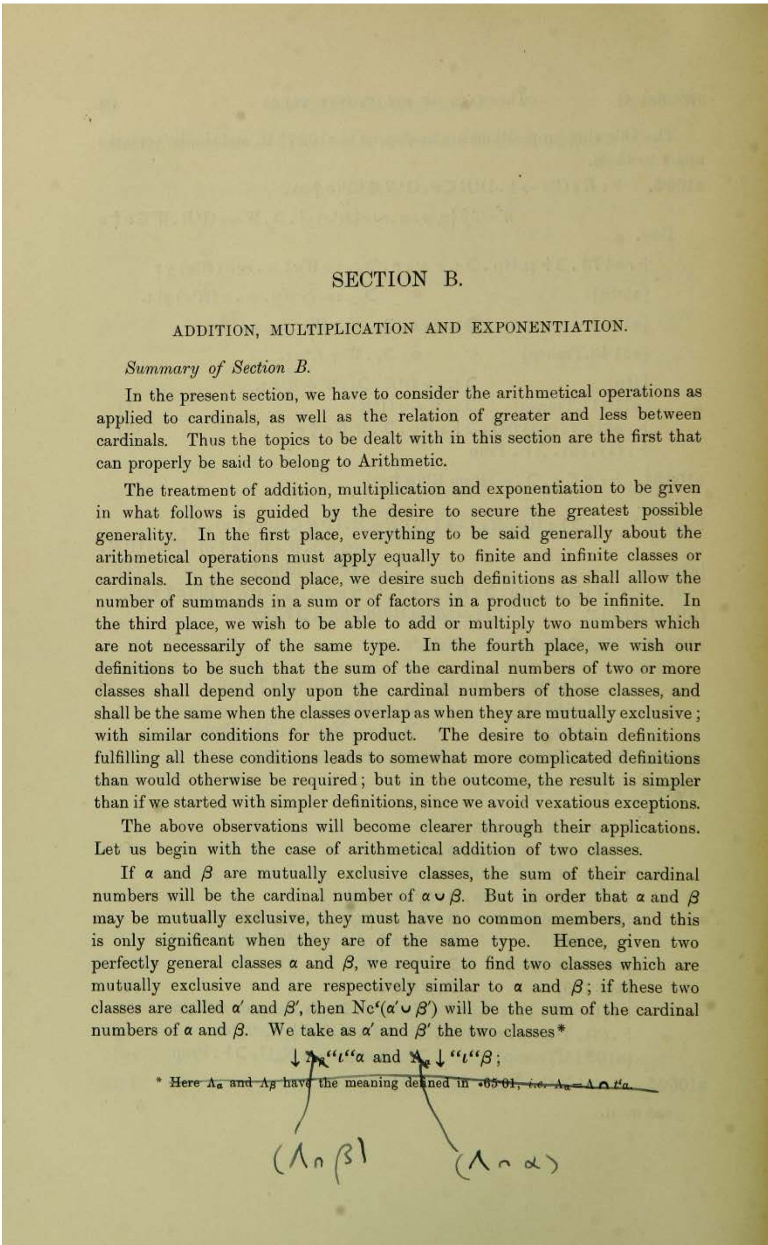
[14] E.g., Whitehead provided "operations" for what he termed the "heading" on page 313 of Volume I (letter to Russell, 23 Aug. 1910, RA1 710).

[15] *B&R* I: 20–2.

[16] Cambridge University Press did not install Monotype hot-metal composing machines until 1913. "Hitherto all setting had been by hand" (Black, *Cambridge University Press* [1984], p. 284).

[17] Whitehead to Russell, 14 July 1910. This concern was with sheets 13 and 14: Volume I, pp. 193–224.

[18] Volume III was not reset but photographically reprinted. For the typesetting of the new material in the second edition, see Linsky (2011), pp. 9–10.



SECTION B.

ADDITION, MULTIPLICATION AND EXPONENTIATION.

Summary of Section B.

In the present section, we have to consider the arithmetical operations as applied to cardinals, as well as the relation of greater and less between cardinals. Thus the topics to be dealt with in this section are the first that can properly be said to belong to Arithmetic.

The treatment of addition, multiplication and exponentiation to be given in what follows is guided by the desire to secure the greatest possible generality. In the first place, everything to be said generally about the arithmetical operations must apply equally to finite and infinite classes or cardinals. In the second place, we desire such definitions as shall allow the number of summands in a sum or of factors in a product to be infinite. In the third place, we wish to be able to add or multiply two numbers which are not necessarily of the same type. In the fourth place, we wish our definitions to be such that the sum of the cardinal numbers of two or more classes shall depend only upon the cardinal numbers of those classes, and shall be the same when the classes overlap as when they are mutually exclusive; with similar conditions for the product. The desire to obtain definitions fulfilling all these conditions leads to somewhat more complicated definitions than would otherwise be required; but in the outcome, the result is simpler than if we started with simpler definitions, since we avoid vexatious exceptions.

The above observations will become clearer through their applications. Let us begin with the case of arithmetical addition of two classes.

If α and β are mutually exclusive classes, the sum of their cardinal numbers will be the cardinal number of $\alpha \cup \beta$. But in order that α and β may be mutually exclusive, they must have no common members, and this is only significant when they are of the same type. Hence, given two perfectly general classes α and β, we require to find two classes which are mutually exclusive and are respectively similar to α and β; if these two classes are called α' and β', then $\mathrm{Nc}‘(\alpha' \cup \beta')$ will be the sum of the cardinal numbers of α and β. We take as α' and β' the two classes*

$$\downarrow \cancel{\Lambda_\beta}\,“\iota“\alpha \quad \text{and} \quad \cancel{\Lambda_\alpha} \downarrow “\iota“\beta\,;$$

~~* Here Λ_α and Λ_β have the meaning defined in *65·01, *i.e.* $\Lambda_\alpha = \Lambda \cap t‘\alpha$.~~

$(\Lambda \cap \beta)$ $(\Lambda \cap \alpha)$

Illustration 2. Scan of Volume II, p. 66. Two subscripts to a conjunction are replaced here and elsewhere and a footnote deleted.

II. CORRECTION TO VOLUME II, P. 66, IN THE PROOFS

The most significant of the corrections in these proofs occurs in Volume II, Part III, titled "Cardinal Arithmetic", in the Summary of section B, "Addition, Multiplication and Exponentiation". Two corrections were made in the text (see *Illus. 2*) and a footnote deleted.

The changes occur at the foot of page 66 of the first edition (p. 63 of the second). The paragraph preceding the changes (which is the same in both editions) explains what is intended:

> If α and β are mutually exclusive classes, the sum of their cardinal numbers will be the cardinal number of $\alpha \cup \beta$. But in order that α and β may be mutually exclusive, they must have no common members, and this is only significant if they are of the same type. Hence, given two perfectly general classes α and β, we require to find two classes which are mutually exclusive and are respectively similar to α and β; if these two classes are called α' and β', then $\mathrm{Nc}`(\alpha' \cup \beta')$ will be the sum of the cardinal numbers of α and β. We take as α' and β' the two classes*
>
> $$\downarrow \Lambda_\beta `` \iota `` \alpha \text{ and } \Lambda_\alpha \downarrow `` \iota `` \beta$$
>
> * Here Λ_α and Λ_β have the meaning defined in *65·01, *i.e.* $\Lambda_\alpha = \Lambda \cap t`\alpha$.

The markings on the proofs are on the last two lines. A long line through the note beginning "*Here ... " indicates that the note is to be deleted. A line from Λ_β in the penultimate line indicates that Λ_β is to be replaced by $(\Lambda \cap \beta)$, and a second line indicates that Λ_α is to be replaced by $(\Lambda \cap \alpha)$. This change indicates that the notions expressed by Λ_α and Λ_β will not be defined via *65·01 but instead directly expressed by the simpler expressions $(\Lambda \cap \alpha)$ and $(\Lambda \cap \beta)$, respectively. This simplification of the notation will be explained below, but first it is important to notice that the changes indicated, in the last two lines, are not the whole of what appears in the first edition, where the paragraph above continues as before to the point of "We take as α' and β' the two classes ..." (notice that the asterisk indicating a note disappears). In the published first edition we have:

> ... the sum of the cardinal numbers of α and β. We note that $(\Lambda \cap \alpha)$ and $(\Lambda \cap \beta)$ indicate respectively the Λ's of the same types as α and β, and accordingly we take as α' and β' the two classes

$$\downarrow(\Lambda \cap \beta)\text{“}\iota\text{“}\alpha \text{ and } (\Lambda \cap \alpha)\downarrow\text{“}\iota\text{“}\beta;$$

This shows that these marginal corrections were not the last indication of the changes to be made. Further changes, in particular the addition of the phrase "We note that $\Lambda \cap \alpha$ and $\Lambda \cap \beta$ indicate respectively the Λ's of the same types as α and β ...", indicate that the changes marked in these proofs were not the very last before the final printing.

In the remainder of ∗110 the change from Λ_α to $(\Lambda \cap \alpha)$ was made between the proofs and the published first edition in several places, but not in all. Thus ∗110·01·1·101·11·12·13·115·152·18 were all corrected. Occurrences of Λ_β and Λ_γ in ∗110·71 on page 86 were not corrected. This seems to be a result of the length of the lines in which they occur. It would have been impossible to add the spaces needed for the extra symbols given the structure of the lines. There was probably a query from the printer on this that was later discarded.

The changes on page 66 were not all the changes made before the final version of Volume II was published. For other changes see sections III–IV below. Still other changes in these proofs were made to the erratum lists in the published edition. The bound proofs may contain the corrections that Russell intended to be the last, to be copied over in full to a second set that was returned, sheet by sheet, to the Press prior to the printing of Volume I's 750 or Volume II's 500 copies as a sheet's proofreading was completed and the corrections made.[19]

The present alteration was intended as a simplification of the notation, although it involves changes in the notion of relative types, including the addition of the long "Prefatory Statement of Symbolic Conventions", that were made late in the composition of Volume II. The problem being addressed is to define a notion of the sum of two classes for which the cardinal number of the sum will be the sum of the cardinal numbers. If two classes overlap, say the classes $\{a,b\}$ and $\{a,c\}$, their union $\{a,b,c\}$ will not do as the sum of the classes, for the cardinal of each of these classes is 2, and the sum of the cardinals, 4,

[19] The view that the volume was printed sheet by finished sheet is supported by Whitehead's letter to Russell of 31 May 1910 (RA1 710). The proposition ∗3·03, which is mentioned on page 14 of Volume I (p. 13 of the second edition) "must stand", but alterations can still be made, Whitehead writes, to the mentions of it on page 101 in sheet 7, and page 114. Page 101 no longer has a mention of ∗3·03. On page 114 there appears to be a qualifying sentence: "It is to be understood, like ∗1·72, as applying also to functions of two or more variables."

is not the cardinal of the union of the classes, which is 3. The solution in ∗110 of *PM* is to define the sum as a union of two classes, of types close to those of the original classes, but which do not overlap.[20] The two classes are constructed so that each member of α is paired with the unit set of $\{\beta\}$ and each member of β is paired with $\{\alpha\}$. In modern notation ∗110·01 would be written:

$$\alpha + \beta = (\beta \times \{\alpha\}) \cup (\alpha \times \{\beta\}).$$

The additional feature of taking the intersections of α and β with Λ, the empty class ($\varnothing$ in contemporary notation), is used to make sure that the types of the two summed classes are unchanged. The intersection of Λ with α is an empty class, but of the type of classes whose members are in α. An expression like "Λ" for the empty class will be *typically ambiguous*. There is an empty class of individuals, an empty class of classes of individuals, and so on, for each type. In the version of *PM* that is corrected in these proofs the notion of the members of Λ_α is defined at ∗65·01 as: $\Lambda_\alpha = \Lambda \cap t‘\alpha$. The latter intersected class $t‘\alpha$ is defined in turn at ∗63·01 by $\{\alpha\} \cup - \{\alpha\}$, the union of $\{\alpha\}$ and its complement. In contemporary set theory the complement of a set (everything not a member of the set) is too large to be a set, and will form a *proper class*. In type theory every class α of a given type t will have a complement—α, the class of everything of type t that is not in α. (It is informative to see that despite the impression of talking about types as somehow metalinguistic, or otherwise inexpressible in the language of *PM*, there is in fact no problem with defining this notion of the *type* of a class.) The upshot of this chain of eliminating two definitions, ∗63·01and ∗65·01, is that the notion of the empty class of the type of classes which have α as a member, Λ_α, is to be replaced by the expression $\Lambda \cap \alpha$, the empty class of the lower type of the members of α—thus saving the two steps of definition. The change does not appear to affect the results about addition that follow.[21] This is precisely the simplification indicated in the corrections to the proofs on page 66 of Volume II.

[20] The sum will be just one higher than the types of α and β.

[21] We are grateful to Gregory Landini for directing our attention to this difference of types with the new definition.

III. ADDITIONS TO THE FIRST EDITION AFTER THESE PROOFS

Two related changes involve material added to the first edition that are not marked in the proofs. This strongly suggests that these were not the last proofs that the authors corrected of the sheets concerned.

The first change is at Volume II, page 68, line 11, where the proofs have:

> This illustrates what is required generally where typical ambiguity occurs, namely that, though it is often desirable that our symbols should be typically ambiguous, it is always essential to right symbolism that their value should be unique as soon as their type is assigned.

This is replaced in the first edition with:

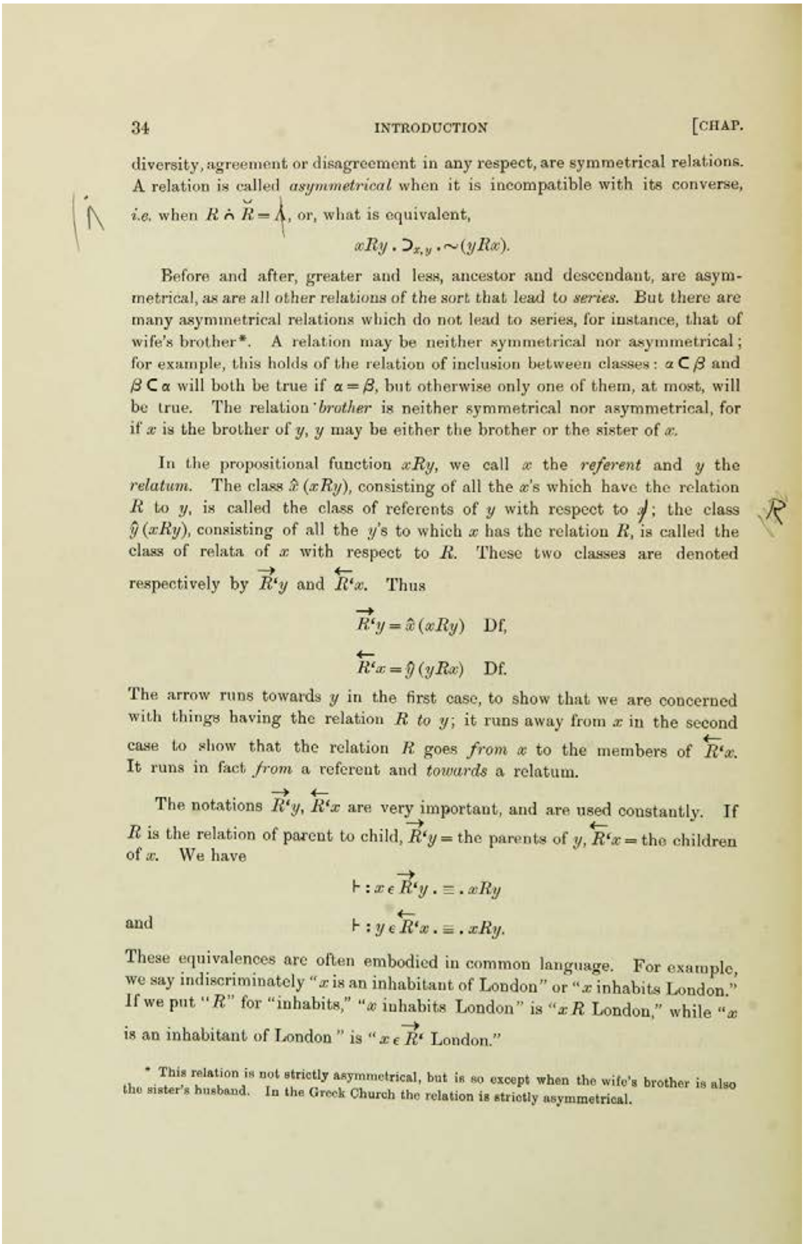
> It is always essential to right symbolism that the values of typically ambiguous symbols should be unique as soon as their type is assigned. The scope of these definitions and of the corresponding definitions for multiplication and exponentiation (∗13·04·05 . ∗116·03·04) is extended by convention IIT of the prefatory statement.

The second change is at Volume II, page 76, line 4, where this addition is made to the end of the paragraph:

> Also in connection with these definitions and the corresponding definitions ∗113·04 and ∗116·03·04 and in ∗117·02·03, the convention IIT of the prefatory statement must be noted.

There are two concerns about types that effect the discussion in this section related to the "Prefatory Statement of Symbolic Conventions" that delayed the appearance of Volume II. The restriction of the theory of cardinal numbers to classes of common types was met with the notion of "homogeneous cardinals" and the notation N_0 in the Introduction to Part III. The "Prefatory Statement" discussed a further restriction on theorems that depend on the existence of a classes of cardinalities that are not assured by the Axiom of Infinity introduced in ∗120. The "convention IIT" indicates the restriction of propositions and definitions to cases where the assumption is made that the cardinalities are "adequate" to avoid this.

These changes indicate that the adjustment of Volume II to meet with the Axiom of Infinity were still troubling the authors at the very last stages of proof correction.



34 INTRODUCTION [CHAP.

diversity, agreement or disagreement in any respect, are symmetrical relations. A relation is called *asymmetrical* when it is incompatible with its converse, i.e. when $R \mathbin{\dot\frown} \breve{R} = \dot\Lambda$, or, what is equivalent,

Λ

$$xRy \,.\, \supset_{x,y} .\, \sim(yRx).$$

Before and after, greater and less, ancestor and descendant, are asymmetrical, as are all other relations of the sort that lead to *series*. But there are many asymmetrical relations which do not lead to series, for instance, that of wife's brother*. A relation may be neither symmetrical nor asymmetrical; for example, this holds of the relation of inclusion between classes: $\alpha \subset \beta$ and $\beta \subset \alpha$ will both be true if $\alpha = \beta$, but otherwise only one of them, at most, will be true. The relation *brother* is neither symmetrical nor asymmetrical, for if x is the brother of y, y may be either the brother or the sister of x.

In the propositional function xRy, we call x the *referent* and y the *relatum*. The class $\hat{x}(xRy)$, consisting of all the x's which have the relation R to y, is called the class of referents of y with respect to x; the class $\hat{y}(xRy)$, consisting of all the y's to which x has the relation R, is called the class of relata of x with respect to R. These two classes are denoted respectively by $\overrightarrow{R}\text{‘}y$ and $\overleftarrow{R}\text{‘}x$. Thus R

$$\overrightarrow{R}\text{‘}y = \hat{x}(xRy) \quad \text{Df},$$

$$\overleftarrow{R}\text{‘}x = \hat{y}(yRx) \quad \text{Df}.$$

The arrow runs towards y in the first case, to show that we are concerned with things having the relation R *to* y; it runs away from x in the second case to show that the relation R goes *from* x to the members of $\overleftarrow{R}\text{‘}x$. It runs in fact *from* a referent and *towards* a relatum.

The notations $\overrightarrow{R}\text{‘}y$, $\overleftarrow{R}\text{‘}x$ are very important, and are used constantly. If R is the relation of parent to child, $\overrightarrow{R}\text{‘}y =$ the parents of y, $\overleftarrow{R}\text{‘}x =$ the children of x. We have

$$\vdash : x \in \overrightarrow{R}\text{‘}y \,.\equiv .\, xRy$$

and

$$\vdash : y \in \overleftarrow{R}\text{‘}x \,.\equiv .\, xRy.$$

These equivalences are often embodied in common language. For example, we say indiscriminately "x is an inhabitant of London" or "x inhabits London." If we put "R" for "inhabits," "x inhabits London" is "$x R$ London," while "x is an inhabitant of London" is "$x \in \overrightarrow{R}\text{‘}$ London."

* This relation is not strictly asymmetrical, but is so except when the wife's brother is also the sister's husband. In the Greek Church the relation is strictly asymmetrical.

Illustration 3. Scan of Volume 1, p. 34, with a Lambda dot restoration and an *R* for *x* correction over a scribble in pencil.

IV. CORRECTIONS TO VOLUME I, P. 34 AND *25, IN THE PROOFS

A seemingly trivial typographical correction relating to the empty relation $\dot{\Lambda}$ reflects an important issue about relations and the theory of types, and leads to the discovery of places where the first edition of Volume 1 of *PM* was corrected by hand! On page 34 of Volume 1 (see *Illus. 3*), a capital Lambda, Λ, is corrected to a dotted Lambda, $\dot{\Lambda}$. The list of errata says for page 218, "[owing to brittleness of the type, the same error is liable to occur elsewhere]."[22] It is interesting that this remark does not come as an erratum for the Introduction, at page 34.

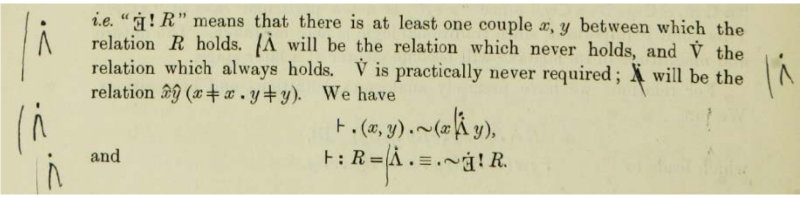
Whitehead noticed an earlier $\dot{\Lambda}$ discrepancy on page 30 of the Introduction at the same time he noticed the discrepancy in *25:

> In sheet 16 *as printed off* there are some very annoying misprints of Λ for $\dot{\Lambda}$, especially in the *Definition*, which are *not in the 2nd* proof. I have written to the Press to ask about it. Also on p 30 of the Introduction Λ comes for $\dot{\Lambda}$, but I can't find my proofs of sheets 1 and 2, so am not sure if we or the Press are to blame. Please look it up—this habit of the Press will be disastrous. (RA1 710, 23 Aug. 1910)

> The Press tell me that the loss of the dots (Λ for $\dot{\Lambda}$) on pp 241–243 is due to breakage in printing. They will put them in by hand. I think we must ask them to do so in that number—as it involves the definition and general introduction of $\dot{\Lambda}$. (RA1 710, 26 Aug. [1910])

This printing fault shows that, as might be expected, the authors did not routinely subject the printed-off sheets to a close re-examination. Whitehead discovered the fault only because it recurred much later, at this point overlooking the one on page 218. The missing dots were drawn in on pages 30 and 34 (*cf.* the 2nd ed., pp. 29 and 32). Someone (perhaps Russell) first dotted the Lambdas, and then Russell (definitely) made marginal insertions to restore them, as we see below:

[22] There are dotted Lambdas on at least three dozen pages in Volume 1. Whether there are Lambdas that permanently lost their dots remains undetermined. The known Lambdas could be compared with their appearance in *PM*'s second edition; but in the absence of a digitally searchable edition, there could be no assurance that one had inspected them all.



i.e. "$\dot{\exists}\,!\,R$" means that there is at least one couple x, y between which the relation R holds. $\dot{\Lambda}$ will be the relation which never holds, and $\dot{V}$ the relation which always holds. $\dot{V}$ is practically never required; $\dot{\Lambda}$ will be the relation $\hat{x}\hat{y}\,(x \neq x \,.\, y \neq y)$. We have

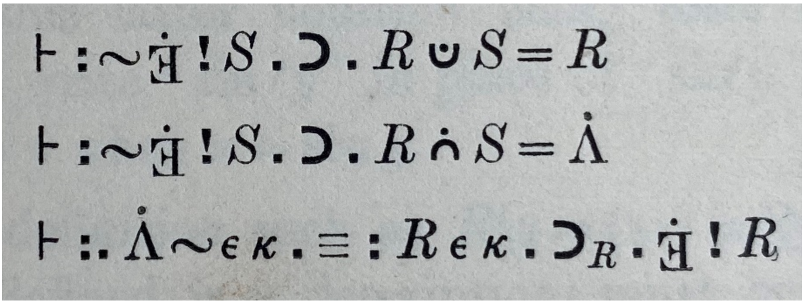
$$\vdash .\, (x, y) \,.\, \sim(x \,\dot{\Lambda}\, y),$$

and

$$\vdash : R = \dot{\Lambda} \,.\, \equiv .\, \sim \dot{\exists}\,!\,R.$$

Illustration 4. From Volume I, p. 30, of the *PM* proofs.

It is quite possible that dots broke off different Lambdas at different times in the printing of certain sheets of *PM*. Perhaps, therefore, no two copies of Volume I of *PM* are identical. The inking of the restored dots is sometimes greyish, and the dots are often off-centre in the first edition. *Cf.* under magnification the dot above the Λ at I: 243: 2 up. In the RA Supporting Library copy, the dot is off-centre to the right, whereas in the line below it the dot is not off-centre. In Russell's copy (*Illus.* 5 below), both dots are misshapen, not a perfect circle like the dot above the ∃ on each line.



$$\vdash : \sim \dot{\exists}\,!\,S \,.\, \supset .\, R \,\dot{\cup}\, S = R$$

$$\vdash : \sim \dot{\exists}\,!\,S \,.\, \supset .\, R \,\dot{\cap}\, S = \dot{\Lambda}$$

$$\vdash :.\, \dot{\Lambda} \sim\in \kappa \,.\, \equiv : R \in \kappa \,.\, \supset_R .\, \dot{\exists}\,!\,R$$

Illustration 5. Last three lines, *25·62·63 of Volume I, p. 243, showing two hand-drawn Lambda dots (first edition, Russell's library copy).

Whitehead and Russell were acutely aware of the significance of the dot over the Lambda.[23] It is easy for one to think that because classes and relations are extensional, the empty class will be the same as the

[23] Thus we regard the drawn-in dots as restorations, rather than authors' corrections. On some proof pages the dot appears to be present but is nevertheless marked for restoration (*Illus. 3*), while on others Russell seems first to have drawn in the missing dot and then made a marginal insertion (*Illus. 4*).

empty relation. However, the dot over the Lambda marks the difference between the empty class and the empty relation, between the class that has no members and the empty relation that holds between no pairs of things. These belong to two distinct logical types, as monadic and two-place propositional functions belong to different types. The warning in the Errata about brittle type was inserted after proofs of the Introduction had been passed by the authors, the respective sheets printed off in their 750 copies, and the dots hand-supplied on the unbound sheets. Following Whitehead's complaint, stronger dotted Lambdas must have been ordered from the type foundry, or some other adjustment made. No Lambdas were marked for restoration after page 243 of Volume I, including Volume II.

Norbert Wiener's famous paper "A Simplification of the Logic of Relations"[24] proposes to represent relations as classes of ordered pairs, those being defined as classes of a certain complex form, $<x, y> = \{\{\{x\}, \Lambda\}, \{\{y\}\}\}$, in which Λ serves as a tag, as in Russell's new definition of $+$, in order to introduce an asymmetry that distinguishes the first member of the pair from the second. Ordered pairs, and consequently relations, are all of the same types as monadic functions of functions, etc. One consequence that Wiener notes at the end of the paper is that with his definition there is no distinction between the empty class and an empty relation, and so $\Lambda = \dot{\Lambda}$.

Wiener, then, was aware of the use of Λ as a technical device to tag one element of a pair, and to the difference between Λ and $\dot{\Lambda}$. This knowledge may have come from some explicit reference to these issues by Russell.

V. THE DZIEWICKI CORRESPONDENCE WITH RUSSELL

Michael Henry Dziewicki (1851–1928) was born in England to a Polish immigrant father and English Quaker mother, studied at Jesuit colleges in France, and moved to Poland in 1880, where he was known as "Michał Henryk", and married. Dziewicki was associated with the Jagiellonian University as an English instructor, although he wrote on and edited the medieval logician John Wyclif (or Wycliffe) and translated Polish fiction into English. Dziewicki was known for a distinctive and successful approach to teaching English to Poles and in a letter

[24] See WIENER (1914).

says that he had tutored Leon Chwistek in English.[25] Twenty-four letters from Dziewicki to Russell between 1913 and 1923 are preserved in the Bertrand Russell Archives, and are the basis of our record of him.[26] On 10 May 1915 Russell wrote to Ludwig Wittgenstein,[27] who then was serving on a gunboat on the Vistula river near Kraków,[28] and suggested that Wittgenstein visit him. Russell describes Dziewicki as "a lonely old logician" who has studied *Principia Mathematica*. Postcards that Dziewicki sent to Wittgenstein describe visits in June of 1915. In a letter after the war (20 Sept. 1919) Dziewicki described Wittgenstein as "a most genial young man", and (14 Feb. 1923) that they discussed philosophical issues related to time and space and the nature of belief during their visits.[29]

A study of this correspondence is not strictly inconsistent with the hypothesis that the volumes acquired by the Russell Archives are the proofs that Russell sent to Dziewicki in 1913, but there are difficulties in accepting the hypothesis. The correspondence also suggests that there was a circle of logicians and philosophers in Kraków who were influenced by Russell's works.

21 February 1913. Dziewicki reports to Russell that:

> I had got as far as "Implication and Formal Implication" [*PoM*, Chap. II] in your great work, when the *Problems of Philosophy*, which I had sent for, arrived. I at once interrupted my work and set to read the smaller book, which I have just finished.

In a later letter (1 Feb. 1922), Dziewicki, who is usually precise, states:

[25] See Bremer, "Michał Dziewicki" (2016); Bremer tells us Dziewicki was childless (p. 211). Dziewicki, "The Standpoint and First Conclusions of Scholastic Philosophy" (1889–90). Dziewicki to Russell, 19 Dec. 1922, RA3 Rec. Acq. 1,027.

[26] Several of the letters are worn and soiled, as if Russell had carried them about on his person. Few have his "Ans" notation in a corner, indicating that he had answered a letter; it is evident from Dziewicki's letters that Russell replied more often than he indicated. He had some of them typed and corrected the typing, but did not add notes.

[27] *Wittgenstein in Cambridge* (2012), p. 81.

[28] Kraków (or Cracow) was then part of Austria. In his first letter after the war, Dziewicki relocates Kraków in Poland.

[29] Brian McGuinness (*Wittgenstein in Cambridge*, p. 81), reports on the postcards, but reads Dziewicki's letter of 20 September 1919 differently and assumes, incorrectly, that an insertion about Wittgenstein expecting to be killed in the war is in Russell's hand rather than Dziewicki's. The ink is merely different.

"It is now ten years or more since I wrote to you for considerable help in my studies of mathematical logic." That assistance included sending Dziewicki a personal copy of *The Principles of Mathematics* (1903). At this point in 1913 there was clearly an established correspondence between Dziewicki and Russell, although he makes no mention of *PM*.

19 March 1913. Dziewicki points out a remark in the *Principles of Mathematics*, page 25, that "Between any two terms there is a relation not holding between any other two terms." But, he says, Russell adds: "This principle … is incapable of proof." Dziewicki constructs an artificial relation between arbitrary classes *A* and *B* involving those things that are neither *A* nor *B*, which in fact satisfies Russell's description. The letter concludes with a request for a book with a system of symbolic logic that "would correspond to an elementary algebra". He notes that he has studied Jevons' logic,[30] but does not find it sufficient.

26 April 1913. Dziewicki thanks Russell (again) for his "article on Mathematical Logic", and complains that he "cannot make out the use of the *dots*" as he cannot access the work of Peano in which this notation originated.[31]

24 May 1913. The letter begins with a very low-key way of referring to a gift of two bound volumes of proofs, if that indeed was what Dziewicki received:

> I have once more and most heartily, to thank you for your extreme kindness. I have received both your letter and the proofs; and though I am heavily handicapped by my ignorance of mathematics, I hope that "Labor omnia vincit improbis" will prove true in my case.
>
> What I want to find out is, whether your system of Symbolic Logic can be adequate (or made adequate) for the purposes of Metaphysics. And for that reason, I must first learn to use the Symbols and reason by their means in a continued series of arguments.
>
> What you note in your letter as to pure mathematics and real actual space, I had already gathered from your *Principles of Mathematics*, but

30 STANLEY JEVONS, *The Principles of Science: a Treatise on Logic and Scientific Method* (1877).

31 RUSSELL, "Mathematical Logic as Based on the Theory of Types" (1908); in *Papers* 5. The dot notation does not come into use until section VI of this paper. In note 21 there, Russell refers to Peano and two works by Whitehead for the usage of dots.

> was not quite sure whether you would admit the (to me) evident conclusion, that there may be conceptions or series of conceptions, which though absolutely immaterial—i.e. not apprehended by sense—may form "spaces" of *n* dimensions....
>
> You will be (perhaps interested to learn that you are a good deal discussed here in the University, both by pupils and professors, and much appreciated by mathematical specialists in the Academy of Sciences here.

He suggests they meet in the south of France in the summer. There is no confirmation that they did so.

18 June 1913. Dziewicki writes a five-page letter to Russell discussing bound (real) variables and the theory of types, and alludes to a remark on "p. 43 of *Principia Mathematica*", which is in the Introduction.

The letters so far suggest that Dziewicki impressed Russell as a serious student of the *Principles of Mathematics*, then read "Mathematical Logic as Based on the Theory of Types" (1908), and, it seems, received proofs of *Principia Mathematica* from Russell. Wittgenstein's report that Dziewicki discussed the philosophy of space in their meeting in 1915 also seems to fit.

The correspondence continued with letters to Russell (dated 5 September 1913, 27 and 28 October 1913, and 24 February 1914), the last when Russell was about leave for Harvard University. In these letters Dziewicki discusses the theory of descriptions from *14 of *PM*, suggesting that his reading is proceeding.

[*June 1914*]. A puzzling remark appears in the next letter:

> It is almost a year since you kindly sent me proofs of *Principia Mathematica*; and until the present time, I have been constantly working at them, so far as my occupations and capacity for strenuous mental work allowed me. I had great difficulties at first; the proofs,[32] not being a complete set, gave me only an imperfect idea of the system of dots, indispensable for the understanding of your notation. Luckily, I managed in November to get a copy of the book itself; and at present I may say that, so far as mere understanding goes, I have mastered the formal part of the work as far as *38.[33] Perhaps you will not smile when I add that I am rather proud of the feat.

[32] In discussing *PM*'s proof sheets in 2005, we referred to this passage (Linsky and Blackwell, p. 150 n. 11).

[33] I.e., to page 311 of Volume 1 of the first edition.

There follows a long, eighteen-page discussion of descriptions and classes, accurately presented in *PM* notation, and so confirming the claim to have studied up to ∗38, which comes close to being the whole of Part I on "Mathematical Logic", with Part II called "Prolegomena to Cardinal Arithmetic".

This letter suggests that the "proofs" that Russell sent were "incomplete", so much so that the system of dots is not made clear by what Dziewicki received. This is confusing. The system of dots is explained in the Introduction, pages 9–11, and so much earlier than passages clearly included in the material he read before November 1913. Possibly he was unable to *apply* the dot principles. Russell and Whitehead say in concluding their account of the dot system:

> Other uses of dots follow the same principles, and will be explained as they are introduced. In reading a proposition, the dots should be noticed first, as they show its structure. In a proposition containing several signs of implication or equivalence, the one with the greatest number of dots before or after it is the *principal* one: everything that goes before this one is stated by the proposition to imply or be equivalent to everything that comes after it. (*PM*, 2nd ed., I: 10–11)

This explanation is in the sixteen pages of sheet 1 of *PM*'s first volume, so possibly it was missing from the proofs Russell sent. (It is an odd sheet to be missing from his gift.) Dziewicki then reports that "in November", presumably November 1913, he was able to "get a copy of the book itself" and then proceeded to correspond with Russell about issues in Volume I of *PM*.[34] Calling the three-volume set of *PM* "the book itself", if that's what he was referring to, is a little odd; and he never explicitly mentions Volumes II and III and yet tells Russell in 1922 that he owns Volume I. The June 1914 letter is thus doubly confusing about the proofs, and perhaps the evidence it provides should be discounted for that. The formulas in his letter show that he became fluent in the dot notation.

19 December 1922. After the war the correspondence continued, with Dziewicki frequently expressing his debt to Russell. He lists the books

[34] In his letter of 11 April 1920 Dziewicki notes that he has studied "only" the first 330 pages of Volume I, i.e. down to and including ∗40, Products and Sums of Classes. His rate of progress since 1914 obviously was very slow, although he credits *PM* for taking his mind off the "slaughter and famine" of the war (1 Feb. 1922).

by Russell in his library and makes special mention of the copy of the *Principles* Russell sent him. There is no mention of the two bound volumes of *PM* proofs:

> I do not know the *Tractatus Logico-Philosophicus* of Wittgenstein, nor is it possible to get it here: but I should be thankful if you would send it me, as you offered.[35] At present I have the following volumes of your works: *Problems of Philosophy*; *Philosophical Essays*; *Leibnitz*; *The External World*; *Principles of Mathematics* (which you sent me, intimating you might perhaps have to ask for it again); and Vol. 1., *Principia Mathematica*. These works, if I were not busy giving lessons, might be a library for me to study till the end of my life.

What, then, is the full provenance of the two volumes of proofs acquired by the Russell Archives? It is clear that Russell sent Dziewicki proofs of part of *PM* in 1913. The antiquarian bookdealer from whom McMaster University Library purchased the books hypothesized that they had been in the possession of the "Dziewicki family" in Australia, specifically a son, and then found their way via the collection of a Sydney "professor of mathematics" to an antiquarian dealer in Sydney.[36] But Michael Dziewicki was childless.[37] It is our hypothesis that these are *not* the proofs that Russell undoubtedly sent Dziewicki in May of 1913. The bound proofs have all that Russell and Whitehead have to say in explaining the dot system. They cannot be the "copy of the book itself" that Dziewicki was able to obtain in November 1913, which was probably the copy of Volume 1 that he reported owning in 1922. No other correspondence in the Russell Archives, as far as anyone is aware, mentions a gift of proofs of the first edition of *PM*. The bound volumes closely resemble the published first edition, and previous owners may not have realized what they had. Perhaps we should simply say that we refer to these as "the Dziewicki proofs" to honour the relationship between Russell and Dziewicki, without claiming that they are the proofs sent by Russell in May 1913.[38]

35 Russell did send it.

36 Email to Blackwell from Christian Westergaard, Sophia Rare Books, 2 Oct. 2018. His catalogue entry for *PM* (which calls the bound page proofs "galley proofs") may be consulted at sophiararebooks.com/sep2018.pdf, pp. 216–21.

37 Bremer, pp. 209–10.

38 *Acknowledgements*: We wish to acknowledge helpful comments and conversations with Nick Griffin, Gregory Landini, the Ready Division preservationist Audrie

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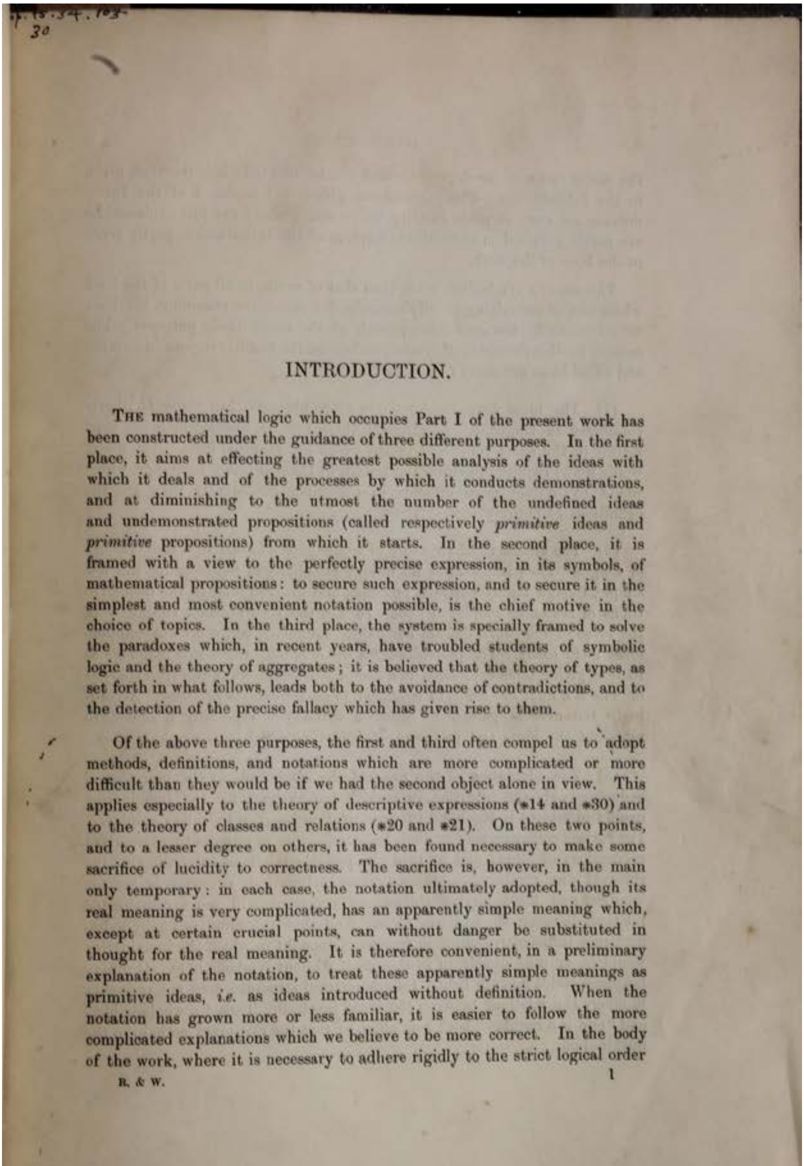
Schell, and the Division's Digital Archives Librarian Bridget Whittle. Thanks go also to McMaster Library for helping to acquire the bound proof volumes of *Principia Mathematica*. Christian Westergaard, proprietor of Sophia Rare Books, København, Denmark, disclosed for us what he could about his discovery of the volumes in a bookstore in Sydney, Australia. F. Michael Walsh and John G. Slater alerted Blackwell to their availability.

APPENDIX
CORRECTIONS AND MARKINGS IN THE PROOF VOLUMES

Images of the corrections and other markings can be viewed in the digitized proof volumes at digitalarchive.mcmaster.ca/islandora/object/macrepo%3A90174. Because of the nature of the markings, non-literal descriptions are often resorted to in the second column.

PAGE	CORRECTION OR MARKING	EDITORIAL COMMENT	IN ERRATA? CORRECTED IN 1ST EDITION?
Vol. 1: p. 1: top left corner of sheet 1 when folded (*Illus. 6*). This page is discoloured or soiled, presumably from being topmost when the sheet was folded.	pp. ~~15~~ . 34 . ~~103~~ 30	Both strike-throughs refer to pages that were not corrected but appear in the Errata. Corrections to the other 2 pp. were made in the 1st ed. Since p. 103 is in sheet 5, Russell seems to have had a stack of at least sheets 1–5.	
1: 15: 2 lines up	Delete "of" before "$\phi\hat{x}$".	Not changed.	Yes. Corrected: no.
1: 17: top left corner of sheet 2 when folded	p. 30	The note is on the 1st p. of sheet 2.	
1: 30: 1–2, 4–5 lines up	Four Lambda restorations of "Λ" to "$\dot{\Lambda}$" (*Illus. 4*).	See sec. IV. Note the Errata warning re p. 218: 2 lines up: "[owing to brittleness of the type, the same error is liable to occur elsewhere]."	No. Corrected: yes.



p. 15. 54. 103.
30

INTRODUCTION.

THE mathematical logic which occupies Part I of the present work has been constructed under the guidance of three different purposes. In the first place, it aims at effecting the greatest possible analysis of the ideas with which it deals and of the processes by which it conducts demonstrations, and at diminishing to the utmost the number of the undefined ideas and undemonstrated propositions (called respectively *primitive* ideas and *primitive* propositions) from which it starts. In the second place, it is framed with a view to the perfectly precise expression, in its symbols, of mathematical propositions: to secure such expression, and to secure it in the simplest and most convenient notation possible, is the chief motive in the choice of topics. In the third place, the system is specially framed to solve the paradoxes which, in recent years, have troubled students of symbolic logic and the theory of aggregates; it is believed that the theory of types, as set forth in what follows, leads both to the avoidance of contradictions, and to the detection of the precise fallacy which has given rise to them.

Of the above three purposes, the first and third often compel us to adopt methods, definitions, and notations which are more complicated or more difficult than they would be if we had the second object alone in view. This applies especially to the theory of descriptive expressions (*14 and *30) and to the theory of classes and relations (*20 and *21). On these two points, and to a lesser degree on others, it has been found necessary to make some sacrifice of lucidity to correctness. The sacrifice is, however, in the main only temporary: in each case, the notation ultimately adopted, though its real meaning is very complicated, has an apparently simple meaning which, except at certain crucial points, can without danger be substituted in thought for the real meaning. It is therefore convenient, in a preliminary explanation of the notation, to treat these apparently simple meanings as primitive ideas, *i.e.* as ideas introduced without definition. When the notation has grown more or less familiar, it is easier to follow the more complicated explanations which we believe to be more correct. In the body of the work, where it is necessary to adhere rigidly to the strict logical order

R. & W. 1

Illustration 6. Scan of Volume 1, p. 1, with the first sheet signed "1" at the lower right. Noted at the top left are pages with corrections.

PAGE	CORRECTION OR MARKING	EDITORIAL COMMENT	IN ERRATA? CORRECTED IN 1ST EDITION?
I: 33 top left corner of sheet 3 when folded	p. 34	The note is on the 1st p. of sheet 3.	
I: 34: line 3 (*Illus. 3*).	A Lambda restoration of "Λ" to "$\dot{\Lambda}$".	See sec. IV.	No. Corrected: yes.
I: 34: line 15	"*x*" is struck through and "R" inserted in ink in margin over pencil scribbling.	The pencil scribbling's purpose was, perhaps, to mark the passage for consideration later.	Yes. Corrected: no. In BR's copy of the 1st ed., "*x*" is vertically struck through with "R" inserted in the right margin. Yes, in the 2nd ed., p. 33.
I: 97 top right corner of sheet 7 when folded	p. 103 p. 84 p. 487	There is nothing noted on p. 84, and collation with the 1st ed. found nothing.	
I: 103: line 7	er	In "assumption" "ump" is struck vertically through; "er" is in margin to make the word "assertion".	Yes. Corrected: no. 2nd ed., p. 99, yes.
I: 103: line 25	"*r*" in left margin to replace final "*q*".		Yes. Corrected: no. 2nd ed., p. 99, yes.
I: 241: top left corner of sheet 16 when folded	pp. 241, 242	See sec. IV. None of the 24 Lambda dots on p. 242 seem drawn in, but some are on p. 243.	

PAGE	CORRECTION OR MARKING	EDITORIAL COMMENT	IN ERRATA? CORRECTED IN 1ST EDITION?
I: 241: 3, 10 lines up	Two Lambda restorations of "Λ" to "Λ̇".		No. Corrected: yes.
I: 243: 1, 2, 14 lines up	Three Lambda restorations of "Λ" to "Λ̇".	See sec. IV and *Illus. 4*.	No. Corrected: yes.
I: 304: line 19, right margin of last page of sheet 19 when folded	Long arrow "→" pointing slightly up to right edge.	304 is the last p. of sheet 19. There are no handwritten marks on the "next" page over on this sheet (p. 289) or the "next" p. of the next folded sheet, i.e. p. 305 of sheet 20.	
I: 371: 5 lines up	& ·641	In pencil. No caret but the correction must be meant for insertion in "(*83·731)".	No. Corrected: no; nor in 2nd ed.
I: 487: line 13	In "*95", "4" to replace "5".		Yes.
I: 497: top right corner of sheet 32 when folded	p. 503	In pencil. P. 503 is on the same sheet as p. 497.	
I: 503: line 14	In "88·38", the final "8" is deleted and "6" written in left margin.	In pencil.	Yes. Corrected: no.
Vol. II: p. 1: top right corner of sheet 1 when folded	p. 101	In pencil. See 101: 12 lines up. Since p. 101 is on sheet 7, Russell seems to have had a stack of at least sheets 1–7 of Vol. II.	

PAGE	CORRECTION OR MARKING	EDITORIAL COMMENT	IN ERRATA? CORRECTED IN 1ST EDITION?
II: 66: 2 lines up (*Illus. 2*).	Re two subscripts to the intersection: "A_β" is to be replaced by "$(\Lambda \cap \beta)$" and "Λ_α" by "$(\Lambda \cap \alpha)$".	Inserted in 1st ed.: "We note that $\Lambda \cap \alpha$ and $\Lambda \cap \beta$ indicate respectively the Λ's of the same types as α and β, and accordingly" [here the original wording "We take …" continues but without the concluding asterisk].	No. Corrected: yes.
II: 66: note	Footnote deleted.	The footnote was: "Here Λ_α and Λ_β have the meaning defined in $*65{\cdot}91$, *i.e.* $\Lambda_\alpha = \Lambda \cap t\text{‘}\alpha$."	No. Corrected: yes.
II: 67: line 3	Re two subscripts to the conjunction.	See 66: 2 lines up for the replacements.	No. Corrected: yes.
II: 75: line 5	Re two subscripts to the intersection: "β" on left should read: "$(\Lambda \cap \beta)$"; "Λ_α" on right should read "$(\Lambda \cap \alpha)$".	Only "β" is visible on the scan because the binding is too tight. (See corrections to II: 66.) In the 1st ed. 2 lines are brought back from p. 76, and a new sentence is added at the foot of 76: "These definitions are extended by IIT of the prefatory statement." (See II: xxv for IIT.)	No. Corrected: yes.

PAGE	CORRECTION OR MARKING	EDITORIAL COMMENT	IN ERRATA? CORRECTED IN 1ST EDITION?
II: 76: 3 lines up	Re two subscripts to the conjunction, followed by "[& so throughout]".	See II: 66. Trimming of the fore-edge removed the upstroke on the handwritten "[". This may be a note to himself—with the fair copy of the corrected proofs perhaps marking each instance of the subscripts to be changed.	No. Corrected: yes.
II: 77: 3–		The 1st ed. is revised on several lines in accordance with changes at 75: line 5 and 76: 3 lines up. The lines affected are lengthened, and most are no longer indented or quite as much. P. 86, 3–4 lines up, of the next sheet of the 1st ed. beginning "[*73·1]", escaped correction. These may not be errors as the subscripts are still defined, and so the results may be provable as asserted.	No. Corrected: yes.
II: 101: 12 lines up	":" is to be replaced by "."	Before "∃!", BR deleted the top dot of the colon.	Yes. Corrected: no.
II: 616: 11 lines up	Subscript "P" to "limin" is to be subscript "Q".		No. Corrected: no.